Effective vortex mass from microscopic theory

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We calculate the effective mass of a single quantized vortex in the Bardeen–Cooper–Schrieffer superconductor at finite temperature. Based on effective action approach, we arrive at the effective mass of a vortex as integral of the spectral function $J(\omega)$ divided by ω^3 over frequency. The spectral function is given in terms of the quantum-mechanical transition elements of the gradient of the Hamiltonian between two Bogoliubov–deGennes (BdG) eigenstates. Based on self-consistent numerical diagonalization of the BdG equation we find that the effective mass per unit length of vortex at zero temperature is of order $m(k_f\xi_0)^2$ (k_f=Fermi momentum, ξ_0=coherence length), essentially equaling the electron mass displaced within the coherence length from the vortex core. Transitions between the core states are responsible for most of the mass. The mass reaches a maximum value at $T\approx 0.5T_c$ and decreases continuously to zero at T_c.

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I. INTRODUCTION

Phase coherence is the defining characteristic of the superfluid matter, and vortices—the quantized twist of the underlying phase texture—are the unique elementary excitations of the condensate. Their ubiquity, not to mention their role in transport and phase transition, makes the dynamics of an isolated vortex or their array an integral part of our understanding of superfluidity.

The motion of a single vortex may be phrased as a Newtonian equation

$$M_v\frac{d^2\mathbf{r}_v}{dt^2}+\eta\frac{d\mathbf{r}_v}{dt}=\mathbf{F}_v, \tag{1}$$

where $\mathbf{F}_v$ is the force acting on the vortex, and M_v is its effective mass. The position of the vortex is represented by $\mathbf{r}_v$. The mass of a vortex is *entirely effective* in nature.[1] A vortex is unable to sustain its existence outside the superfluid medium, hence the only sensible definition of the mass, *the intrinsic mass*, must be zero. Now, imagine a magnet with its north and south poles placed on either side of a thin superconducting slab as shown in Fig. 1. Dragging the magnet parallel to the surface of the slab requires a force that exceeds $M_m(d^2\mathbf{r}_v/dt^2)$ (M_m=mass of the magnet) because in the process, a $h/2e$ flux quantum attached to the magnet interacts with the surrounding quasiparticles. The virtual transitions among the quasiparticle states caused by the vortex motion lead to renormalization of mass, which in this case will account for the entire vortex mass. Real transitions between the states, on the other hand, lead to dissipation, giving rise to a damping term represented by η in Eq. (1).

The Newtonian description of the vortex motion is a limiting case of the more general quantum-mechanical formulation. Quantum-mechanical law of motion of an object embedded in a medium with which it interacts is most easily derived in the effective action approach.[2] In essence, one divides the dynamics into those of quasiparticles and the vortex, then by integrating out the quasiparticle degrees of freedom, one is left with the effective dynamics of the vortex alone. Equations of vortex dynamics were written down in this way first in the work of Šimánek[3] and later extensively developed by Ao and Zhu.[4,5]

In Ref. 5, the functional integral theory of vortex dynamics was applied to calculate the friction coefficient η of a moving vortex, and to calculate the transverse force on it in both clean and dirty limits. Wentzel–Kramers–Brillouin (WKB) approximation for the extended eigenstates was used to calculate the transition probability that led to friction at temperature T. The friction coefficient thus derived was consistent with earlier results derived by other approaches.[5]

Based on a similar approach, Han, Ao, and Zhu (HAZ) tried to calculate the effective mass of a slowly moving vortex within the same WKB approximation for the eigenstates of a single vortex.[6] The calculation of the effective mass of a quantized vortex has been repeatedly tackled by theorists in the past. The conclusions roughly fall into two categories, with one group of theories predicting a "small" mass of roughly one electron mass per atomic length,[7] whereas another group predicts a "large" mass, of order $m(k_f\xi_0)^2$ (k_f =Fermi momentum, ξ_0=coherence length at $T=0$).[8] In either case the theories were limited to zero temperature. The work of HAZ extends the calculation to an arbitrary finite temperature by working with the imaginary-time action.

