Low-energy quasiparticle states at superconductor/charge-density-wave interfaces

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Quasiparticle bound states are found theoretically on transparent interfaces of *d*-wave superconductors with charge-density-wave (CDW) solids, as well as *s*-wave superconductors with *d*-density waves (DDWs). These bound states represent a combined effect of Andreev reflection from the superconducting side, unconventional quasiparticle *Q* reflection from the density-wave solid, and standard specular reflection. If the order parameter for a density-wave state is much less than the Fermi energy, bound states with almost zero energy take place for an arbitrary orientation of symmetric interfaces. For larger values of the order parameter, dispersionless zero-energy states are found only on (110) interfaces. Two dispersive energy branches of subgap quasiparticle states are obtained for (100) symmetric interfaces. Andreev low-energy bound states, taking place in junctions with CDW or DDW interlayers, result in anomalous junction properties, in particular, the low-temperature behavior of the Josephson critical current.

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I. INTRODUCTION

Low-energy quasiparticle states play an important role in forming electron transport in mesoscopic hybrid superconducting systems at low temperatures. In transparent superconductor–normal-metal–superconductor (S-N-S) junctions, subgap states originate entirely in Andreev reflection processes. In the presence of finite interface transparencies, both Andreev and conventional reflections come into play in forming subgap bound states. Zero-energy Andreev surface states in *d*-wave superconductors also represent a combined effect of Andreev and specular quasiparticle reflections.

Interesting possibilities for forming low-energy subgap states arise in hybrid systems involving gapped solids with various electronic ordering like charge-density waves (CDWs) or itinerant antiferromagnets (AFs). In the absence of any potential barriers and/or a Fermi velocity mismatch, the standard specular reflection from a plane interface vanishes. However, normal-metal quasiparticles moving with subgap energies towards the gapped phase will be reflected from the transparent interface. If the gap in the quasiparticle spectrum originates in the electronic ordering, a nonspecular quasiparticle reflection on various plane crystal interfaces can arise in accordance with the order-parameter structure. Andreev quasiparticle retroreflection on transparent S-N interfaces is a remarkable and well-known effect of this kind, but it is not the only one. Unconventional quasiparticle reflection resulting in low-energy quasiparticle bound states arises, for example, at CDW-N interfaces,[1–3] as well as on AF-N interfaces.[4]

Normal-metal subgap quasiparticles change their momenta by the wave vector $\boldsymbol{Q}$ of the charge-density-wave pattern, in an unconventional reflection process on interfaces with the gapped CDW phase.[1] Since the nesting condition $\varepsilon_f(\boldsymbol{k}_f+\boldsymbol{Q})=-\varepsilon_f(\boldsymbol{k}_f)$ is presumably satisfied in the CDW solid, at least with the quasiclassical accuracy, the velocity $\partial\varepsilon_f/\partial\boldsymbol{k}$ changes its sign in a *Q-reflection* event. Then *Q* reflection represents a retroreflection of quasiparticles. A neutral electron-hole pair with the transferred momentum $\boldsymbol{Q}$ arises in the *Q*-reflection event and forms the condensate in the CDW solid. No electric current appears in this process, while in Andreev reflection an incoming electron is reflected as a hole and a Cooper pair carries the electric charge 2*e* into the bulk of the superconductor. On the other hand, the retroreflection leads to a possibility for forming quasiparticle bound states in CDW-N-CDW systems with the same spectrum as for Andreev bound states in S-N-S structures.[2,3] *Q* reflection contributes also to the conductance of N-CDW-N junctions.[5] The excess resistance in CDW-N junctions at low voltages has been observed experimentally and attributed to *Q* reflection processes, when the incident electron returns along its original path with its charge unchanged.[6] As this has been demonstrated recently in Ref. 4, normal-metal quasiparticles experience a spin-dependent *Q* reflection from interfaces with itinerant antiferromagnets. Quasiparticle subgap states located near interfaces with AFs, have been found, in particular, near S-AF interfaces.

In the present paper we determine subgap states representing a combined effect of Andreev and *Q* reflections on transparent interfaces between a semiconductor with charge-density waves and a superconductor (CDW-S). For simplicity, we consider symmetric interfaces having identical crystal orientations on both sides. All phases are assumed to be (quasi-) two-dimensional, taking place on a square lattice. In particular, we study below solids with a two-dimensional CDW ordering, as well as with a *d*-density-wave phase, which has been suggested regarding the pseudogap state in cuprates (see Refs. 7–11 and references therein). We demonstrate that quasiparticle subgap states arise on *d*-wave superconductor (*d*SC)-CDW and *s*-wave superconductor *d*-density-wave (*s*SC-DDW) (and do not appear on *s*SC-CDW and *d*SC-DDW) interfaces. We discuss an interface between a low-temperature *s*-wave superconductor and a *d*-density-wave phase implying a low doping range in cuprates. If the order parameter for a density-wave state is much less than the Fermi energy, the quasiclassical theory can be applied to describing the state. Within this framework, zero-energy bound states take place for an arbitrary interface orientation. For larger values of the order parameter, the

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S-matrix approach is developed to solving the problem in question. Then dispersionless zero-energy states are found only on (110) interfaces. Two dispersive energy branches of subgap quasiparticle states are obtained for (100) interfaces. Andreev low-energy bound states, taking place in junctions with CDW or DDW interlayers, result in anomalous junction properties, in particular, the anomalous low-temperature behavior of the Josephson critical current.

