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Enhanced backscattering from a metallic cylinder with a random rough surface

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The scattering of a TE-polarized electromagnetic wave from a metallic cylinder with a slightly random rough surface is studied by means of the stochastic functional approach. The incoherent scattering distribution can be calculated from the Wiener coefficients up to the second order and demonstrates the enhanced backscattering peak, which is attributed to the participation of the surface plasmon waves supported by the cylindrical metal surface as the intermediate scattering processes in multiple scattering.

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I. INTRODUCTION

The scattering of waves from a random rough surface is a very common physical phenomenon; for instance, the scattering of radio waves from irregular ground, the sea surface, or irregular-shaped objects; radar clutter echo; diffuse scattering of light by a rough plane; excitation of a surface plasmon by a random metallic surface; random scattering in optical waveguides; etc. These are not only important in practical topics but also interesting theoretical problems, and a number of works have been published on related theories using various techniques.

The phenomenon of enhanced backscattering, manifested as a well-pronounced peak in the antispecular direction in the angular distribution of waves incoherently scattered by random rough surfaces, has attracted a great deal of attention in recent years.[1–6] The backscattering enhancement associated with the excitation of surface plasmon polaritons was first studied theoretically by various approaches[7–13] and lately has been demonstrated by the experimental observations.[14–17] The delay is due largely to the difficulties encountered in the fabrication of rough surfaces producing adequate polariton coupling. In a recent experimental study[18] on light scattering in periodic β-aligned carbon nanotubes, which have potential for a wide range of applications, such as optical switching[19] and power limiting,[20] the enhanced backscattering effect has also been observed and was explained using a mechanism of electromagnetically induced excitation of surface plasmon polaritons. The existence of plasmon polaritons in the systems of carbon nanotubes has been reported.[20,21]

It has been believed that the mechanism responsible for the enhanced backscattering is the coherent interference in the multiple-scattered waves, and for slightly rough surfaces this has been attributed to the participation of unstable or stable surface electromagnetic (EM) waves as the intermediate scattering processes. Most of the studies, however, have been concentrated on random rough surfaces in a planar geometry. A few works[22–28] have treated the problems of scattering from cylindrical rough surfaces, but no enhanced backscattering peak has been demonstrated.

In this paper, the scattering of an electromagnetic wave from a metallic cylinder with a slightly random rough surface is treated by means of the stochastic functional approach,[24–27] and the effect of surface plasmon waves supported by the cylindrical metal surface on the enhanced backscattering is studied. In the stochastic functional approach, the cylindrical random surface is assumed to be a homogeneous Gaussian random field, homogeneous with respect to the group of motions on the cylinder—i.e., translations along the axis and rotations around the axis. The random scattered-wave field is regarded as a nonlinear stochastic functional of the cylindrical random surface and can be represented as a Wiener-Ito expansion in terms of the Wiener-Hermite differentials of the cylindrical wave functions. The expansion coefficients up to the second order are determined by the approximate boundary conditions for small roughness. Various statistical characteristics of the scattered wave are obtained from the stochastic-wave field by making use of the orthogonality of the Wiener-Hermite differentials. Some numerical calculations are shown for the angular distribution of incoherent scattering, in which the enhanced backscattering peak can be clearly observed.

II. SCATTERING FROM A CYLINDRICAL SMOOTH SURFACE

We first consider the scattering of an EM plane wave from a metallic smooth cylinder. An incident EM plane wave whose electric field has a horizontal polarization (TE case) is expanded in terms of the cylindrical TE waves. The scattered fields are determined by making use of the boundary conditions on the surface of the cylinder.

A. Expansion of an incident plane wave in terms of cylindrical wave

We consider the incident EM plane wave with the incident wave vector $\vec{k}_2=(\vec{k}_{t2}(\beta),\varphi_i,\beta)$, where φ_i denotes the incident angle in the x-y plane with respect to the x axis and $\beta=k_2\cos\theta_i$, $|\vec{k}_{t2}(\beta)|=\gamma_0=\sqrt{k_2^2-\beta^2}=k_2\sin\theta_i$, θ_i being the incident angle with respect to the z axis. Assume that the electric field has horizontal polarization (TE case), then we have the following expansion in terms of the cylindrical TE waves for the incident electric and magnetic field:

$$\vec{E}_{in}=\sum_{m=-\infty}^{+\infty} i^m e^{im(\varphi-\varphi_i)+i\beta z}\left[\frac{m}{\gamma_0 r}J_m(\gamma_0 r)\vec{e}_r+iJ'_m(\gamma_0 r)\vec{e}_\varphi\right], \tag{1}$$

$$\vec{H}_{in}=\sum_{m=-\infty}^{+\infty}\left(-\frac{i^m}{Z_0}\right)e^{im(\varphi-\varphi_i)+i\beta z}\left[\frac{i\beta}{k_2}J_m'(\gamma_0 r)\vec{e}_r-\frac{\beta}{k_2}\frac{m}{\gamma_0 r}J_m(\gamma_0 r)\vec{e}_\varphi+\frac{\gamma_0}{k_2}J_m(\gamma_0 r)\vec{e}_z\right], \quad (2)$$

where k_2 is the wave number and Z_0 the wave impedance in the space outside the cylinder.