In neither work, Refs. 5 and 6, was the validity of the WKB approximation carefully checked against an exact diagonalization of the Bogoliubov–deGennes (BdG) equation. Also lacking in previous works was the consideration of the self-consistent nature of the pair potential, which in particular will affect the behavior at T close to the transition temperature T_c. After the pioneering work of Gygi and Schlüter, the self-consistent numerical diagonalization of the BdG equation for a single vortex or an array of vortices became standard.[12]

In this paper, we employ a self-consistent numerical method to solve the BdG equation with a single vortex. The eigenstates thus obtained are used to calculate the effective

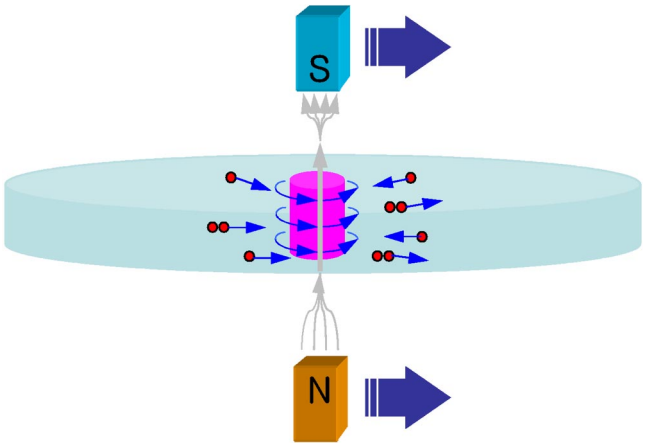

FIG. 1. (Color online) A magnet generates a quantized flux inside a thin slab of superconductor (blue). When a magnet is dragged, the flux interacts with the surrounding medium.

mass of a vortex, according to the formula originally derived in Ref. 6. Employing self-consistent calculation of the pair potential at finite temperatures we compute the variation of the effective mass in the full temperature range $0<T<T_c$.

We proceed by revisiting the formulation of the effective mass of Ref. 6 in Sec. II. The mass formula is then rewritten in a form that is convenient for efficient numerical work. In Sec. III numerical results for the temperature-dependent mass are presented. We conclude with a summary and outlook in Sec. IV.

II. FORMULATION

The effective action $S_{\rm eff}$ of a single vortex of a Bardeen–Cooper–Schrieffer (BCS) superconductor, centered at $\mathbf{r}_v(\tau)$ at imaginary time τ, is given by[3–5,9]

$$\frac{1}{8}\int_0^\beta d\tau\int_{-\infty}^{\infty}d\tau'\int_0^\infty d\omega J(\omega)e^{-\omega|\tau-\tau'|}|\mathbf{r}_v(\tau)-\mathbf{r}_v(\tau')|^2. \quad (2)$$

The spectral function is the quantity

$$J(\omega)=\sum_{ab}\delta(\omega-|E_a-E_b|)|f(E_a)-f(E_b)|\left|\left\langle a\left|\frac{\partial H_0}{\partial \mathbf{r}_v}\right|b\right\rangle\right|^2, \quad (3)$$

which encodes the density of available low-energy excitations created by, in this case, the change in the vortex position $\mathbf{r}_v$. Fermi distribution function with energy x is denoted $f(x)$. There is an additional action in $S_{\rm eff}$, pertaining to the transverse motion of the vortex, which we do not consider here.[5] The states a, b are the eigenstates of the Bogoliubov–deGennes (BdG) Hamiltonian H_0 in the presence of vortex at $\mathbf{r}_v$, with energies E_a and E_b respectively. When we assume that the dynamics is sufficiently *local in time*,[10] we may approximate $\mathbf{r}_v(\tau)-\mathbf{r}_v(\tau')\approx(\tau-\tau')\dot{\mathbf{r}}_v(\tau)$ and write

$$S_{\rm eff}\approx\frac{1}{2}\times\left(\int_0^\infty d\omega\frac{J(\omega)}{\omega^3}\right)\times\int_0^\beta d\tau\left(\frac{d\mathbf{r}_v}{d\tau}\right)^2. \quad (4)$$