II. CDW-S INTERFACES

We consider a tight-binding model for electrons with a superconducting Δ^{ij} and a density-wave W^{ij} order parameter on a square lattice,

$$H = - t\sum_{\langle ij\rangle\sigma} c_{i\sigma}^{\dagger}c_{j\sigma} + \sum_{i,j} (\Delta^{ij}c_{i\uparrow}^{\dagger}c_{j\downarrow}^{\dagger} + \text{H.c.}) + \sum_{\langle ij\rangle\sigma} W^{ij}c_{i\sigma}^{\dagger}c_{j\sigma} - \mu\sum_{i} c_{i\sigma}^{\dagger}c_{i\sigma}. \quad (1)$$

Assume nearest-neighbor hopping, and consider either s-wave pairing $\Delta^{ij}=-V_s\langle c_{i\downarrow}c_{i\uparrow}\rangle\delta_{ij}=\delta_{ij}\Delta_s^i$ or d-wave pairing $\Delta^{ij}=-V_d\langle c_{i\downarrow}c_{j\uparrow}\rangle=\Delta_d^{ij}\delta_{|i-j|,1}$ such that $\Delta_d^{ii\pm\hat{a}}=-\Delta_d^{ii\pm\hat{b}}$. Here $\hat{a}$ and $\hat{b}$ are basis vectors for the square lattice with the lattice constant a. The order parameter for a two-dimensional CDW is taken in the form $W^{ij}=(-1)^{i_a+i_b}W_s^i\delta_{ij}=-(V^{CDW}/2)\langle n_{i\uparrow}+n_{i\downarrow}\rangle\delta_{ij}$, whereas for a d-density wave $W^{ij}=i(-1)^{i_a+i_b}W_d^{ij}\delta_{|i-j|,1}=(V^{DDW}/2)\Sigma_\sigma\langle c_{i\sigma}^{\dagger}c_{j\sigma}-\text{H.c.}\rangle$ and $W_d^{ii\pm\hat{a}}=-W_d^{ii\pm\hat{b}}$. Thus we study only the simplest model for pinned two-dimensional charge-density waves with the characteristic wave vector $\boldsymbol{Q}=(\pi,\pi)$ on a square lattice. Although realistic two-dimensional CDW ordering usually takes place in more complicated situations,[12–14] the main conclusions of the present paper can be qualitatively applicable to them as well. Thus if the nesting condition is valid only on a part of the Fermi surface, just respective electrons will participate in the density-wave ordering and the Q reflection will take place for respective regions of momentum directions. Further, the quasiclassical superconducting d-wave order parameter $\Delta_d(\boldsymbol{k}_f,x_i)=2\Delta_d^{ii+\hat{a}}[\cos(k_{fa}a)-\cos(k_{fb}a)]$, taken for incoming $\boldsymbol{k}_f$ and outgoing $\boldsymbol{k}_f+\boldsymbol{Q}$ momenta in a Q-reflection event, would have opposite signs for a wide range of possible wave vectors $\boldsymbol{Q}$, not only for the particular value (π,π).

In describing plane interfaces, it is convenient to work in a coordinate system where axes x and y are chosen perpendicular and parallel to the interface, respectively. For a (100) interface x and y coincide with the crystal axes. Then the normal-state electron band $\xi(\boldsymbol{k})=-\mu-2t(\cos k_a+\cos k_b)$ and the respective Brillouin zone is spanned by $k_{a,b}\in[-\pi,\pi]$, where momenta are given in units of a^{-1}. For a (110) interface we have $\xi(\boldsymbol{k})=-\mu-4t\cos(k_x/\sqrt{2})\cos(k_y/\sqrt{2})$ and $k_x\in[-\sqrt{2}\pi,\sqrt{2}\pi]$, $k_y\in[-\pi/\sqrt{2},\pi/\sqrt{2}]$, on account of the periodic conditions along the surface.

A density-wave order parameter W is taken to be nonzero only on one semi-infinite half space $x<0$, while Δ may be nonzero on the other. For simplicity, no interface potential barrier is introduced in the problem and we consider only identical crystal-to-interface orientations of both half spaces, as if they formed one and the same square lattice. A deviation from half filling will be assumed, first, to be equal to zero ($\mu=0$) or negligibly small everywhere. This guarantees the validity of the nesting condition $\varepsilon_f(\boldsymbol{k}_f+\boldsymbol{Q})=-\varepsilon_f(\boldsymbol{k}_f)$ in the normal-metal state of both solids. We assume always that the superconducting order parameter is much less than the Fermi energy $\Delta\ll\varepsilon_f$, so that the quasiclassical theory of superconductivity applies to the problem in question. If a density-wave order parameter W is sufficiently large $W\gg\Delta$, then the S-matrix approach can be applied to describing the interface Andreev bound states at CDW-S and DDW-S boundaries. There is no need to consider the parameter W/ε_f to be small within the S-matrix approach. Interface states can appear, since quasiparticles with energies below the CDW or DDW gap do not penetrate in the bulk of solids with density waves. At the same time, Andreev reflection does not permit subgap quasiparticles to enter into the bulk of the superconductor.

