Competition between BCS superconductivity and ferromagnetic spin fluctuations in MgCNi₃

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The low temperature specific heat of the superconductor MgCNi₃ and a nonsuperconductor MgC_{0.85}Ni₃ is investigated in detail. An additional contribution is observed from the data of MgCNi₃ but absent in MgC_{0.85}Ni₃, which is demonstrated to be insensitive to the applied magnetic field even up to 12 T. A detailed discussion on its origin is then presented. By subtracting this additional contribution, the zero field specific heat of MgCNi₃ can be well described by the BCS theory with the gap ratio (Δ/k_BT_c) determined by the previous tunneling measurements. The conventional *s*-wave pairing state is further proved by the magnetic field dependence of the specific heat at low temperatures and the behavior of the upper critical field.

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Since the discovery of the intermetallic perovskite superconductor MgCNi₃,¹ plenty of efforts have been focused on the superconducting pairing symmetry in this material because its conduction electrons are derived predominantly from Ni which is itself a ferromagnet.²⁻⁵ However, up to now, there is still not a consensus on this issue. The measured penetration depth,⁶ critical current behavior,⁷ and earlier tunneling spectra⁸ suggested an unconventional superconductivity, the later tunneling data⁹ supported the *s*-wave pairing symmetry and gave a reasonable interpretation on the contradiction to the result in Ref. 8. The s-wave pairing has also been demonstrated by the ¹³C NMR experiments¹⁰ and the specific heat measurements.^{1,8,11–14} To our knowledge, all the previous reports on the specific heat of MgCNi3^{1,8,11-14} were characterized in the framework of a conventional phonon-mediated pairing. However, there is an obvious deviation of the experimental data from the prediction of BCS theory in the low temperature,^{8,15} i.e., the entropy conservation rule is not satisfied. Such deviation has been interpreted by the presence of unreacted Ni impurities in Refs. 8 and 15, whereas it is still prominent in the samples without Ni impurities.¹⁴ On the other hand, strong spin fluctuations have been observed in MgCNi₃ by NMR experiment,¹⁰ which is suggested to be able to severely affect the superconductivity in MgCNi₃^{2,5,10,11,16} or even induce some exotic pairing mechanism.² Consequently, the behavior of the specific heat will inevitably be changed by the spin fluctuations. Therefore, before a real pairing mechanism being concluded from the specific heat data, we have to carefully investigate how the ferromagnetic spin fluctuations contribute to the specific heat of MgCNi₃.

In this work, we elaborate on the specific heat (*C*) of MgC_xNi₃, system both in normal state and superconducting state. A low temperature upturn is clearly distinguished in the C/T vs T^2 curves and found to be insensitive to the applied magnetic field. By doing some quantitative analysis, we present the evidence of most possible mechanisms responsible for this upturn. After subtracting this additional contribution, a well defined BCS-type electronic specific heat is extracted. The temperature dependence of the upper critical field and the field dependence of the low temperature specific heat also supports such conventional BCS superconductivity

in MgCNi₃. These analyses indicate that although the spin fluctuations may suppress the pairing strength in MgCNi₃, the superconductivity is certainly not induced by any exotic mechanism.

Polycrystalline samples of MgC_xNi₃ were prepared by the powder metallurgy method. Details of the preparation were published previously.¹⁷ The superconductor MgCNi₃ has a T_c of 6.7 K and the nonsuperconductor MgC_{0.855}Ni₃ was synthesized by continually reducing the carbon component until the diamagnetism was completely suppressed. The heat capacity data presented here were taken with the relaxation method¹⁸ based on an Oxford cryogenic system Maglab in which the magnetic field can be achieved up to 12 T. Details of the sample information and the measurements can be found in recent report.¹¹ It should be emphasized here that the Cernox thermometer used for calorimetry has been calibrated at 0, 1, 2, 4, 8, and 12 T, and the calibration for the intermediate fields is performed by an interpolation using the result of the adjacent fields. Therefore, any prominent field dependence of the specific data should reflect the intrinsic properties of the measured sample.