The effective vortex mass is the quantity in parenthesis, given by

$$M_v=\sum_{ab}\left|\frac{f(E_a)-f(E_b)}{(E_a-E_b)^3}\right|\left|\left\langle a\left|\frac{\partial H_0}{\partial \mathbf{r}_v}\right|b\right\rangle\right|^2. \quad (5)$$

This formula, first derived by HAZ, allows an explicit calculation of the vortex mass at temperature T using the eigenenergies and wave functions of the BdG Hamiltonian.

The eigenstates of the BdG equation in the cylindrical coordinates are written in the general form[11]

$$\psi_a(\mathbf{r})=\frac{1}{\sqrt{2\pi L}}e^{ik_z z}e^{i\mu\theta}\begin{pmatrix}e^{-i\theta/2}u_\alpha(r)\\ e^{+i\theta/2}v_\alpha(r)\end{pmatrix} \quad (6)$$

in the presence of a gap function $\Delta(\mathbf{r})=\Delta(r)e^{-i\theta}$ for a rectilinear vortex of length L centered at $r=0$. The radial functions $[u_\alpha(r),v_\alpha(r)]$ are obtained as eigenfunctions of the coupled differential equation (r dependence in u, v and Δ is implicit)

$$r^2u''+ru'+\left[(k_r^2+2mE)r^2-\left(\mu-\frac{1}{2}\right)^2\right]u=2mr^2\Delta v,$$

$$r^2v''+rv'+\left[(k_r^2-2mE)r^2-\left(\mu+\frac{1}{2}\right)^2\right]v=-2mr^2\Delta u, \quad (7)$$

where $k_r^2=k_f^2-k_z^2$. Since the gradient operator does not mix different k_z states, states characterized by different k_z's make additive contributions to the mass. Therefore, we assume that the effective mass is $\sim(k_fL)$ times the mass obtained from $k_z=0$ sector alone. Hence, we restrict ourselves to $k_z=0$ in the numerical diagonalization. Now each eigenstates a is labeled by $a=(\mu_\alpha,E_\alpha)$ and $b=(\mu_\beta,E_\beta)$ where $E_\alpha(E_\beta)$ denotes energies of states having the angular momentum $\mu_\alpha(\mu_\beta)$.

Self-consistency requires that the r-dependent gap function obey the relation $\Delta(r)=V\Sigma_a u_a(r)v_a(r)[1-2f(E_\alpha)]$ for some choice of the pairing interaction strength V. Uniqueness of the wave function requires that μ be half odd integers. Using these eigenstates, transition amplitudes can be worked out

$$-\left\langle\alpha\left|\frac{\partial H_0}{\partial \mathbf{r}_v}\right|\beta\right\rangle=\frac{\hat{x}\mp i\hat{y}}{2}\int_0^R\left\{(u_\alpha v_\beta+u_\beta v_\alpha)r\frac{d\Delta}{dr}\right.$$
$$\left.\pm(u_\alpha v_\beta-u_\beta v_\alpha)\Delta\right\}dr. \quad (8)$$

Upper and lower signs correspond to $\mu_\alpha=\mu_\beta\pm1$. The upper limit of integration is chosen at $r=R$ where a hard-wall boundary condition is imposed: $u_\alpha(r=R)=0=v_\alpha(r=R)$. The two eigenstates differ by one unit of angular momentum due to the gradient operator which connects them.