Quasiparticles in the superconducting half space can be described in terms of standard Andreev equations for Andreev amplitudes $\tilde{\psi}^T(x,\boldsymbol{k}_f)\equiv(u(x,\boldsymbol{k}_f),v(x,\boldsymbol{k}_f))$

$$[-iv_{f,x}\hat{\tau}_3+\hat{\Delta}(\boldsymbol{k}_f,x)]\tilde{\psi}(x,\boldsymbol{k}_f)=\varepsilon\tilde{\psi}(x,\boldsymbol{k}_f), \quad (2)$$

complemented with the suitable boundary conditions at CDW-S or DDW-S interfaces. Here $v_{f,x}=[\partial\xi(\boldsymbol{k})/\partial k_x]|_{\boldsymbol{k}=\boldsymbol{k}_f}$ is the x component of the electron normal-state Fermi velocity, $\hat{\tau}_\alpha$ are Pauli matrices in particle-hole space. The superconducting order-parameter matrix $\hat{\Delta}(\boldsymbol{k}_f,x)$ represents both s-wave or d-wave order parameters, which are defined as $\hat{\Delta}_s(x)=\Delta_s(x)\hat{\tau}_+/2+\Delta_s^*(x)\hat{\tau}_-/2$, $\hat{\Delta}_d(\boldsymbol{k}_f,x)=\Delta_d(\boldsymbol{k}_f,x)\hat{\tau}_+/2+\Delta_d^*(\boldsymbol{k}_f,x)\hat{\tau}_-/2$. Continuous coordinate x in quasiclassical equations (2) originated from the x components of site positions: $x_j=jd$. Here $d=a,a/\sqrt{2}$ for (100) and (110) interfaces, respectively.

Boundary conditions for Andreev equations (2) can be obtained by solving the scattering problem for quasiparticles on the Fermi surface moving from the bulk of the normal metal towards CDW-N or DDW-N interfaces. For this purpose one can put $\Delta_{s,d}=0$ in the Bogoliubov–de Gennes equations, which follow from the Hamiltonian (1), and solve respective interface problems. The difference $\boldsymbol{Q}$ between the outgoing $\tilde{\boldsymbol{k}}_f$ and the incoming $\boldsymbol{k}_f$ momenta takes place for a quasiparticle Q reflection both from the CDW or DDW phase. The wave vector of the density wave on the square lattice is $\boldsymbol{Q}=(\pi,\pi)$, with respect to the crystal axes. In the x,y-coordinate system, $\boldsymbol{Q}=(\pi,\pi)$ for (100) interface and $\boldsymbol{Q}=(\sqrt{2}\pi,0)$ in the (110) case. So, a quasiparticle going towards the (100) boundary of an electronically ordered phase can change its parallel to the interface momentum component k_y by $Q_y=\pi$ in the Q-reflection process or keep k_y unchanged in the process of specular reflection. For (110) boundary, by contrast, parallel to the interface component k_y does not change both in Q and specular reflections. For this reason the boundary conditions for (100) and (110) interfaces differ from each other.

For the (110) interface the boundary conditions take the form

$$\tilde{\psi}^{out}(0,k_y) = \left(r^e(k_y)\frac{1+\hat{\tau}_3}{2} + r^h(k_y)\frac{1-\hat{\tau}_3}{2} \right)\tilde{\psi}^{in}(0,k_y). \quad (3)$$

Andreev amplitudes $\tilde{\psi}^{in}(x,\boldsymbol{k}_f)$ involve solutions with $v_{f,x} < 0$, in contrast with $\tilde{\psi}^{out}(x,\boldsymbol{k}_f)$.

Reflection amplitudes for electrons r^e and holes r^h, which enter the quasiclassical boundary conditions, are taken for the normal-state phase of the superconducting region. At the CDW-N interface we find the following relation between electron and hole amplitudes: $r^h_{CDW} = r^e_{CDW}$. This differs from the relation, which takes place at the DDW-N interface, $r^h_{DDW} = -r^e_{DDW}$. The latter equality is a consequence of specific time-reversal symmetry breaking in the DDW phase.[7] Solving standard Andreev equations (2) for *s*-wave and *d*-wave superconductors with boundary conditions (3) on the (110) interface, we find simple results for the subgap spectrum of quasiparticle interface bound states. There are only zero-energy quasiparticle bound states at CDW-*d*SC and DDW-*s*SC interfaces. At the same time, there are no subgap states at all at CDW-*s*SC and DDW-*d*SC (110) interfaces. Andreev amplitudes related to the bound states decay exponentially in the bulk of the CDW (DDW) and superconducting half spaces on the scale of characteristic coherence lengths $\xi_{CDW,DDW} \sim v_{f,x}/W_{s,d}$ and $\xi_{s,d} \sim v_{f,x}/\Delta_{s,d}$, respectively.

