

Kondo screening cloud in a double quantum dot system

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We analyze the transport properties of two artificial magnetic impurities coupled together via a tunable Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction mediated by conduction electrons of a finite-size one-dimensional wire. We show that the sign of the RKKY interaction can be controlled by gating the wire. We investigate the interplay between finite-size effects and RKKY interaction and found that the two artificial impurities start to interact to each other as soon as the Kondo screening cloud length becomes larger than the length of the wire. This should allow one to give a lower experimental estimate of the Kondo screening cloud length.

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I. INTRODUCTION

One of the most remarkable triumphs of recent progress in nanoelectronics has been the observation of the Kondo effect in a single semiconductor quantum dot.¹⁻³ When the number of electrons in the dot is odd, it can behave as an $S=1/2$ magnetic impurity, interacting via magnetic exchange with the conduction electrons. One of the main signatures of the Kondo effect is that the conductance through the quantum dot reaches the named unitary limit $2e^2/h$ at low enough temperature $T < T_K^0$, where T_K^0 is the Kondo temperature. In this temperature regime, the impurity spin is screened and forms a singlet with a conduction electron belonging to a very extended many-body wave function known as the Kondo screening cloud. The size of this screening cloud may be evaluated as $\xi_K^0 \approx \hbar v_F / T_K^0$, where v_F is the Fermi velocity. In a quantum dot, the typical Kondo temperature is of order 1 K, which leads to $\xi_K^0 \approx 1 \mu\text{m}$ in semiconducting heterostructures. Finite-size effects related to the final extent of this length scale have been predicted recently in two different geometries: a quantum dot embedded in a ring threaded by a magnetic flux⁴⁻⁶ and a quantum dot embedded between two finite-size wires connected to external leads.⁷⁻⁹ In the former geometry, it was shown that the persistent current induced by a magnetic flux is particularly sensitive to screening-cloud effects and is drastically reduced when the circumference of the ring becomes smaller than ξ_K^0 .⁴ In the latter geometry, signature of the finite-size extension of the Kondo cloud was found in the temperature dependence of the conductance through the whole system (wires + dot).⁷⁻⁹

The previous results were obtained by considering a single impurity confined in some mesoscopic finite-size environment. Similar effects have also been predicted in related geometries.^{10,11}

Nevertheless, a more appropriate and certainly relevant question for bulk materials would be what is the lower bound on the average impurity separation necessary to apply the single impurity theory? As described in Ref. 12, there are at least three different ways of estimating such a length scale: (i) One can first simply require that the minimum interimpurity distance to be larger than ξ_K^0 . (ii) One may require that

the density of electron states within an energy T_K^0 of the Fermi surface be at least as large as n_i , the impurity concentration (this constitutes the so-called Nozières exhaustion principle). (iii) Following the mean-field treatment of the Kondo lattice by Doniach,¹³ one may also require that the energy scale related to the average Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction to be small compared to T_K^0 .

In one dimension, definitions (i) and (ii) provide $R \sim \xi_K^0$ for the minimum distance at which impurities can be treated independently, whereas definition (iii) provides $R \sim (\rho^0 J)^2 \xi_K^0$, where J is the Kondo coupling between the impurity spin and the conduction electrons and ρ^0 the density of states. Definition (iii) gives, therefore, a lower estimate for the interimpurity distance to account for multi-impurity effects.

Recent progress in nanolithography allows one to design complex geometries able to mimic the behavior of a many-impurity problem. In particular, Craig *et al.*¹⁴ have recently manufactured a system in which two Kondo quantum dots are connected to a common open conducting region. Here both the RKKY and Kondo interactions compete, thus providing an experimental realization of the two-impurity Kondo problem.

In this paper, we propose to use a rather similar geometry to estimate the Kondo screening cloud size. We consider the following device: two quantum dots connected to a quantum wire of length $2l$. We assume that the quantum dots can be both tuned via independent plunger gate voltages in the Kondo regime, and that the finite-size wire (FSW) can be gated. We also assume the FSW to be weakly connected to a two-dimensional reservoir. The whole geometry is depicted in Fig. 1. The setup we want to study is therefore related to a two-impurity Kondo problem in a confining geometry, a question recently addressed by Galkin *et al.*¹⁵ using slave boson mean-field theory. Galkin *et al.* have shown that the low-energy behavior of this system strongly depends on the position of the Fermi energy as for the single-impurity problem in a confined geometry.¹⁵

We intend to show that when $2l \gg \xi_K^0$, the impurities behave almost independently, whereas for $2l \lesssim \xi_K^0$, two-impurity effect takes place. The analysis of transport through one of the quantum dots allows one to discriminate between

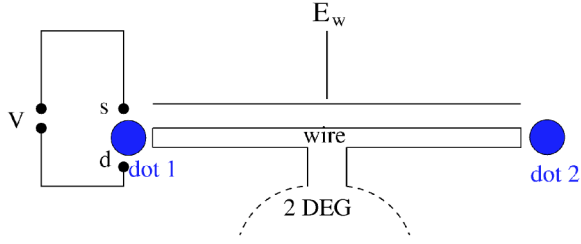


FIG. 1. Schematic representation of the device under study. The two quantum dots are connected to a finite-size open wire. The Fermi level in the wire is controlled by a gate voltage E_w .

a single-impurity behavior and a two-impurity behavior and may therefore provide an estimate of ξ_K^0 . The plan of the paper is the following: in Sec. II we present our model Hamiltonian and derive how the finite size effects (FSEs) renormalize both the Kondo temperature and the RKKY interaction. In Sec. III we show how FSE quantitatively affects the transport properties through the quantum dot. Finally in Sec. IV we give a brief conclusion and discuss the generality of our results.