In general, the low temperature specific heat C(T, H) of a superconductor consists of four main contributions by neglecting the component of the nuclear moments,^{19,28} each has a different dependence on *T* and two of which depend on *H*, also in different ways

$$C(H,T) = C_{mag}(H,T) + C_{DOS}(H,T) + \gamma_0 T + C_{ph}(T), \quad (1)$$

where $\gamma_0 T$ represents a spare zero-field linear term and C_{ph} is due to the lattice or phonon contribution. The Debye phonon specific heat $C_{ph} = \beta T^3$ can usually describe the lattice contribution at low temperatures. However, the departure from T^3 law has often been observed, which is due to the fact that the density of modes of the phonon in real solid does not follow the assumed ω^2 law, here ω is the angular frequency of a harmonic wave associated with the lattice vibration. In such case, the deviations may be expanded in higher order terms such as T^5 , T^7 , etc. The *H*-dependent terms in Eq. (1), i.e., $C_{mag}(H)$ and $C_{DOS}(H)$, are the contributions associated with magnetism and the electronic density of states (DOS), respectively. If there is no magnetism associated contribution, the normal state specific heat at low temperature can be



FIG. 1. The low-temperature specific heat of $MgCNi_3$ at various magnetic fields from 0 to 12 T. The thick solid line denotes the universal background including all the normal state data for different fields.

approximatively described as $C_n(T) = \gamma_n T + \beta T^3$ in the framework of metal theory. Therefore, a linear relation can be obtained by plotting the normal state data as $C_n(T)/T$ vs T^2 , and its intercept and slope correspond to γ_n and β , respectively.

The low temperature specific heat at various magnetic fields up to 12 T is plotted as C(T)/T vs T^2 in Fig. 1. Two important features should be emphasized here. First, all the normal state data at various magnetic fields merge into one,¹¹ which is consistent with the results reported by other groups.^{14,15} Second, this common normal state background remarkably deviates from the linear relation as discussed earlier. In Fig. 2, only the normal state data are replotted in a magnified scale. In order to survey the normal behavior at very low temperature, the magnetic field up to 12 T was applied in measurements, which exceeds the highest upper



FIG. 2. A plot of C/T vs T^2 for MgCNi₃ and MgC_{0.85}Ni₃ in different magnetic fields. The data of MgC_{0.85}Ni₃ can be well defined by a straight line while that of MgCNi₃ remarkably deviate the linearity. The dashed line is a linear extrapolation of the high temperature data of MgCNi₃ and the upper solid line is the theoretical fit considering higher order phonon contributions.



FIG. 3. Comparison between the field-insensitive low-temperature upturn in the specific heat of MgCNi₃ ($\Delta C = C - \gamma T - \beta T^3$) and the calculated field dependent Schottky anomaly.

critical field of our sample and is 4 T higher than that used by other groups.^{14,15} The specific heat of the nonsuperconductor MgC_{0.85}Ni₃ is also presented in Fig. 2 as a comparison. It is obvious that the perfect linear relation of C(T)/T vs T^2 is satisfied for MgC_{0.85}Ni₃, which is a striking contrast with the case of MgCNi₃.

The obvious upturn in the low temperature C(T)/T vs T^2 curves of MgCNi₃ cannot be associated with Ni impurites since the x-ray diffraction pattern shows no indication for Ni impurites.¹¹ To say the least, if there is still extreme small content of Ni impurites leading to the prominent low temperature upturn of C/T, the field dependence of its specific heat should also be obvious, which is clearly inconsistent with our experimental results. Moreover, if this upturn is due to the excess free Ni in MgCNi₃, it should also be observed in MgC_{0.85}Ni₃ because of the similar process of synthesizing these two samples. Quantitatively, taking the data from Refs. 20 and 21 yields for 10% of superfluous Ni an upturn which is at least two orders of magnitude smaller than the observed one. Therefore, the contribution of the excessive Ni can be neglected comparing with the whole specific heat. Furthermore, the possible Schottky anomaly is presented in Fig. 3, its field dependence is obviously too strong to compare with our experimental result (nearly field independent).