The above results can be qualitatively understood as follows. There are no subgap states at CDW-*s*SC interfaces, due to the absence of interface-induced pair-breaking processes in this case. Since at (110) interface Q reflection is quite analogous to specular one, zero-energy bound states arise at (110) CDW-*d*SC interfaces for the same reason as well-known zero-energy states at an impenetrable (110) surface of a *d*-wave superconductor. Indeed, due to the energy gap in the CDW solid, CDW-*d*SC interface is impenetrable for low-energy quasiparticles even in the absence of any interface potential barriers. The analogy with an impenetrable (110) surface of a *d*-wave superconductor works, in a more complicated way, also for the zero-energy bound states at (110) DDW-*s*SC interfaces. This is a pair-breaking interface, due to a time-reversal symmetry breaking in the DDW solid. An important role in forming subgap states on SC-DDW interfaces plays the difference π between phases Θ_e and Θ_h of reflection amplitudes $r^{e(h)}_{DDW}$ for electrons and holes $r^{e(h)}_{DDW} = e^{i\Theta_{e(h)}}$. The phase difference $\Theta_e - \Theta_h$ can be effectively ascribed to the variation of the phase of the superconducting order parameter in a reflection event. In order to see this, one can introduce auxiliary quantities $\tilde{u}(x,\tilde{\boldsymbol{k}}_f,\varepsilon) = u(x,\tilde{\boldsymbol{k}}_f,\varepsilon)e^{-i\Theta_e/2}, \tilde{v}(x,\tilde{\boldsymbol{k}}_f,\varepsilon) = v(x,\tilde{\boldsymbol{k}}_f,\varepsilon)e^{-i\Theta_h/2}$ into Andreev equations and boundary conditions, taken for the outgoing momentum $\tilde{\boldsymbol{k}}_f$. Andreev amplitudes for incoming momentum $\boldsymbol{k}_f$ are kept unchanged. Then the problem becomes, formally, identical to the one for specularly reflecting impenetrable boundary and the effective order parameter for the outgoing momenta $\Delta_{eff}(\tilde{\boldsymbol{k}}_f,x) = e^{-i(\Theta_e-\Theta_h)}\Delta(\tilde{\boldsymbol{k}}_f,x)$. In general, if only a phase difference Θ takes place between the order parameters for incoming and outgoing quasiparticles, Andreev bound states will appear with the energy $|\varepsilon| = |\Delta\cos(\Theta/2)|$. Since the *s*-wave order parameter itself does not change its sign in a reflection process, an outgoing quasiparticle sees an effective superconducting order parameter with an additional phase $\Theta_e - \Theta_h = \pi$ as compared with the phase on the incoming trajectory. This directly results in the zero-energy interface states at (110) DDW-*s*SC interfaces. However, in the case of (110) DDW-*d*SC interface, there is also a sign change of the *d*-wave order parameter in a reflection event: $\Delta_d(\tilde{\boldsymbol{k}}_f,x) = -\Delta_d(\boldsymbol{k}_f,x)$. As a whole, an outgoing quasiparticle sees an effective superconducting order parameter with an extra phase $\pi - (\Theta_e - \Theta_h)$ as compared with the phase on the incoming trajectory. Since $\Theta_e - \Theta_h = \pi$, the total phase variation of the effective order parameter in a reflection event vanishes. For this reason there are no pair-breaking processes at transparent (110) DDW-*d*SC interface and no interface bound states there. We note that the *d*-wave order parameter changes its sign in a Q-reflection event for any interface-to-crystal orientation: $\Delta_d(\boldsymbol{k}_f + \boldsymbol{Q},x) = -\Delta_d(\boldsymbol{k}_f,x)$. Thus there are no subgap states on a transparent DDW-*d*SC interface with an arbitrary orientation, if only Q reflection takes place there.

Consider now (100) interfaces, for which boundary conditions can be written as follows:

$$\begin{pmatrix} \tilde{\psi}^{out}(0,k_y) \\ \tilde{\psi}^{out}(0,k_y+Q_y) \end{pmatrix} = \check{S} \begin{pmatrix} \tilde{\psi}^{in}(0,k_y) \\ \tilde{\psi}^{in}(0,k_y+Q_y) \end{pmatrix}. \quad (4)$$

The S matrix for a CDW-N or a DDW-N boundary takes the form $\check{S} = \hat{S}^e[(1+\hat{\tau}_3)/2] + \hat{S}^h[(1-\hat{\tau}_3)/2]$, where

$$\hat{S}^{e,h} = \begin{pmatrix} r^{e,h}_{k_y,k_y} & r^{e,h}_{k_y+Q_y,k_y} \\ r^{e,h}_{k_y,k_y+Q_y} & r^{e,h}_{k_y+Q_y,k_y+Q_y} \end{pmatrix}. \quad (5)$$

Since $Q_y \neq 0$ for (100) interface, Q and specular reflections represent physically different reflection channels. Reflection amplitudes depend now on two parallel to the surface momentum components of incoming and outgoing quasiparticles, respectively. The momentum components coincide with each other for specular reflection and differ by Q_y for Q reflection. One can show that $\hat{S}^h_{CDW} = \hat{S}^e_{CDW}$ for the CDW-N interface and $\hat{S}^h_{DDW} = \hat{\rho}_3\hat{S}^e_{DDW}\hat{\rho}_3$ for the DDW-N interface. We define Pauli matrices ρ_α in space of two quasiparticle trajectories $(\boldsymbol{k},\boldsymbol{k}+\boldsymbol{Q})$. The S matrix satisfies the unitarity condition $\check{S}\check{S}^\dagger = 1$, which follows from the conservation of the probability current for each of independent quasiparticle solutions. The unitarity of the S matrix leads, in particular, to the following equations: $|r^{e,h}_{k_y,k_y}|^2 = |r^{e,h}_{k_y+Q_y,k_y+Q_y}|^2 = R^{sp}(k_y)$, $|r^{e,h}_{k_y+Q_y,k_y}|^2 = |r^{e,h}_{k_y,k_y+Q_y}|^2 = R^Q(k_y)$. Here R^Q and R^{sp} are reflection coefficients for Q and specular reflections, respectively. For subgap quasiparticles $R^{sp}(k_y) + R^Q(k_y) = 1$.