B. Primary scattered fields by a smooth cylinder

Even though the incident electric field is TE polarized, the scattered electric field is no longer TE polarized. Both the longitudinal z components of the electric and magnetic fields exist for the scattered wave. Since the transverse components of the fields can be expressed in terms of the longitudinal components of the fields as

$$\vec{E}_t=\frac{[i\beta\nabla_t E_z-i\omega\mu\vec{e}_z\times\nabla_t H_z]}{|\vec{k}_t(\beta)|^2}, \quad (3)$$

$$\vec{H}_t=\frac{[i\beta\nabla_t H_z-i\omega\varepsilon\vec{e}_z\times\nabla_t E_z]}{|\vec{k}_t(\beta)|^2}, \quad (4)$$

where

$$\nabla_t=\vec{e}_r\frac{\partial}{\partial r}+\vec{e}_\varphi\frac{1}{r}\frac{\partial}{\partial r}=\vec{e}_r\frac{\partial}{\partial r}+\vec{e}_\varphi\frac{im}{r} \quad (5)$$

denotes the transverse operator in the cylindrical coordinates, merely the longitudinal components E_z and H_z have to be determined. Expanding the longitudinal components E_z^s and H_z^s of the scattered fields in terms of the cylindrical waves,

$$E_z^s(H_z^s)=\begin{cases}\displaystyle\sum_{m=-\infty}^{+\infty}A_m^0(C_m^0)\left[\frac{J_m(\varsigma_0 r)}{J_m(\varsigma_0 a)}\right]e^{im\varphi+i\beta z} & (r<a),\\ \displaystyle\sum_{m=-\infty}^{+\infty}B_m^0(D_m^0)\left[\frac{H_m(\gamma_0 r)}{H_m(\gamma_0 a)}\right]e^{im\varphi+i\beta z} & (r>a),\end{cases} \quad (6)$$

and the continuous conditions of E_z, H_z, E_φ, and H_φ on the boundary $r=a$ where a is the radius of the cylinder, then we obtain the expansion coefficients A_m^0, B_m^0, C_m^0, and D_m^0 of the mth cylindrical wave as

$$A_m^0=B_m^0=\frac{i\eta_0 m\beta}{a}\left(\frac{1}{\gamma_0^2}-\frac{1}{\varsigma_0^2}\right)\frac{[\Lambda_m(\gamma_0 a)-\psi_m(\gamma_0 a)]}{\Delta_m(\beta)}, \quad (7)$$

$$C_m^0=D_m^0-\left(\frac{\gamma_0}{k_2}\right)J_m(\gamma_0 a)=\frac{[\Lambda_m(\gamma_0 a)-\psi_m(\gamma_0 a)]\left[\frac{k_1}{\varsigma_0}\Lambda_m(\varsigma_0 a)-\frac{k_2}{\gamma_0}\psi_m(\gamma_0 a)\right]}{\Delta_m(\beta)}, \quad (8)$$

where $\Lambda_m(x)=J_m'(x)/J_m(x)$, $\psi_m(x)=H_m'(x)/H_m(x)$, and

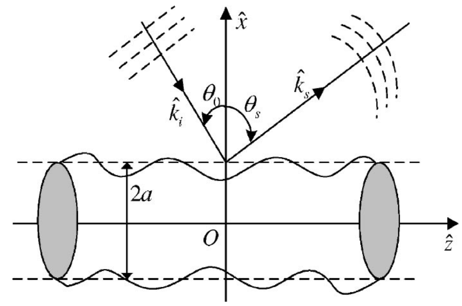

FIG. 1. Geometry of the scattering problem.

$$\Delta_m(\beta)=\left(\frac{m\beta}{a}\right)^2\left[\frac{1}{\gamma_0^2}-\frac{1}{\varsigma_0^2}\right]^2-k_2^2\left[\frac{\Lambda_m(\varsigma_0 a)}{\varsigma_0}-\frac{\psi_m(\gamma_0 a)}{\gamma_0}\right]\times\left[\frac{k_1\Lambda_m(\varsigma_0 a)}{\varsigma_0}-\frac{k_2\psi_m(\gamma_0 a)}{\gamma_0}\right] \quad (9)$$

is the dispersion equation for the surface plasmon modes.