II. FINITE-SIZE EFFECTS AND RKKY INTERACTIONS

In such a geometry, the two artificial impurities are connected to both extremities of a quantum gated wire. Since the wire is also weakly connected to a two-dimensional reservoir, its spectrum is continuous. In such a confined geometry, FSEs are to play an important role, first because they affect the renormalization of the Kondo couplings of each impurity as was shown in Ref. 7, but also the RKKY interaction. To take both effects into account, we choose to use a simple tight-binding model that should capture the main physical ingredients of this system.

A. Model Hamiltonian

In order to treat this problem, we may write the following simple one-dimensional (1D) tight-binding Hamiltonian:

$$H = H_D + H_W + H_R + H_{tun,D} + H_{tun,R} \quad (1)$$

with

$$H_D = \sum_{i=1,2;\sigma} \epsilon_{d,i} d_{i,\sigma}^\dagger d_{i,\sigma} + U_i n_{d,i,\uparrow} n_{d,i,\downarrow}, \quad (2)$$

$$H_W = -t \sum_{j=1;\sigma} (c_{j,\sigma}^\dagger c_{j+1,\sigma} + \text{h.c.}) + E_w \sum_{j=1;\sigma} n_{c,j,\sigma}, \quad (3)$$

$$H_R = -t \sum_{l=1}^{\infty} \sum_{\alpha=1,2;\sigma} (a_{l,\alpha,\sigma}^\dagger a_{l+1,\alpha,\sigma} + \text{h.c.}), \quad (4)$$

$$H_{tun,D} = -t' \sum_{\sigma} (c_{1,\sigma}^\dagger d_{1,\sigma} + c_{2l,\sigma}^\dagger d_{2,\sigma} + \text{h.c.}), \quad (5)$$

$$H_{tun,R} = -t_w \sum_{\sigma} (c_{l+1,\sigma}^\dagger a_{l+1,\sigma} + \text{h.c.}), \quad (6)$$

where $d_{i,\sigma}^\dagger$ creates an electron with spin σ in dot i , $c_{j,\sigma}^\dagger$ creates an electron in the wire at site j with spin σ , and, finally, $a_{l,\alpha,\sigma}^\dagger$ creates an electron in the reservoir at site l , in the channel α with spin σ . In Eq. (2), the two quantum dots are described by two Anderson impurity models, where $\epsilon_{d,i}$, U_i are, respectively, the energy level and the Coulomb repulsion energy in dot i . Equation (3) describes a 1D noninteracting finite-size wire (FSW) of length $2l$. In Eq. (4), we described the reservoir that coupled to the FSW by an infinite two-channel wire. This assumption is for further technical convenience, the only ingredient required being that the FSW is tunnel coupled to a continuum (see Ref. 8 for details). The last two equations (5) and (6) describe tunneling of electrons, respectively, between the dots and the FSW and between the reservoir and the FSW. In the following we also assume a symmetric geometry, i.e., we suppose $\epsilon_{d,1} = \epsilon_{d,2} = \epsilon_d$ and $U_1 = U_2 = U$. In fact, we only require in the sequel that the Kondo temperatures associated with each dot are comparable, i.e., $T_K^0 = T_{K,1}^0 \sim T_{K,2}^0$. We have also assumed that the FSW is connected to a gate controlled by E_w . In order to study transport properties, we need to weakly connect at least one dot to extra leads (which are represented with dashed lines in Fig. 1). Such a way of probing the quantum dot transport properties is more in the spirit of the scanning-tunnel-microscope- (STM)-type measurements since it directly probes the local dot density of states.

We assume that the two dots can be cast in the Kondo regime. Such a situation has been recently realized experimentally in Ref. 14. We can perform a Schrieffer Wolff transformation in order to write $H_D + H_{tun,D}$ in the following form:

$$H_D + H_{tun,D} \rightarrow J(\vec{S}_1 \cdot \sigma_1 + \vec{S}_2 \cdot \sigma_2), \quad (7)$$

with $J = 2t'^2[(1/\epsilon_d) + (1/U + \epsilon_d)]$ and $\vec{\sigma}_{1/2l} = c_{1/2l}^\dagger (\vec{\tau}/2) c_{1/2l}$ is the local spin density at sites $1/2l$ ($\vec{\tau}$ are the Pauli matrices). We have neglected potential scattering terms that are not playing an important role in what follows. Before taking into account both the Kondo and RKKY interactions, we need to evaluate these two interactions in a confined geometry.