Solving Andreev equations for *s*-wave and *d*-wave superconductors with boundary conditions (5) on the (100) interface, we find the following results. There are no bound states at CDW-*s*SC and DDW-*d*SC interfaces, analogously to the case of the (110) interface. However, on CDW-*d*SC and DDW-*s*SC interfaces there are two dispersive energy branches of quasiparticle Andreev bound states, which are symmetric with respect to the zero level:

$$\varepsilon_{CDW-dSC}(\boldsymbol{k}_f) = \pm|\Delta_d(\boldsymbol{k}_f)|\sqrt{R^{sp}_{CDW-N}(\boldsymbol{k}_f)}, \quad (6a)$$

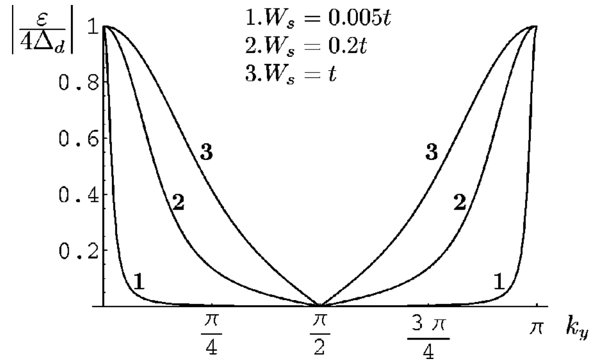

FIG. 1. The dispersive bound-state energy $|\varepsilon_{CDW-dSC}(\boldsymbol{k}_f)|$, normalized to the maximal value $4\Delta_d$ of the d-wave order parameter, vs k_y at (100) CDW-dSC interface. Curves are taken for three values of the CDW order parameter: 1. $W_s=0.005t$; 2. $W_s=0.2t$; 3. $W_s=t$.

$$\varepsilon_{DDW-sSC}(\boldsymbol{k}_f) = \pm|\Delta_s|\sqrt{R^{sp}_{DDW-N}(\boldsymbol{k}_f)}. \tag{6b}$$

Here we assume spatially constant order parameters, so that $\Delta_d^{ii+\hat{a}}=\Delta_d, \Delta_s^i=\Delta_s, W_d^{ii+\hat{a}}=W_d, W_s^i=W_s$ coincide with their bulk values. Quantities $|\varepsilon_{CDW-dSC}(\boldsymbol{k}_f)|$ and $|\varepsilon_{DDW-sSC}(\boldsymbol{k}_f)|$, taken for various values of W_d and W_s, are shown in Figs. 1 and 2, respectively, as functions of parallel to (100) interface quasiparticle momentum component.

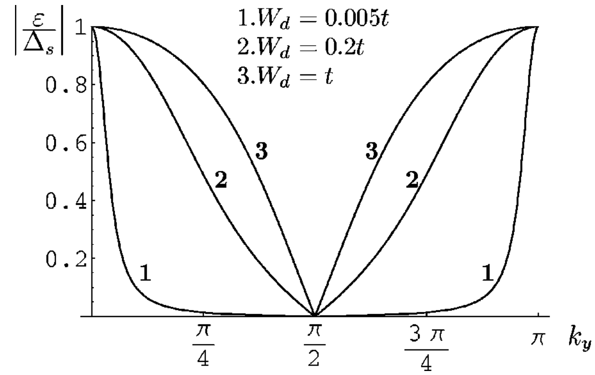

FIG. 2. The dispersive bound-state energy $|\varepsilon_{DDW-sSC}(\boldsymbol{k}_f)|$, normalized to the s-wave order parameter, vs k_y at (100) DDW-sSC interface. Curves are taken for three values of the DDW order parameter: 1. $W_d=0.005t$; 2. $W_d=0.2t$; 3. $W_d=t$.

We do not present explicit analytical expressions for the reflection coefficient $R^{sp}_{DDW-N}(\boldsymbol{k}_f)$ and the dispersive subgap energies $\varepsilon_{DDW-sSC}(\boldsymbol{k}_f)$ at (100) interfaces, since they are too cumbersome. The explicit expression for the energy subgap spectrum at (100) CDW-dSC interface takes the form

$$\varepsilon_{CDW-dSC}(\boldsymbol{k}_f) = \pm|\Delta_d(\boldsymbol{k}_f)| \times \left(\frac{A(k_y) + \sqrt{A^2(k_y) + 4\left(\frac{W_s}{2t}\right)^2}}{A(k_y) + 2\sin^2 k_y + \sqrt{A^2(k_y) + 4\left(\frac{W_s}{2t}\right)^2}} \right)^{1/2},$$