It is found that this upturn can be well fitted if the above mentioned T^5 term is considered (see the upper solid line in Fig. 2). In other words, the departure from the T^3 behavior may be due to the non-Debye phonon DOS, which is consistent with the notable difference of the Debye temperature between MgCNi₃ and MgC_{0.85}Ni₃.¹¹ If the electron-phonon coupling is indeed the origin of superconductivity in MgCNi₃, it is reasonable to associate the disappearance of superconductivity in MgC_{0.85}Ni₃ with the remarkable difference of its phonon DOS from that of MgCNi₃. However, some careful work is needed to understand such obvious difference of phonon structure between these two samples, since they have similar crystal lattices and chemical components.

Another possible explanation of the earlier mentioned low

temperature upturn is the existence of strong spin fluctuations due to the higher DOS at Fermi energy $[N(E_F)]$ of MgCNi₃ than that of MgC_{0.85}Ni₃,¹¹ consequently, the coupling between the electrons and spin fluctuations in MgCNi₃ should also be stronger. The ferromagnetic spin fluctuations have been demonstrated by NMR experiments.¹⁰ Doniach and Engelshberg²² and Berk and Schrieffer²³ showed that the absorption and re-emission of spin fluctuations renormalizes the electronic self-energy, leading to an enhanced effective mass at low temperatures. This effect manifests itself as a low-temperature enhancement of the electronic specific-heat coefficient, λ_{sf} , which depends on temperature as $T^2 \ln(T/T_{sf})$ (here T_{sf} is the characteristic spin-fluctuation temperature) at low temperature. Considering the presence of ferromagnetic spin fluctuations, the normal state specific heat of MgC_xNi₃ can be expressed as follows:

$$C_n(H=0,T) = A[1 + \lambda_{ph} + \lambda_{sf}(T)]T + \gamma_0 T + \beta T^3, \quad (2)$$

where βT^3 are the contributions of phonon excitations, $\lambda_{sf}T$ and $\lambda_{ph}T$ represent the contributions of effective mass renormalization due to the electron-spin fluctuation coupling and the electron-phonon coupling, respectively, and *A* is a constant correlated with $N(E_F)$. It can be seen from Eq. (2) that the deviation from the linear dependence of C(T)/T on T^2 is due to the temperature dependence of λ_{sf} . Moreover, Béal-Monod, Ma, and Fredkin²⁴ have estimated the shift $\delta C/T$ caused by an applied field *H* to be

$$\delta C/T \approx 0.1 \left(\frac{\mu H}{k_B T_{sf}}\right)^2 \frac{S}{\ln S},$$
 (3)

where S is Stoner factor. Equations (2) and (3) indicate that the possible magnetic field dependence of the normal state specific heat is completely determined by the spin fluctuations. For simplicity, Eq. (2) can be rewritten as $C_n(H=0,T) = \gamma_n(T)T + \beta T^3$, in which $\gamma_n(T) = A[1+\lambda_{ph} + \lambda_{sf}(T) + \gamma_0/A]$. Therefore, the $\gamma_n \sim T$ relation directly reflects the temperature dependence of λ_{sf} . In Fig. 4, we present the determined $\gamma_n(T)$ by selecting various β values. Fitting the $\gamma_n(T)$ relations to the formula of $A[1+BT^2 \ln(T/T_{sf})]$ yields T_{sf} varying from 13 to 16 K. By inserting the determined T_{sf} , calculated Stoner factor S^2 and the highest field value in our measurements into Eq. (3), we can estimate the shift $\delta C/T$ caused by the applied field to be less than 2%, which is in agreement with our experimental results. However, if this explanation is correct, we must understand the collapse of the entropy conservation around T_c caused by considering such additional electronic specific heat, as discussed later. Therefore, the specific-heat contribution of the spin fluctuations themselves may be another candidate responsible for the low temperature upturn in specific heat of MgCNi₃.