B. Kondo temperatures

Let us first estimate the Kondo temperature of a *single* artificial impurity assuming that one quantum dot is disconnected of the system. This can be achieved by pinching the wire-dot tunnel junction off. The remaining dot is coupled to the FSW, and its Kondo temperature can be estimated following Ref. 8. In order to calculate T_K , we first diagonalize the Hamiltonian at $J=0$, i.e., we diagonalize $H_0 \equiv H_W + H_R + H_{tun,R}$. If $t_w=0$, then the wave functions and eigenvalues of the wire are

$$\psi_{(j)} = (1/\sqrt{2l+1}) \sin(k_{w,n} j),$$

$$k_{w,n} = \pi n / (2l+1); \quad 1 \leq n \leq 2l,$$

$$\epsilon(k_{w,n}) = -2t \cos k_{w,n} + E_w. \quad (8)$$

For nonzero t_W , the spectrum of H_0 becomes continuous. In order to study how the Kondo interaction renormalizes, we express c_1, c_{2l} in terms of the eigenstates, c_ϵ of H_0

$$c_j = \int_{-2t}^{2t} d\epsilon [f_j^e(\epsilon)c_{e,\epsilon} + f_j^o(\epsilon)c_{o,\epsilon}], \quad (9)$$

with $j=1, 2l$ here and the indices e, o are for even or odd components under parity symmetry under the middle of the line. Indeed H_0 can be decomposed as $H_0^e + H_0^o$, which are obtained by a folding procedure around a vertical axis located at the point $j=l+1/2$. We have normalized the operators $c_{e/o,\epsilon}$ such that $\{c_{e/o,\epsilon}^\dagger, c_{e/o,\epsilon'}\} = \delta(\epsilon - \epsilon')$ and $\{c_{e/o,\epsilon}^\dagger, c_{o/e,\epsilon'}\} = 0$. The local density of states seen by dot 1 is defined by

$$\rho_1(\epsilon) = |f_1^e(\epsilon)|^2 + |f_1^o(\epsilon)|^2 = \rho_1^e(\epsilon) + \rho_1^o(\epsilon) \quad (10)$$

and is normalized according to $\int_{-2t}^{2t} d\epsilon \rho_1(\epsilon) = 1$. In this basis the Kondo Hamiltonian (7) can be written as $H = H_0 + H_K$ with

$$H_0 = \int d\epsilon \epsilon (c_{e,\epsilon}^\dagger c_{e,\epsilon} + c_{o,\epsilon}^\dagger c_{o,\epsilon}), \quad (11)$$

and

$$\begin{aligned} H_K = & \int \int d\epsilon d\epsilon' \{ [f_1^e(\epsilon) f_1^e(\epsilon')] c_{e,\epsilon}^\dagger \vec{\sigma} c_{e,\epsilon'} \\ & + f_1^o(\epsilon) f_1^o(\epsilon') c_{o,\epsilon}^\dagger \vec{\sigma} c_{o,\epsilon'} \} \cdot (\vec{S}_1 + \vec{S}_2) + f_1^e(\epsilon) f_1^o(\epsilon') \\ & \times [c_{e,\epsilon}^\dagger \vec{\sigma} c_{o,\epsilon'} + c_{o,\epsilon}^\dagger \vec{\sigma} c_{e,\epsilon'}] \cdot (\vec{S}_1 - \vec{S}_2). \end{aligned} \quad (12)$$

To obtain this equation, we have used the fact that $f_{2l}^e(\epsilon) = f_1^e(\epsilon)$ and $f_{2l}^o(\epsilon) = -f_1^o(\epsilon)$.

The local density of states $\rho_1(\epsilon)$ can be computed exactly for our tight-binding model following Ref. 7 and reads

$$\rho_1^{\text{even/odd}}(k) = \frac{1}{\pi t} \frac{\gamma_W^2 \sin^2 k_w \sin k}{D_{e/o}^2}, \quad (13)$$

with

$$\begin{aligned} D_{e/o}^2 = & [\cos kl(\sin k_w(l+1) \mp \sin k_w l) - \gamma_W^2 \cos k(l+1) \\ & \times \sin(k_w l)]^2 + [\sin kl(\sin k_w(l+1) \mp \sin k_w l) \\ & - \gamma_W^2 \sin k(l+1) \sin(k_w l)]^2. \end{aligned}$$

Note that we have defined $\gamma_W = t_W/t$ and k_w is related to k by

$$-2t \cos k = -2t \cos k_w + E_W. \quad (14)$$

If $\gamma_W = 0$, the even local density of states (LDOS) ρ_1^e is simply composed of l Dirac peaks and levels separated by the level spacing $2\Delta \sim \hbar v_F/l$. ρ_1^o has a similar structure but is shifted by Δ . When $\gamma_W \neq 0$, these $2l$ finite-size levels in the wire acquire a finite width and their position is slightly shifted because of the coupling to the continuum spectrum. $\rho_1^{e/o}$ now become continuous. An example of such a local density of states has been plotted for $l=11$ and $\gamma_W=0.5$ in Fig. 2. Note that even for $\gamma_W=1$, i.e., a perfect coupling between the source and the leads, we get a succession of