where

$$A(k_y) = \left(\frac{W_s}{2t}\right)^2 - \sin^2 k_y. \tag{7}$$

As is seen in Figs. 1 and 2, as well as from Eq. (7), the coefficient of specular reflection from (100) interface in the absence of potential barriers can become significant only if the dimensionless parameter $W_{s,d}/t$ is of the order of unity. The appearance of specular reflection modifies effects of Q reflection and leads to a splitting of zero-energy interface bound states. The larger the parameter $W_{s,d}/t$, the higher the absolute value of the bound-state energy. This effect is not present at (110) interfaces, since Q and specular reflections are physically indistinguishable there, unless an interface potential barrier and/or a Fermi velocity mismatch result in a finite phase difference $0<|\Theta_e-\Theta_h|<\pi$. The barrier and the mismatch open a channel of specular reflection. This results in splitting of the zero-energy bound states at (100) CDW-insulator (I)-dSC and DDW-I-sSC interfaces. The bound-state energies reach the edge of the continuous spectrum in the limit of impenetrable insulating interlayer. The same effect takes place at DDW-I-sSC (110) interfaces. By contrast, bound states at (110) CDW-dSC interfaces keep their zero energy even in the presence of any interface potential barriers and/or a mismatch of Fermi velocities.

If parameters $W_{s,d}/t$ are sufficiently small as compared with unity, Andreev bound states have very low energies almost in the whole range of k_y, except for narrow vicinities of $k_y=0,\pi$. The condition $W_{s,d}/t \ll 1$ allows us to apply the quasiclassical approach to describing the density-wave phases. Similarly, the condition $\Delta \ll t$ is necessary for justifying the applicability of the quasiclassical theory to superconductors. Then the characteristic lengths of the phases significantly exceed the lattice spacing $\xi_{s,d} \equiv \hbar v_F/\Delta_{s,d} \gg a, \hbar v_F/W_{s,d} \gg a$, so that the density-wave amplitudes W_s^i, W_d^{ij} are also slowly varying functions as compared with the atomic scale a. Below we represent a joint quasiclassical approach to the superconducting and the density-wave phases, as well as respective results on subgap spectra.

A specific feature of the density-wave phases, which is important in the derivation of quasiclassical equations, is associated with a rapidly oscillating order parameter $W^{ji} \propto (-1)^{j_a+j_b} = \exp(i\boldsymbol{Qj})$ in the coordinate space. This prevents one from using a standard quasiclassical approach, analogously to the case of itinerant antiferromagnets considered in this respect in Ref. 4. Since $2\boldsymbol{Q}$ coincides with a basis vector of the reciprocal lattice of the crystal in the absence of the density waves, the quasiclassical equations can be written for pairs of coupled quasiparticle trajectories $\boldsymbol{k}$ and $\boldsymbol{k}+\boldsymbol{Q}$. This contrasts with the standard equations (2), which are written separately for each quasiparticle trajectory. The quasiclassical theory, modified along this way, allows arbitrary relation between $W_{s,d}$ and $\Delta_{s,d}$, taking into account all terms of the first order in parameters $W_{s,d}/t, \Delta_{s,d}/t$. Since the gap in the energy spectrum of electrons and holes in the CDW (DDW) phases takes place only for $|\mu| < W_s[W_d(\boldsymbol{k}_f)]$, we assume that the deviation from half filling in the CDW (DDW) solid can be finite, but not large $\mu \ll \varepsilon_f$, so that the nesting condition $\varepsilon_f(\boldsymbol{k}_f+\boldsymbol{Q})=-\varepsilon_f(\boldsymbol{k}_f)$ holds in the system within the quasiclassical accuracy. A small parameter $(\mu/\varepsilon_f) \ll 1$ can be considered on the same footing as other small quasiclassical parameters like $\Delta_{s,d}/\varepsilon_f$ and $W_{s,d}/\varepsilon_f$. Then μ should be included in the quasiclassical equations and hence into slowly varying Andreev amplitudes. In this case the Fermi momenta in rapidly oscillating exponentials $\exp(\pm i\boldsymbol{k}_f\boldsymbol{r})$, which enters the relation between Bogoliubov and Andreev amplitudes, should be taken at $\mu=0$.

It is now convenient to collect into a Nambu four-spinor the Andreev amplitudes $\psi_j^T \equiv (u_j(\boldsymbol{k}_f), u_j(\boldsymbol{k}_f+\boldsymbol{Q}), v_j(\boldsymbol{k}_f), v_j(\boldsymbol{k}_f+\boldsymbol{Q}))$. Then the Andreev equations take the form

$$\left(-\mu\tau_3\rho_0 - i\tau_3\rho_3 v_{f,x}\frac{\partial}{\partial x} + \check{W}(x) + \check{\Delta}(x)\right)\psi(x) = \varepsilon\psi(x). \quad (8)$$

Here $v_{f,x}$ is the Fermi velocity at half filling, $\check{\Delta}(x) = \check{\Delta}_s(x) + \check{\Delta}_d(\boldsymbol{k}_f, x)$, $\check{\Delta}_s(x) = \rho_0\Delta_s(x)(\tau_+/2) + \rho_0\Delta_s^*(x)(\tau_-/2)$, $\check{\Delta}_d(\boldsymbol{k}_f,x) = \Delta_d(\boldsymbol{k}_f,x)\rho_3(\tau_+/2) + \Delta_d^*(\boldsymbol{k}_f,x)\rho_3(\tau_-/2)$. $\check{W}(x) = \check{W}_s(x) + i\check{W}_d(\boldsymbol{k}_f,x)$, $\check{W}_s(x) = W_s(x)\rho_1\tau_3$, $\check{W}_d(\boldsymbol{k}_f,x) = W_d(\boldsymbol{k}_f,x)i\rho_2\tau_0$. DDW gap function $W_d(\boldsymbol{k}_f,x) = 2W_d^{ii+\hat{a}}[\cos k_{fa} - \cos k_{fb}]$ has the same form as the d-wave superconducting order parameter.