III. SCATTERING FROM A CYLINDRICAL RANDOM ROUGH SURFACE

We now consider the scattering of an incident TE-polarized wave from a metallic cylinder with a slightly random rough surface shown in Fig. 1 using the stochastic functional approach.[24–26] The spectral representation of a cylindrical random rough surface is given at first. The scattered fields are expanded in terms of the cylindrical wave functions and represented as a Wiener-Ito expansion in terms of the Gaussian random measure. The Wiener expansion coefficients up to the second order are obtained by applying the approximate boundary conditions at the interface.

A. Spectral representation of a cylindrical random rough surface

The geometry of the problem is indicated in Fig. 1. For brevity, we have here assumed that the cylindrical rough surface is homogeneous in the φ direction; that is, the corrugation of the rough surface proceeds only along the axis of the cylinder, all cross sections being circular, but of radii viewed as a stochastic process. Let (r,φ,z) denote the cylindrical coordinates, and let the random cylindrical surface with mean radius a be described by

$$r=a+f(z;\omega), \quad \langle f(z;\omega)\rangle=0, \quad (10)$$

where f is a homogeneous random function over the cylindrical surface, ω denotes a sample point in the sample space which is the ensemble of the realizations of f, and $\langle\cdots\rangle$ indicates the probabilistic average over the sample space. Then, as shown in our previous works,[24–26] we have the spectral representation of $f(z;\omega)$ in terms of a Wiener integral:

$$f(z;\omega)=\int_{-\infty}^{+\infty}e^{i\lambda z}F(\lambda)dB(\lambda), \quad (11)$$

where we have put $dB(\lambda)\equiv dB(\lambda;\omega)$ and will omit ω for brevity in what follows. Here $dB(\lambda)$ denotes the complex

Gaussian random measure. The correlation function of the cylindrical random rough surface is then obtained by

$$R(z) = \langle f(z+z';\omega) f(z';\omega)\rangle = \int_{-\infty}^{+\infty} e^{i\lambda z}|F(\lambda)|^2 d\lambda, \quad (12)$$

and the variance that describes the random surface roughness

$$\sigma^2 = R(0) = \int_{-\infty}^{+\infty} |F(\lambda)|^2 d\lambda, \quad (13)$$

where we have used the relation $F(\lambda)=F^*(-\lambda)$. Here $|F(\lambda)|^2$ is called the power spectrum of the random cylindrical surface. $|F(\lambda)|^2=0$ and then $\sigma^2=0$ corresponds to an ideal smooth boundary.

B. Wiener-Ito expansion of the scattered fields

The scattered electromagnetic fields can be expanded in terms of the cylindrical wave functions, and at the same time the fields are nonlinear functionals of the Gaussian random rough surface. Since a nonlinear stochastic functional can be represented as a Wiener-Ito expansion in terms of the Gaussian random measure $dB(\lambda)$, then we have the following expansion for the z components of the electric and magnetic fields inside the metallic cylinder,

$$\begin{Bmatrix} E_{z1} \\ H_{z1} \end{Bmatrix} = \sum_{m=-\infty}^{+\infty}\sum_{n=0}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}\begin{Bmatrix} A_m^n(\beta,\lambda_1,\ldots,\lambda_n) \\ C_m^n(\beta,\lambda_1,\ldots,\lambda_n)\end{Bmatrix} \times J_m(\varsigma_n r)e^{i\eta_n z+im\varphi}\hat{h}_n[dB(\lambda_1),dB(\lambda_2),\ldots,dB(\lambda_n)], \quad (14)$$

and outside the cylinder,

$$\begin{Bmatrix} E_{z2} \\ H_{z2} \end{Bmatrix} = \sum_{m=-\infty}^{+\infty}\sum_{n=0}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}\begin{Bmatrix} B_m^n(\beta,\lambda_1,\ldots,\lambda_n) \\ D_m^n(\beta,\lambda_1,\ldots,\lambda_n)\end{Bmatrix} \times H_m(\gamma_n r)e^{i\eta_n z+im\varphi}\hat{h}_n[dB(\lambda_1),dB(\lambda_2),\ldots,dB(\lambda_n)], \quad (15)$$

where $\hat{h}_n[\cdots]$ denotes the nth-degree complex Wiener-Hermite differential, which is to be understood as a generalization of Hermite polynomial (notice $\hat{h}_0=1$), the integrals in the above equations represent the n-tuple complex Wiener integrals, and the coefficients A_m^n, B_m^n, C_m^n, and D_m^n are the unknown expansion coefficients to be determined by applying the boundary condition on the random rough boundary. The parameter $\eta_n=\beta+\lambda_1+\ldots+\lambda_n$ is the composed axial wave number which originates from the scattering by the random rough boundary. ς_n and γ_n are defined as $\varsigma_n=\sqrt{k_1^2-\eta_n^2}$ and $\gamma_n=\sqrt{k_2^2-\eta_n^2}$ with k_1 and k_2 being the wave numbers inside and outside the cylinder. The transverse components of the scattered fields in terms of the longitudinal z components of the fields can be obtained as given in Eqs. (3)–(5).