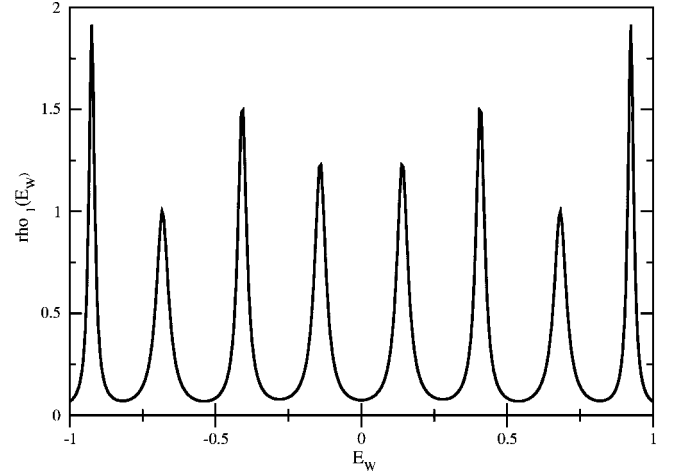


FIG. 2. Example of the local density of states seen by the dot 1. We took $2l=22$ and $\gamma_W^2=0.25$.

peaks of broader widths. We have modeled the reservoir by a two-channel infinite wire in order that both even and odd sectors couple to the continuum. This density profile is quite general and does not depend on our particular model Hamiltonian. It occurs as soon as we couple a finite-size system to a continuum.

Different energy scales emerge in the problem, the level spacing Δ , which needs to be compared to the bare Kondo temperature T_K^0 of the dots and the peak width δ . By bare Kondo temperature T_K^0 , we mean the Kondo temperature of a single independent dot in the Kondo regime, i.e., the Kondo temperature obtained in the limit $l \rightarrow \infty$. When $T_K^0 \sim \Delta$, FSE start taking place. The genuine Kondo temperature of a single dot T_K starts to depend on the fine structure of the local density of states and can be, in general, very different from T_K^0 . In this situation, the Kondo temperature T_K depends on the wire gate voltage E_W being set on a resonance (which gives a Kondo temperature T_K^R) or off a resonance (which gives a Kondo temperature T_K^{NR}). This discussion has been largely detailed in Refs. 7 and 8. In the limit $\gamma_W^2 \ll 1$ one can evaluate

$$T_K^R \approx \delta \left(\frac{T_K^0}{D_0} \right)^{\gamma_W^2} = O(\delta_n), \quad (15)$$

and

$$T_K^{NR} \approx \Delta \left(\frac{T_K^0}{D_0} \right)^{1/2 \gamma_W^2}. \quad (16)$$

For strong finite effects, one has $T_K^{NR} \ll T_K^R \sim O(\delta)$. Such dramatic dependence of the single-impurity energy scale with the gate voltage position clearly affects all transport and thermodynamic properties.^{7,8}

When both impurities are taken into account, the previous results can be extended starting from the Hamiltonian in Eq. (12). If the RKKY interactions were switched off, both impurities would be now coherently screened in a two-stage procedure. As above, FSEs take place when $T_K^0 \sim \Delta$. When this situation is met, the two energy scales associated with

the two-stage screening process depend strongly on the even/odd local densities of states being on or off resonances. This situation is similar to the one occurring for a ferromagnetic RKKY interaction and will be discussed in Sec. II D.

C. RKKY interaction

In Sec. II B we focused on the Kondo temperature of a single dot ignoring their mutual interactions. Nevertheless, the two dots also interact through the RKKY interaction. The RKKY interaction reads $H_{\text{RKKY}} = I \vec{S}_1 \cdot \vec{S}_2$. This interaction is automatically generated at second order in the Kondo coupling J and corresponds to the coherent process where an electron participating in the screening of the artificial first impurity can also participate in the screening of the second impurity. Remember that a more dynamical type of screening is involved here rather than the one occurring for charge impurities in a Fermi liquid.

1. Usual situation: $T_K^0 \gg \Delta$

Let us first consider the simple case of a very long wire connecting the two dots without the reservoir (i.e., $\gamma_W=0$). At lowest order in J , the RKKY interaction reads

$$I = J^2 \left(\frac{2}{2l} \right)^2 \times \sum_{\epsilon(k_w), \epsilon_F; \epsilon(k'_w), \epsilon_F} \frac{\sin(k_w) \sin(k'_w) \sin(2k_w l) \sin(2k'_w l)}{\epsilon(k_w) - \epsilon(k'_w)}, \quad (17)$$

where the $\epsilon(k_w)$ have been defined in Eq. (8) and ϵ_F is the Fermi energy. For $l \gg 1$ and $\epsilon_F \sim 0$, this expression simplifies and reads¹⁶

$$I = \frac{J^2 \cos[2k_F(2l)]}{4\pi t(2l)} = \frac{\pi \lambda^2 D \cos[2k_F(2l)]}{8(2l)}, \quad (18)$$

where $\lambda = \rho_0 J = J \sin(k_F) / (\pi t)$ is the dimensionless Kondo coupling and $D = 2t$ is the band width. This is the usual form of the RKKY interaction in 1D. The above interaction is always positive at exactly half filling because we take $2l$ sites. Had we taken an odd number of sites, we would get an opposite sign, i.e., a ferromagnetic interaction.