On integrating by parts, Eq. (8) is equal to (assuming $\mu_\alpha=\mu_\beta+1$)

$$(E_\beta - E_\alpha)\int_0^R \left\{ \left(u_\alpha \frac{du_\beta}{dr} + v_\alpha \frac{dv_\beta}{dr} \right) r - \left(\left(\mu_\beta - \frac{1}{2} \right) u_\alpha u_\beta + \left(\mu_\beta + \frac{1}{2} \right) v_\alpha v_\beta \right) \right\} dr + \frac{R}{2m} \left(\frac{dv_\alpha}{dr}\frac{dv_\beta}{dr} - \frac{du_\alpha}{dr}\frac{du_\beta}{dr} \right)_{r=R}. \tag{9}$$

One must be careful not to ignore the boundary term at $r=R$ on the rhs of the above equation, which is generally nonzero. A naive application of Feynman–Hellman theorem, as was done by HAZ, would yield Eq. (8)=Eq. (9) without the boundary term, which is incorrect. When either u_α or v_α is localized over the distance much less than R, the boundary term can be ignored. However, no justification can be provided to ignore the boundary term for a generic pair of extended eigenstates.

We follow earlier works[12] and write the eigenfunctions in the form

$$u_\alpha(r) = \sum_i c_{\alpha i}\phi_{ni}(r), \quad v_\alpha(r) = \sum_i d_{\alpha i}\phi_{n+1,i}(r),$$

$$\phi_{ni}(r) = \frac{\sqrt{2}}{RJ_{n+1}(z_{ni})} J_n\left(z_{ni}\frac{r}{R} \right), \tag{10}$$

where z_{ni} is the ith zero of the Bessel function J_n. Integer value n is related to the angular momentum by $\mu_\alpha = n+\frac{1}{2}$. Coefficients $(c_{\alpha i}, d_{\alpha i})$ are determined from matrix diagonalization.[12] Negative-μ states need not be considered separately, instead one can use a positive-μ eigenstate (u_α, v_α) of energy E_α to construct a negative-μ eigenstate, given by $(v_\alpha, -u_\alpha)$, of opposite energy $-E_\alpha$.

With the eigenstates thus obtained, Eq. (9) can be re-expressed using the coefficients $(c_{\alpha i}, d_{\alpha i})$ and making use of Bessel identities. For $\mu_\alpha = n+\frac{1}{2}$, $\mu_\beta = n-\frac{1}{2}$, $n>0$, Eq. (9) becomes

$$A_{n,\alpha\beta} = \frac{2}{R}\sum_{ij} c_{\alpha i}c_{\beta j}z_{ni}z_{n-1,j}\left(\frac{E_\alpha - E_\beta}{z_{ni}^2 - z_{n-1,j}^2} - \frac{\xi_0}{R^2} \right) + \frac{2}{R}\sum_{ij} d_{\alpha i}d_{\beta j}z_{n+1,i}z_{nj}\left(\frac{E_\alpha - E_\beta}{z_{n+1,i}^2 - z_{nj}^2} + \frac{\xi_0}{R^2} \right) \tag{11}$$

and for $\mu_\alpha = +\frac{1}{2}$, $\mu_\beta = -\frac{1}{2}$, it equals

$$B_{\alpha\beta} = \frac{2}{R}\sum_{ij} c_{\alpha i}d_{\beta j}z_{0i}z_{1j}\frac{J_0(z_{1j})}{J_2(z_{1j})}\left(\frac{E_\alpha + E_\beta}{z_{0i}^2 - z_{1j}^2} - \frac{\xi_0}{R^2} \right) + \frac{2}{R}\sum_{ij} d_{\alpha i}c_{\beta j}z_{1i}z_{0j}\frac{J_0(z_{1i})}{J_2(z_{1i})}\left(\frac{E_\alpha + E_\beta}{z_{0j}^2 - z_{i1}^2} - \frac{\xi_0}{R^2} \right). \tag{12}$$