C. Approximation solutions for Wiener expansion coefficients

To investigate the scattering characteristics, we have to determine the Wiener expansion coefficients by applying the boundary conditions at the random boundary $r=a+f(z;\omega)$. For simplicity, we here confine ourselves to the case that the random boundary is slightly rough—that is, $\sigma^2 \ll 1$. Then the boundary conditions at the interface can be approximated as

$$\begin{cases} E_\varphi + \dfrac{\partial E_\varphi}{\partial r} f + \dfrac{E_r}{r}\dfrac{\partial f}{\partial \varphi}\Big|_{r=a} & \text{continuous}, \\ E_z + \dfrac{\partial E_z}{\partial r} f + \dfrac{E_r}{r}\dfrac{\partial f}{\partial z}\Big|_{r=a} & \text{continuous}, \end{cases} \quad (16)$$

and the same for the components of the magnetic fields.

Substituting the expressions of the fields into the approximate boundary conditions and making use of the recurrence formula and the orthogonality relation[24–26] for $\hat{h}_n$, we consequently obtain a set of hierarchical equations for the Wiener expansion coefficients and can solve them by neglecting the higher-order kernels. The hierarchical equations for the Wiener expansion coefficients are

$$\begin{Bmatrix} A_m^1(\beta,\lambda_1) \\ C_m^1(\beta,\lambda_1)\end{Bmatrix} - \begin{Bmatrix} B_m^1(\beta,\lambda_1) \\ D_m^1(\beta,\lambda_1)\end{Bmatrix} = \left[\varsigma_0\Lambda_m(\varsigma_0 a)\begin{Bmatrix} A_m^0(\beta) \\ C_m^0(\beta)\end{Bmatrix} - \gamma_0\psi_m(\gamma_0 a)\begin{Bmatrix} B_m^0(\beta) \\ D_m^0(\beta)\end{Bmatrix}\right]F(\lambda_1) + \left[\frac{\beta}{\varsigma_0}\Lambda_m(\varsigma_0 a)\begin{Bmatrix} A_m^0(\beta) \\ C_m^0(\beta)\end{Bmatrix} - \frac{\beta}{\gamma_0}\psi_m(\gamma_0 a) \times \begin{Bmatrix} B_m^0(\beta) \\ D_m^0(\beta)\end{Bmatrix}\right]\lambda_1 F(\lambda_1) + \left[\begin{Bmatrix} \left(\dfrac{i\omega\mu m}{\varsigma_0^2 a}\right)C_m^0(\beta) \\ \left(\dfrac{-i\omega\varepsilon_1 m}{\varsigma_0^2 a}\right)A_m^0(\beta)\end{Bmatrix} - \begin{Bmatrix} \left(\dfrac{i\omega\mu m}{\gamma_0^2 a}\right)D_m^0(\beta) \\ \left(\dfrac{-i\omega\varepsilon_2 m}{\gamma_0^2 a}\right)B_m^0(\beta)\end{Bmatrix}\right]\lambda_1 F(\lambda_1) \quad (17)$$

and

$$\frac{m\eta_1}{\varsigma_1^2 a}\begin{Bmatrix} A_m^1(\beta,\lambda_1)\\ C_m^1(\beta,\lambda_1)\end{Bmatrix} - \frac{m\eta_1}{\gamma_1^2 a}\begin{Bmatrix}B_m^1(\beta,\lambda_1)\\ D_m^1(\beta,\lambda_1)\end{Bmatrix} = \left[\frac{m\beta}{\varsigma_0 a}\left(\frac{1}{\varsigma_0 a}-\Lambda_m(\varsigma_0 a)\right)\begin{Bmatrix}A_m^0(\beta)\\ C_m^0(\beta)\end{Bmatrix}\right]F(\lambda_1) - \left[\frac{m\beta}{\gamma_0 a}\left(\frac{1}{\gamma_0 a}-\psi_m(\gamma_0 a)\right)\begin{Bmatrix}B_m^0(\beta)\\ D_m^0(\beta)\end{Bmatrix}\right]F(\lambda_1) - \frac{J''_m(\varsigma_0 a)}{J_m(\varsigma_0 a)}\begin{Bmatrix} i\omega\mu C_m^0(\beta)\\ -i\omega\varepsilon_1 A_m^0(\beta)\end{Bmatrix}F(\lambda_1) - \frac{H''_m(\gamma_0 a)}{H_m(\gamma_0 a)}\begin{Bmatrix} i\omega\mu D_m^0(\beta)\\ -i\omega\varepsilon_2 B_m^0(\beta)\end{Bmatrix}F(\lambda_1) \qquad (18)$$