One may argue that the impurity can no longer be regarded as independent when $I \sim T_K^0$. This result has been obtained, for example, by applying the slave boson mean-field theory approximation for the two-impurity problem by Jones *et al.*¹⁷ From this argument, we have seen in the introduction that at fixed ξ_K^0 , the dots need to be brought rather close (compared to ξ_K^0) to start feeling the RKKY interaction effects. However, we have seen that screening-cloud FSEs occur already when $\xi_K^0 \sim 2l$ with a strong renormalization of T_K . This may suggest that FSEs occur first when approaching the two impurities. Nevertheless, this argument is too naive since FSEs may also affect the RKKY interaction as we will see. A more refined analysis is clearly demanded when $T_K^0 \lesssim \Delta$.

2. Unusual situation: $T_K^0 \ll \Delta$

In this regime, one has already seen how the Kondo temperature is renormalized depending on the density of states being on resonance or off resonance. In our general case of a finite-size wire connected to a source lead, the RKKY interaction can be written at lowest order in J as

$$H_{\text{RKKY}} = I \vec{S}_1 \vec{S}_2 = J^2 \vec{S}_1 \vec{S}_2 \int_{-D}^{\epsilon_F} d\epsilon \int_{\epsilon_F}^D d\epsilon' \frac{f_1(\epsilon) f_{2l}(\epsilon) f_1(\epsilon') f_{2l}(\epsilon')}{\epsilon - \epsilon'}, \quad (19)$$

where the $f_j(\epsilon)$ have been defined in Eq. (9). In the limit $\gamma_W \ll 1$, the LDOS can be well approximated as follows

$$\pi \rho_j(\epsilon) = \pi f_j^2(\epsilon) \approx \frac{2}{2l+1} \sum_n \sin^2(k_{w,n} l) \frac{\delta_n}{(\epsilon - \epsilon_n)^2 + \delta_n^2}, \quad (20)$$

and in a similar way

$$\pi f_1(\epsilon) f_{2l}(\epsilon) \approx \frac{2}{2l+1} \sum_n \sin(k_{w,n}) \sin(2lk_{w,n}) \frac{\delta_n}{(\epsilon - \epsilon_n)^2 + \delta_n^2}. \quad (21)$$

In these expressions, we have defined $\epsilon_n = \epsilon(k_{w,n})$ [$k_{w,n}$ was introduced in Eq. (8)] and $\delta_n \approx \gamma_W^2 t \sin^2 k_{w,n} \sin k/l$ corresponds to the peak width. This faithful approximation allows us to evaluate the RKKY interaction when finite-size effects do occur:

$$I \approx \left(\frac{J}{\pi} \right)^2 \left(\frac{2}{2l+1} \right)^2 \sum_{n,n'} \int_{-D}^{\epsilon_F} d\epsilon \int_{\epsilon_F}^D d\epsilon' \sin^2(k_n) \sin^2(k_{n'}) \times (-1)^{n+n'} \frac{\delta_n \delta_{n'}}{(\epsilon - \epsilon') [(\epsilon - \epsilon_n)^2 + \delta_n^2] [(\epsilon' - \epsilon_{n'})^2 + \delta_{n'}^2]}. \quad (22)$$

The terms that contribute mostly in the above summation are the ones where ϵ_n is in the vicinity of ϵ_F . We need to distinguish two different situations.

(i) Suppose that E_W is tuned such that ϵ_F lies between two resonance peaks of the local density of states. The integral is therefore mainly dominated by these two consecutive resonance peaks that implies that $n+n'$ is odd. The whole integral is therefore positive meaning the RKKY interaction is antiferromagnetic.

(ii) Suppose that E_W is tuned such that ϵ_F is on a resonance. The integral is mainly dominated by this resonance peak and therefore $n=n'=n_F$. The whole integral is therefore negative meaning the RKKY interaction is ferromagnetic.

We can therefore conclude that FSE also plays a crucial role by fixing the sign of the RKKY interaction as already noted in Ref. 15. When the LDOS is tuned off resonance, the RKKY interaction is always antiferromagnetic, when the LDOS is tuned on resonance, the RKKY interaction is always ferromagnetic. This strong sign dependence opens the way for a control of the RKKY interaction.

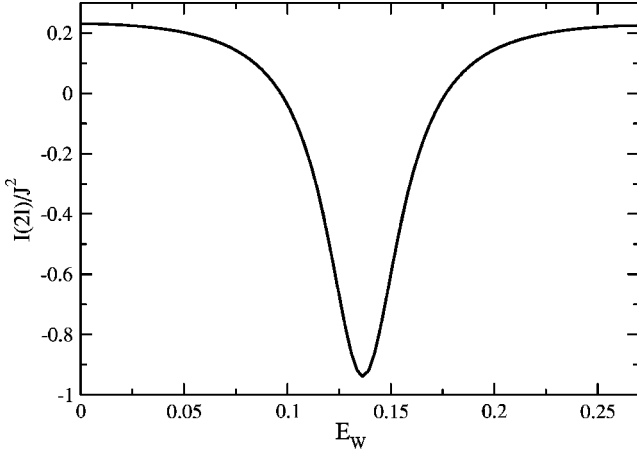


FIG. 3. RKKY interaction as a function of E_W . We took the same parameter as in Fig. 2: $2l=22$ and $\gamma_W^2=0.25$. When E_W is tuned on a resonance, the RKKY interaction becomes positive and gets enhanced by a factor $\sim 1/\gamma_W^2$ as predicted by Eq. (24).