Despite the true mechanism of the low-temperature upturn of C/T, this additional specific heat contribution should be regarded as a part of the normal-state background of the superconducting specific heat below the upper critical field $H_{c2}(T)$. In earlier analysis to the specific heat data,^{8,11,12} this additional part of background has been neglected more or less below $H_{c2}(T)$. We point out here that neglecting this



FIG. 4. Electronic specific heat $\gamma_n(T)$ vs T of MgCNi₃ in the normal state (three different β values are selected in order to avoid artificial errors). The solid lines are theoretical fits to spin-fluctuation model. All the curves except the top two are shifted downwards for clarity.

additional contribution will lead to the collapse of the entropy conservation as reported in Refs. 8 and 15. This opinion is motivated by the subsequent analysis. As shown in Fig. 5(a), the normal state background (as shown in Fig. 2) has been subtracted from the zero-field specific heat data, the entropy difference $\Delta S(T) = \int_0^T dT (\Delta C/T)$ is presented in the inset of Fig. 5(a), here $\Delta C = C_{H=0} - C_n$. It is found that the



FIG. 5. Fitting the specific heat data $(C_{H=0}-C_n)$ measured by different groups (Refs. 11, 14, and 15) to BCS model. The experimental data are denoted by open circles and the fits to the α model are denoted by solid lines. The dotted lines are fits to the original BCS model. The inset in (a) is the entropy difference by integration of $\Delta C/T$ as presented in (a).

TABLE I. Fits to the BCS model for zero-field specific heat.

Groups	Δ (meV)	T_c (K)	$2\Delta/k_BT_c$
Shan et al. ^a	1.20	6.70	4.15
Wälte et al. ^b	1.20	6.80	4.10
Lin et al. ^c	1.15	6.45	4.14

^aSee Ref. 11.

^bSee Ref. 14.

^cSee Ref. 15.

entropy conservation is then well satisfied, indicating that the remainder is the contribution of superconducting state. Such analysis has also been applied on the data measured by Wälte *et al.*¹⁴ and Lin *et al.*,¹⁵ respectively, the entropy is also conserved and the low temperature anomaly as mentioned in Ref. 15 completely vanishes.

When the superconductivity in MgCNi₃ is investigated, the spin fluctuations cannot be neglected because it may compete with superconductivity^{5,11} or even lead to an exotic pairing mechanism other than the conventional s wave.² If the phonon-mediated pairing wins in competing with the spin fluctuations and, hence, the effect of the spin fluctuations only suppress the electron-phonon coupling (or pairing strength),¹¹ the so-called α model²⁵ based on the BCS theory should be a good choice to describe the measured thermodynamic parameters. Comparing with the original BCS model, the only adjustable parameter in this α model is the gap ratio $\Delta(0)/k_BT_c$. This model has been successfully applied to strong coupling systems such as Pb and Hg. Then we try to fit the superconducting part of the zero-field specific heat to the α model, the results are presented in Fig. 5. All the data can be well described by this revised BCS model with the best fitting parameters (i.e., Δ and T_c) listed in Table I. These fits yield a gap ratio $\Delta(0)/k_BT_c \approx 2.06$, corresponding to the maximum gap $\Delta(0) \approx 1.2$ meV which is in good agreement with our previous tunneling measurements.⁹ From the earlier discussions, we can conclude that the coexistence and competition of spin fluctuations and phonons does not change the phonon-mediated pairing mechanism of MgCNi₃.

In order to further verify this picture, we investigate the field dependence of the low temperature specific heat of MgCNi₃. It is known that the electronic specific heat in magnetic fields can be expressed by $C_{el}(T,H) = C_{el}(T,H=0)$ $+\gamma(H)T$. The magnetic field dependence of $\gamma(H)$ is associated with the form of the gap function of the superconductor. For example, in a superconductor with line nodes in the gap function, the quasiparticle DOS (N(E)) rises linearly with energy at the Fermi level in zero field, $N(E) \propto |E - E_F|$, which results in a contribution to the specific heat $C_{DOS} = \alpha T^{2.26}$ In the mixed state with the field higher than a certain value, the DOS near the Fermi surface becomes finite, therefore the quadratic term $C_{DOS} = \alpha T^2$ will disappear and be substituted by the excitations from both inside the vortex core and the delocalized excitations outside the core. For d-wave superconductors with line nodes in the gap function, Volovik et al.²⁷ pointed out that in the mixed state, supercurrents around a vortex core cause a Doppler shift of the quasiparticle excitation spectrum. This shift has important effects upon the low