for $n=1$ and

$$\begin{Bmatrix} A_m^2(\beta,\lambda_1,\lambda_2)\\ C_m^2(\beta,\lambda_1,\lambda_2)\end{Bmatrix} - \begin{Bmatrix} B_m^1(\beta,\lambda_1,\lambda_2)\\ D_m^2(\beta,\lambda_1,\lambda_2)\end{Bmatrix} = \left[\varsigma_1\Lambda_m(\varsigma_1 a)\begin{Bmatrix} A_m^1(\beta,\lambda_1)\\ C_m^1(\beta,\lambda_1)\end{Bmatrix} - \gamma_1\psi_m(\gamma_1 a)\begin{Bmatrix} B_m^1(\beta,\lambda_1)\\ D_m^1(\beta,\lambda_1)\end{Bmatrix}\right]\frac{F(\lambda_2)}{2} + \left[\frac{\eta_1}{\varsigma_1}\Lambda_m(\varsigma_1 a)\begin{Bmatrix} A_m^1(\beta,\lambda_1)\\ C_m^1(\beta,\lambda_1)\end{Bmatrix} - \frac{\eta_1}{\gamma_1}\psi_m(\gamma_1 a)\begin{Bmatrix} B_m^1(\beta,\lambda_1)\\ D_m^1(\beta,\lambda_1)\end{Bmatrix}\right]\frac{\lambda_2 F(\lambda_2)}{2} + \left[\begin{Bmatrix} \left(\frac{i\omega\mu m}{\varsigma_1^2 a}\right)C_m^1(\beta,\lambda_1)\\ \left(\frac{-i\omega\varepsilon_1 m}{\varsigma_1^2 a}\right)A_m^1(\beta,\lambda_1)\end{Bmatrix} - \begin{Bmatrix} \left(\frac{i\omega\mu m}{\gamma_1^2 a}\right)D_m^1(\beta,\lambda_1)\\ \left(\frac{-i\omega\varepsilon_2 m}{\gamma_1^2 a}\right)B_m^1(\beta,\lambda_1)\end{Bmatrix}\right]\frac{\lambda_2 F(\lambda_2)}{2} + \text{the same part with } (\lambda_1 \leftrightarrow \lambda_2) \qquad (19)$$

and

$$\frac{m\eta_2}{\varsigma_2^2 a}\begin{Bmatrix} A_m^2(\beta,\lambda_1,\lambda_2)\\ C_m^2(\beta,\lambda_1,\lambda_2)\end{Bmatrix} - \frac{m\eta_2}{\gamma_2^2 a}\begin{Bmatrix} B_m^2(\beta,\lambda_1,\lambda_2)\\ D_m^2(\beta,\lambda_1,\lambda_2)\end{Bmatrix} = \left[\frac{m\eta_1}{\varsigma_1 a}\left(\frac{1}{\varsigma_1 a}-\Lambda_m(\varsigma_1 a)\right)\begin{Bmatrix} A_m^1(\beta,\lambda_1)\\ C_m^1(\beta,\lambda_1)\end{Bmatrix}\right]\frac{F(\lambda_2)}{2} - \left[\frac{m\eta_1}{\gamma_1 a}\left(\frac{1}{\gamma_1 a}-\psi_m(\gamma_1 a)\right)\begin{Bmatrix} B_m^1(\beta,\lambda_1)\\ D_m^1(\beta,\lambda_1)\end{Bmatrix}\right]\frac{F(\lambda_2)}{2} - \frac{J''_m(\varsigma_1 a)}{J_m(\varsigma_1 a)}\begin{Bmatrix} i\omega\mu C_m^1(\beta,\lambda_1)\\ -i\omega\varepsilon_1 A_m^1(\beta,\lambda_1)\end{Bmatrix}\frac{F(\lambda_2)}{2} - \frac{H''_m(\gamma_1 a)}{H_m(\gamma_1 a)}\begin{Bmatrix} i\omega\mu D_m^1(\beta,\lambda_1)\\ -i\omega\varepsilon_2 B_m^1(\beta,\lambda_1)\end{Bmatrix}\frac{F(\lambda_2)}{2} + \text{the same part with } (\lambda_1 \leftrightarrow \lambda_2) \qquad (20)$$

for $n=2$. The approximate solutions for the Wiener expansion coefficients of the outgoing scattered waves are then obtained as