Furthermore, these integrals can be estimated (in the limit $\gamma_W \ll 1$) using the previous approximations

$$I^{NR} \approx J^2 \frac{2c_1 \sin^4(k_F)}{(2l)^2 \Delta} \approx J^2 \frac{c_1 \sin^2 k_F \rho_0}{2l}, \quad (23)$$

$$I^R \approx -J^2 \frac{c_2 \sin(k_F)}{2l\pi\gamma_W^2 t} \approx -J^2 \frac{c_2 \rho_0}{2l\gamma_W^2}, \quad (24)$$

with $\rho_0 = \sin(k_F)/(\pi t)$ and c_1, c_2 two numerical factors of order 1.

We see that the RKKY interaction I^R is actually enhanced when the LDOS is tuned on resonance ($\gamma_W^2 \ll 1$), whereas $I^{NR} \approx I$ remains almost unchanged. A rather similar behavior occurs for magnetic impurities in quantum corrals.¹⁸ In order to check these predictions, we have numerically calculated the integral given by Eq. (22) and plotted the RKKY interaction I as a function of E_W in Fig. 3 when E_W goes through a resonance. It confirms first that the RKKY interaction can be varied continuously from positive to negative value and second that the RKKY interaction becomes ferromagnetic and considerably enhanced when E_W is nearby a resonance.

D. Discussion

Let us discuss and compare the RKKY interaction with the Kondo interaction. We assume that J and $T_K^0 \sim D \exp[-1/(\rho^0 J)]$ are fixed and the distance between the dots can be varied. When $T_K^0 \gg \Delta$, i.e., $\xi_K^0 \ll (2l)$, we are in the usual situation where the impurities can be regarded as almost independent [the overlap between both screening clouds is $O(2l/\xi_K^0)$].¹² The RKKY interaction becomes important when the distance l between the dots satisfies $l \sim (\rho^0 J)^2 \xi_K^0$, a condition that is clearly not satisfied in this limit.

Suppose now that we bring closer together the quantum dots such that $\xi_K^0 \ll 2l$. In order to analyze this limit, we may integrate out high-energy degrees of freedom resulting from

a change of $J(D)=J$ to $J_{eff}(\Delta)$ in the above expressions for the RKKY interaction.⁷ The two scales that must be compared are now T_K and I .

(i) When the LDOS is tuned off resonance, T_K^{NR} is strongly reduced compared to T_K^0 whereas $I^{NR} \sim I$ increased as $1/2l$ when reducing l . We therefore expect a sharp crossover from an independent regime to a regime where the dots form a singlet (remember that $I > 0$ off resonance). This crossover is mainly due the strong renormalization of T_K^0 into T_K^{NR} rather than to the increase of I .

(ii) When the LDOS is tuned on resonance then $T_K^R = O(\delta)$ with δ the peak width, whereas the RKKY interaction is enhanced by the decrease of l , but especially by the extra factor $\gamma_W^2 \gg 1$. As we decrease l , we also expect here a sharp crossover from a two almost-independent impurity regime to a two ferromagnetically coupled impurity regime as soon as $T_K^R \leq I^R$. When two impurities are coupled ferromagnetically, they form an effective spin-1 impurity. Conduction electrons try to screen this effective spin-1 impurity. Following the work by Jayaprakash *et al.*¹⁹ we can write the following low-energy effective Hamiltonian obtained by integrating out high-energy degrees of freedom from D to $\Gamma < \Delta$ around the resonance, where Γ is the scale at which $T_K^R \leq I^R$

$$\begin{aligned} H_{eff} &\approx H_{el} + J_{eff}(\Gamma) \vec{S} \cdot (\vec{\sigma}_1 + \vec{\sigma}_{2l}) \\ &\approx H_{el} + J_{eff}(\Gamma) \vec{S} \cdot (\sigma_1^{\text{even}} + \sigma_1^{\text{odd}}), \end{aligned} \quad (25)$$

where $\vec{\sigma}_1^{\text{even/odd}}$ correspond to the even or odd part of the local spin density of states coupled to the effective impurity. As we have already seen, this decomposition is obtained by folding the initial model under the vertical axis $j=l+1/2$. The density of states consists of even and odd peaks. Therefore, we expect at very low energy a *two-stage* Kondo effect in this regime. Indeed, suppose we adjust E_W on a resonance, this resonance can be either odd or even under parity. Let us for example assume this resonance to be even and keep on integrating out high-energy degrees of freedom. When we will be close to the resonance, $J_{eff} \rho_1^e$ will grow rapidly, whereas $J_{eff} \rho_1^o$ will remain rather small. In the limit $\gamma_W \ll 1$ these two scales can be evaluated easily and correspond approximately to T_K^R and $(T_K^{NR})^2/\Delta < T_K^{NR}$. We note that the ratio between these two scales is exponential contrary to what was claimed in Ref. 15.

In the previous paragraph we compared ξ_K^0 and $2l$. It is of course not obvious to fine-tune $2l$ the wire length. On the other hand, we may find for each dot different Kondo valleys characterized by different Kondo temperatures and therefore different Kondo screening lengths. A natural way to proceed experimentally would be to select for each dot, at least two distinct Kondo valleys with rather different Kondo scales (ideally by one order of magnitude) in order to probe both regimes $\xi_K^0 = \xi_{K,1}^0 \sim \xi_{K,2}^0 \gg 2l$ and $\xi_K^0 = \xi_{K,1}^0 \sim \xi_{K,2}^0 \leq 2l$. Furthermore, in these Kondo valleys, the Kondo temperatures can also be varied using the dot-plunger gate voltages.