In deriving Eqs. (11) and (12) energy is expressed in units of the zero-temperature gap value at a large distance from the core, denoted $\Delta(0)$, and the length in units of k_f^{-1}. In Eq. (11), α and β refer to eigenstates with a given angular momenta $\mu_\alpha = n+1/2$ and $\mu_\beta = n-1/2$, each. A mapping of negative-μ to positive-μ eigenstates was used to express Eq. (12) solely in terms of the $\mu = +\frac{1}{2}$ eigenstates. Now the vortex mass can be calculated using

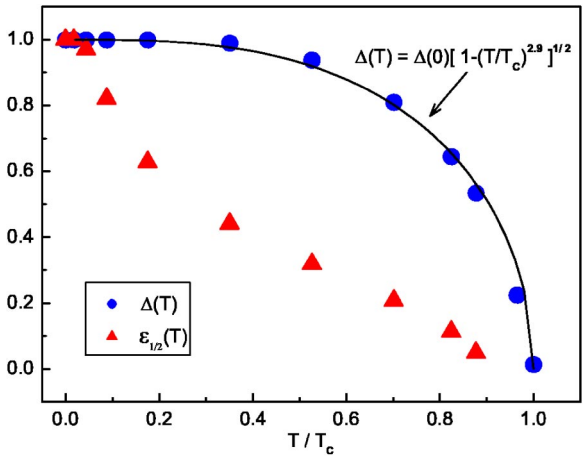

FIG. 2. (Color online) Pair potential well away from the core, $\Delta(T)$ (blue), and the core energy level for $\mu = +\frac{1}{2}$, $\epsilon_{1/2}(T)$ (red), plotted vs T/T_c. Both quantities are normalized by their respective $T=0$ values, with $\epsilon_{1/2}(0)/\Delta(0) = 0.088$ from self-consistent calculation. $\Delta(T)$ shows excellent fit to $\Delta(0)\sqrt{1-(T/T_c)^{2.9}}$ (black curve).

$$M_v = 2m\cdot\frac{\xi_0}{2}\cdot 2\sum_{n>0,\alpha\beta}\left| \frac{f(E_\alpha) - f(E_\beta)}{(E_\alpha - E_\beta)^3} \right| A_{n,\alpha\beta}^2 + 2m\cdot\frac{\xi_0}{2}\sum_{n=0,\alpha\beta}\left| \frac{f(E_\alpha) - f(-E_\beta)}{(E_\alpha + E_\beta)^3} \right| B_{\alpha\beta}^2. \tag{13}$$

Factor 2 multiplying the first term reflects the transitions within the negative angular momentum channels which gives rise to equal mass as the $\mu>0$ transitions.

Equations (11)–(13) are the main results of the paper, allowing us to calculate vortex mass in the superconductor. Using Eqs. (11) and (12) is much more efficient over the brute-force numerical integration of Eqs. (8) and (9) in obtaining the transition amplitude.

III. RESULTS

Equation (7) is diagonalized assuming a coherence length $\xi_0 \equiv E_f/\Delta(0) = 20$ and choosing the radius of the boundary $k_f R = 100$. For each angular momentum μ, eigenstates with energies within $\pm 10\Delta(0)$ were retained. This left us with about 90 eigenstates for $\mu = 1/2$, and a decreasing number of states for larger μ. Calculations were restricted to $k_z = 0$ only as stated earlier. The energy gap vanishes completely at $T \approx 0.571\Delta(0)$, which we take as the transition temperature T_c. We work at several (reduced) temperatures $t \equiv T/T_c$ in the range $0<t<1$.

Figure 2 shows the calculated gap at a large distance from the core, $\Delta(T)$, and the $\mu = +\frac{1}{2}$ core energy level, $\epsilon_{1/2}(T)$, for the temperatures we considered. Both quantities decrease monotonically with T. For $T/T_c \gtrsim 0.9$ the core energy levels are no longer resolved as distinct from the continuum.