$$\left(\frac{m\eta_1}{\varsigma_1^2 a}-\frac{m\eta_1}{\gamma_1^2 a}\right)\begin{Bmatrix} B_m^1(\beta,\lambda_1)\\ D_m^1(\beta,\lambda_1)\end{Bmatrix} = \left\{\left[\frac{m\beta}{\varsigma_0 a}\left(\frac{1}{\varsigma_0 a}-\Lambda_m(\varsigma_0 a)\right) - \varsigma_0\Lambda_m(\varsigma_0 a)\frac{m\eta_1}{\varsigma_1^2 a}\right]\begin{Bmatrix} A_m^0(\beta)\\ C_m^0(\beta)\end{Bmatrix}\right\}F(\lambda_1) - \left\{\left[\frac{m\beta}{\gamma_0 a}\left(\frac{1}{\gamma_0 a}-\psi_m(\gamma_0 a)\right) - \gamma_0\psi_m(\gamma_0 a)\frac{m\eta_1}{\varsigma_1^2 a}\right]\begin{Bmatrix} B_m^0(\beta)\\ D_m^0(\beta)\end{Bmatrix}\right\}F(\lambda_1) - \frac{J''_m(\varsigma_0 a)}{J_m(\varsigma_0 a)}\begin{Bmatrix} i\omega\mu C_m^0(\beta)\\ -i\omega\varepsilon_1 A_m^0(\beta)\end{Bmatrix}F(\lambda_1) - \frac{H''_m(\gamma_0 a)}{H_m(\gamma_0 a)}\begin{Bmatrix} i\omega\mu D_m^0(\beta)\\ -i\omega\varepsilon_2 B_m^0(\beta)\end{Bmatrix}F(\lambda_1) - \left(\frac{m\eta_1}{\varsigma_1^2 a}\right)\left[\frac{\beta}{\varsigma_0}\Lambda_m(\varsigma_0 a)\begin{Bmatrix} A_m^0(\beta)\\ C_m^0(\beta)\end{Bmatrix} - \frac{\beta}{\gamma_0}\psi_m(\gamma_0 a)\begin{Bmatrix} B_m^0(\beta)\\ D_m^0(\beta)\end{Bmatrix}\right]\lambda_1 F(\lambda_1) - \left(\frac{m\eta_1}{\varsigma_1^2 a}\right)\left[\begin{Bmatrix} \left(\frac{i\omega\mu m}{\varsigma_0^2 a}\right)C_m^0(\beta)\\ \left(\frac{-i\omega\varepsilon_1 m}{\varsigma_0^2 a}\right)A_m^0(\beta)\end{Bmatrix} - \begin{Bmatrix} \left(\frac{i\omega\mu m}{\gamma_0^2 a}\right)D_m^0(\beta)\\ \left(\frac{-i\omega\varepsilon_2 m}{\gamma_0^2 a}\right)B_m^0(\beta)\end{Bmatrix}\right]\lambda_1 F(\lambda_1) \qquad (21)$$