We solve Eqs. (8) for superconducting and CDW regions and match the solutions at a transparent interface at $x=0$. As one could expect for an interface with no pair breaking, we find no quasiparticle interface states for a CDW-sSC interface with an arbitrary orientation. At the same time, zero-energy bound states arise on transparent CDW-dSC interfaces for any surface-to-crystal orientations, since the d-wave superconducting order parameter $\Delta_d(\boldsymbol{k}_f) = 2\Delta_d^{ii+\hat{a}}[\cos k_{fa} - \cos k_{fb}]$ always has opposite signs for momenta $\tilde{\boldsymbol{k}}_f = \boldsymbol{k}_f + \boldsymbol{Q}$ and $\boldsymbol{k}_f$. This differs from a specular reflecting impenetrable surface where a fraction of momentum directions, for which the d-wave order parameter changes its sign in a reflection event, strongly depends on a surface orientation.

The quasiclassical energy spectra exactly coincide with more general results, obtained above for (110) interfaces, and represent a good approximation for (100) interfaces under the conditions $W_{s,d} \ll t$, $\Delta_{s,d} \ll t$. One can see in Figs. 1 and 2, that even for $W_{s,d} \ll t$, $\Delta_{s,d} \ll t$ the quasiclassical approximation fails in narrow vicinities of $k_y = 0, \pi$. This agrees with the fact that quasiclassical Eqs. (8) do not apply in vicinities of quasiparticle momenta where $v_{f,x}=0$. In particular, they do not apply near saddle points of quasiparticle energies where van Hove singularities of the normal-metal density of states take place. As it is seen from the expression (7) for dispersive bound-state energies, as well as Figs. 1 and 2, quasiparticle interface states reach the edge of the continuous spectrum (the superconducting gap) for momentum directions where $v_{f,x}=0$.[15] This looks natural, since reflection coefficients $R^{sp}_{CDW-N}(\boldsymbol{k}_f), R^{sp}_{DDW-N}(\boldsymbol{k}_f)$ equal unity for these momentum directions, while the Q reflection disappears. Since we will be interested mostly in a transport across the interface, where the additional factor $v_{f,x}$ arises, these momenta do not contribute to the results noticeably and the conditions turn out not to be restrictive.

Deviations from half filling with $\mu \ll \varepsilon_f$ do not change the zero-energy value of the bound-state energy, within the quasiclassical accuracy. The point is that within this framework relations $\Theta_e^{CDW} - \Theta_h^{CDW} = 0, \Theta_e^{DDW} - \Theta_h^{DDW} = \pi$ are still valid for finite μ. Indeed, we find, assuming also $W \gg \Delta$, that at transparent CDW-N and DDW-N boundaries specular reflection vanishes and there is only Q reflection for arbitrary interface orientation. The respective quasiclassical reflection amplitudes take the form $r^e_{CDW} = r^h_{CDW} = (\mu - i\sqrt{W_s^2 - \mu^2})/W_s$, $r^e_{DDW} = -r^h_{DDW} = [\mu - i\sqrt{W_d^2(\boldsymbol{k}_f) - \mu^2}]/iW_d(\boldsymbol{k}_f)$ and satisfy required relations.

III. S-CDW-S TUNNEL JUNCTION

Consider now Josephson junctions with an interlayer made of gapped CDW (or a DDW) solid. Although we assume no potential barriers in the junction, its effective transparency is finite and tunneling of subgap quasiparticles through the gapped phases substantially depends on the interlayer thickness $l \ll \xi_{s,d}$. Low-energy states on the two CDW-dSC boundaries of dSC-CDW-dSC junctions influence each other, resulting in finite energies of interlayer quasiparticle bound states. Assuming $W_s \gg \Delta_d(\boldsymbol{k}_f)$, we find

$$\varepsilon_B(\boldsymbol{k}_f) = \pm\sqrt{D(\boldsymbol{k}_f)}\Delta_d(\boldsymbol{k}_f)\cos(\chi/2). \quad (9)$$

Here χ is the phase difference of superconducting order parameters on the two banks of the junction and $D = 4K/(1+K)^2$, where $K(\boldsymbol{k}_f) = \exp(2l|W_s|/|v_{f,x}|)$. In the case $\varepsilon_B(\boldsymbol{k}_f) \ll \Delta_d(\boldsymbol{k}_f)$, the self-consistency keeps the expression for bound states unchanged, if one introduces effective order parameters defined in Ref. 16. The Andreev states we study in the present paper arise as a combined effect of Andreev and Q reflections. Contributions of these states to electric transport are quite important. The Josephson current, carried by these states, takes the form

$$J = \int_{-\pi/2}^{\pi/2} \frac{dk_y}{\pi} e\sqrt{D}|\Delta_d(\boldsymbol{k}_f)|\sin\frac{\chi}{2}\tanh\frac{\sqrt{D}|\Delta_d(\boldsymbol{k}_f)|\cos\frac{\chi}{2}}{2T}. \quad (10)$$