Finally, in the previous discussion, we assume $T=0$ and exhibit the main energy scales of the problem. An interesting and controversial issue is to obtain quantitatively the full phase diagram [for example in the (T, J) plane] of the two-

impurity and especially Kondo lattice problem. In order to approach this nontrivial issue, sophisticated techniques are required (see, for example, Ref. 20).

E. Non-Fermi liquid fixed point

In the two-impurities Kondo problem, the two Fermi liquid phases (the impurity singlet or the screened impurity triplet) may be separated at $T=0$ by a non-Fermi liquid (NFL) fixed point depending on some realization of the particle-hole symmetry.^{21–23} One may wonder whether this NFL fixed point can be reached in such a geometry. We have seen that $2l \gg \xi_K^0$ is required in order for the RKKY interaction to take place. In this regime, we need to distinguish between the LDOS being on or off a resonance. When the LDOS is on a resonance, the RKKY interaction is ferromagnetic and no such NFL fixed point is found for this sign of the RKKY coupling. Suppose now the RKKY interaction is tuned in between two resonances, therefore, of different parity. In our 1D geometry, this implies, in particular, $f_1^e(-\epsilon) = f_1^o(\epsilon)$. When this condition is satisfied, it has been demonstrated by Affleck *et al.*,²³ that the Kondo Hamiltonian has indeed the wrong particle-hole symmetry and no nontrivial critical point is expected to occur between the two FL phases. One can therefore conclude that such one-dimensional-like geometry is not suitable to reach the vicinity of the NFL fixed point of the two-impurity Kondo problem. The only way to reach such a NFL fixed point would be to position the wire gate voltage on a resonance (therefore $I < 0$) in order to have the right particle-hole symmetry and to find an extra way to generate an additional antiferromagnetic coupling between the two impurities.

III. TRANSPORT PROPERTIES

A. Zero temperature

The transport properties of the dots can be probed individually by attaching, for example, two very weakly coupled leads as depicted in Fig. 1. The conductance is governed by the local density of the individual dots. Such a three-terminal measurement provides a spectroscopic analysis of the dot density of states much like the STM probes the density of a magnetic impurity adsorbed on a surface.²⁴ From the analysis developed in the previous section, we infer that as soon as $\xi_K^0 \ll (2l)$, the quantum dots are almost independent. This corresponds to the usual situation where the conductance reaches its maximum value. Extending the scattering approach developed by Ng and Lee²⁵ to several leads (see also Refs. 8 and 26), this maximum value reads

$$G_U = \frac{2e^2}{h} \frac{4\Gamma_s\Gamma_d}{(\Gamma_d + \Gamma_s + \Gamma_w)^2} \approx \frac{2e^2}{h} \frac{4\Gamma_s\Gamma_d}{\Gamma_w^2}, \quad (26)$$

where $\Gamma_w = \pi\rho^0 t'^2$ is the junction conductance between the dot and the wire and $\Gamma_{s/d}$ is the junction conductance between the dot and the source-drain lead.

The interesting situation occurs when $\xi_K^0 \gtrsim (2l)$. We have seen that the sign and the amplitude of the RKKY interaction depends on E_w being adjusted to a resonance or off a reso-

nance. This will have strong implications on transport properties. In the former case (on resonance), the impurity is screened through a two-stage procedure and the conductance can reach the maximum value given by Eq. (26). In the latter case, the impurities form a singlet and the conductance is expected to be zero. Note that the conductance is, in fact, nonzero for a generic quantum dot, but instead dominated by potential scattering terms we neglected here that lead to elastic cotunneling processes. At $T=0$, the regime $\xi_K^0 \gg 2l$ can be, therefore, characterized by a variation of the dot conductance between 0 and G_U when tuning the wire gate voltage. It is worth looking at the differential conductance dI/dV , where V is the voltage bias between the two extra leads. A (narrow) peak at zero bias of width T_K^R and height G_U is expected in the former case, whereas no peak is expected in the latter case. Furthermore, two extra small peaks are also expected in the differential conductance at $eV = \pm I$ corresponding to magnetic excitations. These features should allow one to differentiate the single-impurity behavior ($\xi_K^0 \ll 2l$) from the two-impurity behavior ($\xi_K^0 \gtrsim 2l$). The qualitative shape of these extra peaks has been analyzed in Ref. 27.