and

$$\begin{aligned}
\left(\frac{m\eta_2}{\varsigma_2^2 a}-\frac{m\eta_2}{\gamma_2^2 a}\right)\begin{Bmatrix}B_m^2(\beta,\lambda_1,\lambda_2)\\ D_m^2(\beta,\lambda_1,\lambda_2)\end{Bmatrix} &= \left\{\left[\frac{m\eta_1}{\varsigma_1 a}\left(\frac{1}{\varsigma_1 a}-\Lambda_m(\varsigma_1 a)\right)-\varsigma_1\Lambda_m(\varsigma_1 a)\frac{m\eta_2}{\varsigma_2^2 a}\right]\begin{Bmatrix}A_m^1(\beta,\lambda_1)\\ C_m^1(\beta,\lambda_1)\end{Bmatrix}\right\}\frac{F(\lambda_2)}{2}\\
&\quad-\left\{\left[\frac{m\eta_1}{\gamma_1 a}\left(\frac{1}{\gamma_1 a}-\psi_m(\gamma_1 a)\right)-\gamma_1\psi_m(\gamma_1 a)\frac{m\eta_2}{\varsigma_2^2 a}\right]\begin{Bmatrix}B_m^1(\beta,\lambda_1)\\ D_m^1(\beta,\lambda_1)\end{Bmatrix}\right\}\frac{F(\lambda_2)}{2}\\
&\quad-\frac{J_m''(\varsigma_1 a)}{J_m(\varsigma_1 a)}\begin{Bmatrix}i\omega\mu C_m^1(\beta,\lambda_1)\\ -i\omega\varepsilon_1 A_m^1(\beta,\lambda_1)\end{Bmatrix}\frac{F(\lambda_2)}{2}-\frac{H_m''(\gamma_1 a)}{H_m(\gamma_1 a)}\begin{Bmatrix}i\omega\mu D_m^1(\beta,\lambda_1)\\ -i\omega\varepsilon_2 B_m^1(\beta,\lambda_1)\end{Bmatrix}\frac{F(\lambda_2)}{2}\\
&\quad-\left(\frac{m\eta_2}{\varsigma_2^2 a}\right)\left[\frac{\eta_1}{\varsigma_1}\Lambda_m(\varsigma_1 a)\begin{Bmatrix}A_m^1(\beta,\lambda_1)\\ C_m^1(\beta,\lambda_1)\end{Bmatrix}-\frac{\eta_1}{\gamma_1}\psi_m(\gamma_1 a)\begin{Bmatrix}B_m^1(\beta,\lambda_1)\\ D_m^1(\beta,\lambda_1)\end{Bmatrix}\right]\frac{\lambda_2 F(\lambda_2)}{2}\\
&\quad-\left(\frac{m\eta_2}{\varsigma_2^2 a}\right)\left[\begin{Bmatrix}\left(\frac{i\omega\mu m}{\varsigma_1^2 a}\right)C_m^1(\beta,\lambda_1)\\ \left(\frac{-i\omega\varepsilon_1 m}{\varsigma_1^2 a}\right)A_m^1(\beta,\lambda_1)\end{Bmatrix}-\begin{Bmatrix}\left(\frac{i\omega\mu m}{\gamma_1^2 a}\right)D_m^1(\beta,\lambda_1)\\ \left(\frac{-i\omega\varepsilon_2 m}{\gamma_1^2 a}\right)B_m^1(\beta,\lambda_1)\end{Bmatrix}\right]\frac{\lambda_2 F(\lambda_2)}{2}\\
&\quad+\text{the same part with }(\lambda_1\leftrightarrow\lambda_2). \qquad (22)
\end{aligned}$$

IV. INCOHERENT SCATTERING DISTRIBUTIONS

The angular distribution of incoherent scattering can be obtained from the Wiener expansion coefficients.[4,5] Let $P_{ic}(\theta_s,\varphi_s|\theta_i,\alpha)$ denote the incoherent scattering angular distribution—that is, the average power flow scattered incoherently from unit surface area into unit solid angle of the direction (θ_s,φ_s) when the angle of incidence is (θ_i,α)—we then have the expression for the incoherent scattering angular distribution,

$$P_{ic}(\theta_s,\varphi_s|\theta_i,\alpha)=P_{ic}^1(\theta_s,\varphi_s|\theta_i,\alpha)+P_{ic}^2(\theta_s,\varphi_s|\theta_i,\alpha), \quad (23)$$

where the contribution from the first-order Wiener coefficient is given as

$$\begin{aligned}
P_{ic}^1(\theta_s\varphi_s|\theta_i,\alpha) &= \frac{1}{\cos^2\theta_s}\left[\left|\sum_m B_m^1(\sin\theta_i,\sin\theta_s-\sin\theta_i)e^{im(\varphi_s-\alpha)}\right|^2\right.\\
&\quad\left.+\eta_0^2\left|\sum_m D_m^1(\sin\theta_i,\sin\theta_s-\sin\theta_i)e^{im(\varphi_s-\alpha)}\right|^2\right] \quad (24)
\end{aligned}$$

and the contribution from the second-order Wiener coefficient is given as

$$\begin{aligned}
P_{ic}^2(\theta_s,|\varphi_s|\theta_i,\alpha) &= \frac{2}{\cos^2\theta_s}\int_{-\infty}^{+\infty}d\lambda\\
&\quad\times\left[\left|\sum_m B_m^2(\sin\theta_i,\lambda,\sin\theta_s-\sin\theta_i-\lambda)e^{im(\varphi_s-\alpha)}\right|^2\right.\\
&\quad\left.+\eta_0^2\left|\sum_m D_m^2(\sin\theta_i,\lambda,\sin\theta_s-\sin\theta_i-\lambda)e^{im(\varphi_s-\alpha)}\right|^2\right].
\end{aligned} \quad (25)$$

For the purpose of numerical calculations we conveniently assume that the power spectrum $|F(\lambda)|^2$ of the random rough surface has a Gaussian form—that is,

$$|F(\lambda)|^2=\frac{\sigma^2\ell}{\sqrt{\pi}}e^{-\lambda^2\ell^2}, \quad (26)$$

where ℓ is the correlation length of the rough surface. The numerical results of the incoherent scattering distributions are shown in Figs. 2–4 for the different values of the normalized radius of the cylinder $ka=0.5$, 1.0, and 2.0, respectively, as the normalized roughness $k\sigma=0.1$ and the normal-