In the particular case of large interlayer width, $K^{-1}, D \ll 1$, there are low-energy states $\varepsilon_B(\boldsymbol{k}_f) \ll \Delta_d(\boldsymbol{k}_f)$ in the junction which dominate the Josephson critical current at low temperatures $T \ll T_c$ and result in its low-temperature anomalous behavior. Qualitatively, deviations from the conventional Ambegaokar-Baratoff result can be associated with the presence of two different characteristic energies $|\varepsilon_B(\boldsymbol{k}_f)|$ and $|\Delta_d(\boldsymbol{k}_f)|$. In the standard case, when the order parameter does not change its sign in quasiparticle reflection and transmission processes in tunnel junctions, $\varepsilon_B(\boldsymbol{k}_f) = \pm|\Delta_d(\boldsymbol{k}_f)|\sqrt{1-D(\boldsymbol{k}_f)\sin^2(\chi/2)} \approx \pm|\Delta_d(\boldsymbol{k}_f)|$ and there is the only energy scale $|\Delta_d(\boldsymbol{k}_f)|$. Then the Josephson critical current quickly saturates in the temperature region $T \ll \Delta_d$, together with the order parameter. However, in the case $|\varepsilon_B(\boldsymbol{k}_f)| \ll |\Delta_d(\boldsymbol{k}_f)|$, we find from Eq. (10) the two regimes in the low-temperature behavior of the critical current, taking place in the temperature regions $\sqrt{D}|\Delta_d| \ll T \ll |\Delta_d|$ and $T \ll \sqrt{D}|\Delta_d|$. In the former case the Josephson critical current anomalously increases $\propto 1/T$ with decreasing temperature. Only in the latter case does the critical current saturate taking its zero-temperature value, which can noticeably exceed the standard one under the condition $\sqrt{D} \gg D$. This behavior is similar to what can happen in tunnel *d*SC-I-*d*SC junctions with *d*-wave superconductors, S-F-S junctions with low-energy interface states, and *s*SC-AF-*s*SC junctions.[4,17–20] At the same time, an important qualitative difference between properties of *d*SC-I-*d*SC and *d*SC-CDW-*d*SC junctions is that, in contrast with *d*SC-I-*d*SC junctions, in *d*SC-CDW-*d*SC junctions low-energy bound states and respective anomalies in the critical current take place, in particular, for (100) interface. More generally, low-energy interface states in *d*SC-I-*d*SC tunnel junctions arise only along quasiparticle trajectories, where the order parameter changes its sign in a specular reflection event. The fraction of such trajectories vanishes for (100) interfaces and increases with misorientation angle up to unity for (110) orientation. Respectively, low-temperature anomalies in the Josephson current is more pronounced for (110) interface and vanishes for (100) orientation.[18] This is not the case for *d*SC-CDW-*d*SC junctions considered above, since the order parameter changes its sign in a Q-reflection event along all quasiparticle trajectories at any interface orientation.

In addition to symmetric junctions studied above, one can also consider the so-called mirror junctions, where the junction barrier is a reflection-symmetry plane for superconducting electrodes with one and the same crystal orientations. The *d*-wave superconducting order parameters in mirror junctions have opposite signs and the same absolute value for incoming and transmitted quasiparticles. Bound-state energies and the Josephson current through mirror junctions can be obtained from respective quantities for symmetric junctions with the substitution $\chi \to \chi + \pi$. We find that the low-temperature critical current acquires opposite sign in a mirror *d*SC-CDW-*d*SC junction, as compared with the symmetric one, even for particular (100)-(010) interface orientation.

As is known, it is substantially more difficult to observe low-temperature anomalies in the Josephson current through *d*SC-I-*d*SC junctions, as compared with the zero-bias conductance peak. Since these junctions are faceted, the macroscopic current is represented as an average over junctions with different misorientations. Opposite signs which the anomalous critical current (10) possesses for various junction orientations, can be the reason for a cancellation of the low-temperature anomalies in the presence of large-scale facets at the junction interface. For this reason only small mesoscopic *d*SC-I-*d*SC junctions, containing few facets or so, manifest deviations in the temperature dependence of the critical current from the standard behavior.[21–23]

Under the condition $W_d(\boldsymbol{k}_f) \gg \Delta_s$, energies of quasiparticle bound states and Josephson current in *s*SC-DDW-*s*SC junctions are obtained from Eqs. (9) and (10) after the substitutions $\Delta_d(\boldsymbol{k}_f) \to \Delta_s, W_s \to W_d(\boldsymbol{k}_f)$.

IV. CONCLUSIONS

We have shown that subgap Andreev bound states are formed at CDW-*d*SC and DDW-*s*SC interfaces as a combined effect of Andreev, specular, and Q reflection. These states are dispersionless zero-energy bound states at (110) interfaces, whereas they are dispersive for (100) interface-to-crystal orientation. If the density-wave order parameter is not too large $W_{s,d} \ll \varepsilon_f$, the Q reflection dominates the specular one. Then Andreev bound states have almost zero energies also at (100) CDW-*d*SC and DDW-*s*SC interfaces. At the same time there are no bound states at CDW-*s*SC and DDW-*d*SC interfaces. In *d*SC-CDW-*d*SC and *s*SC-DDW-*s*SC Josephson junctions, the interface low-energy bound states are splitted and strongly influence the Josephson current, which manifests low-temperature anomalies as compared with the standard tunnel Ambegaokar-Baratoff result.

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