FIG. 6. The magnetic field dependence of the specific heat coefficient $\gamma(H) - \gamma(0)$ at low temperatures.

energy excitation around the nodes, where its value is comparable to the width of the superconducting gap. For $H \gg H_{c1}$, it is predicted that $N(E_F) \propto H^{1/2}$ and $C_{DOS} = \Delta \gamma(H) T = ATH^{1/2}$ at low temperatures.²⁷ This prediction has been well proved for hole doped cuprates.²⁸ Whereas in a conventional s-wave superconductor, the specific heat in the vortex state is dominated by the contribution from the localized quasiparticles in the vortex cores. From the Bogoliubov equations assuming noninteracting vortices, the DOS associated with the bound excitations is derived as $N(E) \propto B(H)$ (Ref. 29), hence, the contribution of the vortex cores to the specific heat is $C_{DOS} \propto B(H)T$.^{30,31} It is also theoretically derived that the experimentally observed downward curving C(H) is caused by the flux line interactions near H_{c1} and the possible expansion of the vortex cores.^{30,31} According to the earlier discussions, the specific heat coefficient $\gamma(H)$ of conventional s-wave superconductor should linearly depend on the magnetic field well above H_{c1} .

Figure 6 shows the field dependence of $\gamma(H) - \gamma(0)$ of MgCNi₃ below 3 K. The data reported by different groups^{11,14,15} merge into each other by timing a prefactor A close to unity. It is found that γ linearly depends on H above 0.5 T and persists up to 8 T which is close to the upper critical field of MgCNi₃. The legible linearity of $\Delta \gamma \sim H$ relation at higher field and its negative curvature below 0.5 T are in good agreement with the earlier mentioned behaviors of conventional s-wave superconductors. It may be argued that the low temperature limit of about 2 K in our measurements is not low enough to distinguish the d wave's $\Delta \gamma \sim H^{1/2}$ law. However, it should be emphasized that the observed $\Delta \gamma \sim H$ relation is nearly universal at low temperatures below upper critical field, which is very similar to the behavior of V₃Si,³¹ a typical conventional s-wave superconductor.

Finally, we compare the temperature dependence of the upper critical field $H_{c2}(T)$ with the prediction of BCS theory in which the $H_{c2}(T)$ can be expressed as follows:

$$H_{c2}(T) \approx 1.74 H_c(0) (1 - T/T_c) \quad (T_c - T \ll T_c),$$
 (4a)



FIG. 7. The comparison of the temperature dependence of the upper critical field $H_{c2}(T)$ with the BCS-like descriptions in lower and higher temperature limits.

$$H_{c2}(T) \approx H_{c2}(0)[1 - 1.06(T/T_c)^2]$$
 (at low T). (4b)

As shown in Fig. 7, the best fitting to the BCS model is denoted by solid lines. At lower temperature, the experimental data can be well described by Eq. (4b). For the higher temperature near T_c , a prefactor of 1.65 is obtained instead of

the theoretical prediction of 1.74 as expressed in Eq. (4a). Nonetheless, the BCS model is still a preferred description for $H_{c2}(T)$ of MgCNi₃ considering the stronger electronphonon coupling and the presence of ferromagnetic spin fluctuations.

In summary, we have investigated the specific heat data of MgC_xNi_3 system. A remarkable field independent contribution is found in MgCNi₃, reflecting the departure of normalstate specific heat from the T^3 law. By removing this contribution, the zero-field data is well described by the α model (a slightly revised BCS model). The conventional *s*-wave superconductivity is further supported by the linear field dependence of specific heat coefficient $\gamma(H)$ and the BCS-like temperature dependence of upper critical field $H_{c2}(T)$. It is then concluded that, although electron-magnon (spin fluctuations) coupling coexists and competes with electron-phonon coupling effect in MgCNi₃, it only acts as pair breakers while does not induce an exotic superconductivity.

Note added: Most recently, the carbon isotope effect in superconducting $MgCNi_3$ observed by T. Klimczuk and R. J. Cava indicates that carbon-based phonons play a critical role in the presence of superconductivity in this compound.³²

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