B. Finite temperature

The previous results can be extended at finite temperature. When $\xi_K^0 \ll 2l$, the dots behave independently and the finite temperature conductance of dot 1 simply reads

$$G_1 = G_U f(T/T_K^0), \quad (27)$$

where f is a universal scaling function of the ratio T/T_K . Its asymptotic behavior reads²⁹

$$f(x) = 1 - (\pi x)^2 \quad \text{for } x \ll 1, \quad (28)$$

$$f(x) = \frac{3\pi^2/16}{\ln^2(x)} \quad \text{for } x \gg 1. \quad (29)$$

On the other hand, in the limit $\xi_K^0 \gg 2l$, the conductance depends on the artificial impurities forming a singlet or a triplet state. It is not obvious to read from the transport properties in which of these two states the impurities are, as it was recently highlighted by the experiment of Craig *et al.*¹⁴ Indeed for $I < 0$ (ferromagnetic RKKY interaction), the conductance takes the form

$$G_1 = G_U \frac{\pi^2/2}{\ln^2(T/T_K^R)} \quad \text{for } |I| \gg T \gg T_K^R, \quad (30)$$

and is therefore rather small compared to G_U . The conductance for $I > 0$ being dominated by finite-temperature cotunneling processes is also rather small. Therefore the linear conductance does not really allow one to discriminate in which states the impurities are in this temperature range. However, as it was analyzed recently in Refs. 27 and 28, transport spectroscopy in a finite in-plane magnetic field should enable one to distinguish between both impurity ground state (either singlet or triplet). From an experimental point of view, it is, therefore, crucial to gate the wire because it may offer an opportunity to go continuously from one regime to another way.

IV. DISCUSSION OF THE RESULTS

We have seen that when two quantum dots are brought closer together, they significantly deviate from the single-impurity behavior as soon as $\xi_K^0 \gg 2l$. This deviation from the single-impurity behavior occurs mainly because of the strong renormalization of the Kondo temperature but also from the renormalization of the RKKY interaction. When FSEs play an important role, we also predict that the gating of the wire should allow one to control the sign of the RKKY interaction and therefore the wire gate voltage E_W might be used to control the ground state of our system (singlet or triplet). By individually probing the spectral properties of each dot, one can differentiate the single-impurity transport properties from the two-impurity ones and therefore provide a lower estimate of the Kondo screening length.

In order to obtain these results we made some approximations that we want to discuss here. First we assume that the wire is thin enough such that we can restrict a single transverse channel. If the wire contains N different transverse channels, the results derived here can be extended following Ref. 7. In particular, FSEs start to play a role when $T_K^0 \leq \Delta/N$, i.e., for $\xi_K^0 \geq Nl$.⁸ For thick wires, such a condition is difficult to meet experimentally and the renormalization and no renormalization of the Kondo temperature occurs when we vary the wire gate voltage. To see the effects discussed in this paper, it is therefore preferable to work with thin wires containing only a few channels.

We have also assumed that the wire is in a ballistic regime, or at least that the mean-free path l_d is the largest length scale by far. If the mean-free path l_d would be, for example, smaller than the length of the short wire, we would then need to compare the Kondo length scale ξ_K^0 with l_d and not with $2l$ the wire length. Moreover, it is very likely that the simple dependences of T_K and of the RKKY interaction with E_W will be washed out. Nevertheless, for a clean semi-conducting wire of typical length $\sim 1 \mu\text{m}$, we may expect $l_d > 2l$.

We have also neglected Coulomb interactions in the wire. Coulomb interactions will first affect both the Kondo temperature and the RKKY interactions. Assuming that the interacting wire can be described by a Luttinger liquid characterized by a dimensionless strength parameter g ($g=1$ corresponds to the noninteracting case), the Kondo tempera-

ture takes a power-law fashion $T_K^0 \sim v_F(\rho_0 J)^{2/(1-g)}$ instead of the conventional exponential dependence.³⁰ It can also be shown that the RKKY interaction decreases as a power law, $I \sim v_F(\rho_0 J)^2/(2l)^g$, with $2l$ the interimpurity distance.³¹ For a Luttinger liquid with repulsive interactions $g < 1$, it implies an increase of the Kondo temperature (therefore a decrease of the screening cloud) and the RKKY interaction becomes longer range compared to the noninteracting case. We have seen in the introduction for the noninteracting case that the criteria for FSE to occur is $2l \leq \xi_K^0$, whereas the criterion $I > T_K^0$ provides the more restrictive condition $2l \leq (\rho_0 J)^2 \xi_K^0$. On the other hand, in the presence of interactions, both criteria provide the same length scale $l^* \sim (\rho_0 J)^{2/(g-1)} = \xi_K^0$ under which FSE and RKKY interaction comes into play. Therefore, we expect here an even sharper crossover from a single-impurity regime to a two-impurity regime. Nevertheless, repulsive interaction also considerably affects transport properties through the dots. In the low-energy limit, processes that transfer one electron between the leads and the interacting wire become irrelevant.³² Therefore the leads and the wire decouple as two independent (very anisotropic) channels. Because the coupling between the dot and the wire is much stronger, the screening of the impurity occurs at low energy in the wire only. The conductance in the leads is dominated in this low-energy limit by potential scattering terms that lead to resonant cotunneling processes (that do not renormalize). The conductance is therefore very small compared to G_U whenever a single- or two-impurity scenario occurs. It seems, therefore, less obvious to differentiate these two cases (and therefore to provide a lower estimate of ξ_K^0) in the presence of interactions at least in the equilibrium conductance. The differential conductance may reveal, on the other hand, interesting features at large bias that may help to distinguish the single- from the two-impurity regime. A full quantitative analysis of transport properties consistently incorporating both Coulomb interactions and finite level spacing is required but goes beyond the scope of the present paper.

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