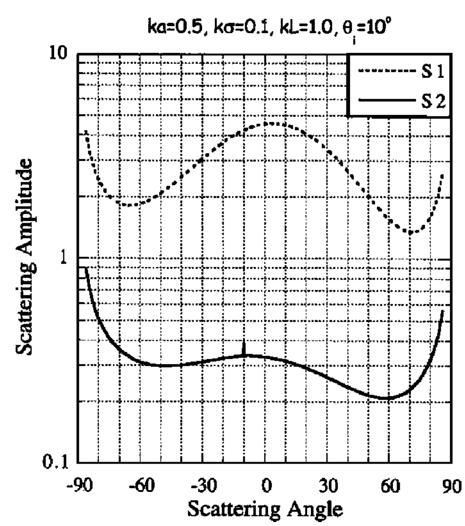

FIG. 2. Incoherent scattering distribution from a cylindrical random rough metal surface that is uniform in the φ direction. The incident angle is $\theta_i=10°$. The dielectric constant of the metal is taken as $\varepsilon_r=-17.55+i0.1$. The normalized roughness and the normalized correlation length are $k\sigma=0.1$ and $k\ell=1.0$. The normalized radius of the cylinder is $ka=0.5$ where the surface can support the zeroth plasmon mode.

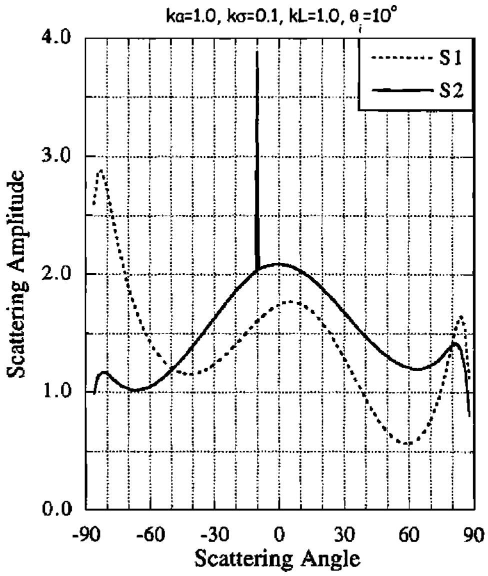

FIG. 3. Same as Fig. 2 but for $ka=1.0$ where the surface can support the zeroth plasmon mode.

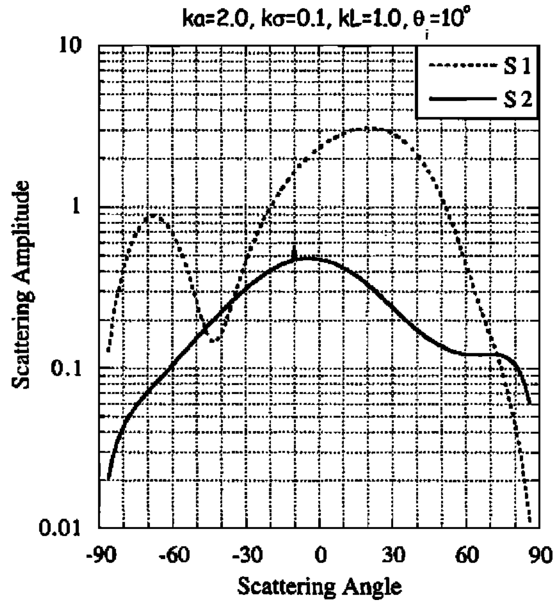

FIG. 4. Same as Fig. 2 but for $ka=2.0$ where the surface can support the zeroth and first plasmon modes.

ized correlation length $k\ell=1.0$. The incident angle is $\theta_i=10°$. The dielectric constant of the metal is taken as $\varepsilon_r=-17.55+i0.1$. The enhanced backscattering can be evidently observed in the incoherent scattering distributions for all three cases and comes from the contribution of the second-order Wiener coefficients. The enhanced peak is much stronger for $ka=1.0$ since in this case the surface plasmon wave is less attenuated when it propagates along the cylindrical rough surface.

V. CONCLUSIONS

In conclusion, we have treated the scattering problem of a TE-polarized electromagnetic wave from a metallic cylinder with its radius being a slightly random rough surface by applying the stochastic functional approach. The enhanced backscattering is observed in the numerical results for the incoherent scattering angular distribution and is attributed to the participation of the surface plasmon waves supported by the cylindrical metal surface as the intermediate scattering processes in multiple scattering. It is expected that much strong enhancement could occur when the cylindrical surface is also rough in φ direction since the higher-order surface plasmon wave can play a more important role in the scattering processes.

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