Referring to Eqs. (5) and (13) we may divide the mass as arising from transitions between the core states (M_v^{cc}), core-to-extended states (M_v^{ce}), and between extended states (M_v^{ee}): $M_v = M_v^{cc} + M_v^{ce} + M_v^{ee}$. No other selection rule is imposed on the initial and final states except the angular momentum dif-

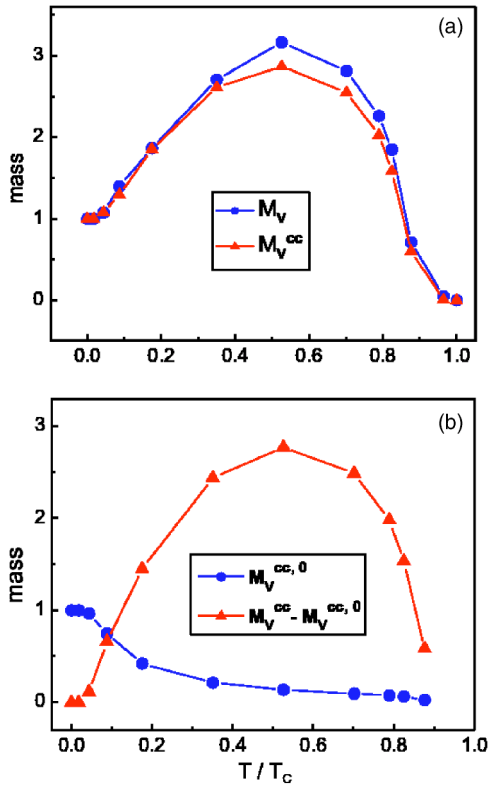

FIG. 3. (Color online) (a) Effective mass $M_v(T)$ and the core-to-core contribution to mass $M_v^{cc}(T)$ at temperature T/T_c, normalized by $M_v(0)$. One finds $M_v(T) \approx M_v^{cc}(T)$ over most of T. (b) $M_v^{cc}(T) = M_v^{cc,0}(T) + \Sigma_{n\neq 0} M_v^{cc,n}(T)$ (see text for definition). $M_v^{cc,0}(T)$ and $M_v^{cc}(T) - M_v^{cc,0}(T)$ are plotted separately. $M_v^{cc,0}(T)$ is monotonically decreasing while the higher-n channels give a maximum mass at $T/T_c \approx 0.5$. The core levels are not well resolved for T too close to T_c, which explains the absence of data points for $T \approx T_c$.

ference which must be one. As it turns out the transition element is vanishingly small between a core and an extended state, and thus the mass is effectively $M_v \approx M_v^{cc} + M_v^{ee}$. In Fig. 3(a) we show the total mass $M_v(T)$ and the core-to-core mass, $M_v^{cc}(T)$. As is evident from the figure, $M_v^{cc}(T) \gg M_v^{ee}(T)$ for all temperatures except very near T_c where core levels are not resolved, but here the total mass is vanishingly small anyway. Both the total mass M_v and the level-resolved masses M_v^{cc} and M_v^{ee} reach a maximum value at $T/T_c \sim 0.5$.

The core-to-core mass can be further grouped according to the angular momentum channels $(\mu_\alpha = n + \frac{1}{2}) \leftrightarrow (\mu_\beta = n - \frac{1}{2})$ over which the transition takes place. We denote such angular-momentum-resolved, core-to-core mass, $M_v^{cc,n}(T)$. Figure 3(b) shows $M_v^{cc,0}(T)$ and $\Sigma_{n\neq 0} M_v^{cc,n}(T) = M_v^{cc}(T) - M_v^{cc,0}(T)$. We find that $M_v^{cc,0}(T)$, between $\mu = \pm\frac{1}{2}$ core levels, survives at zero temperature and monotonically decreases at higher T. The $n \neq 0$ channels give zero mass for $T=0$, reaches a maximum at some intermediate T, and decreases to zero at $T=T_c$. This behavior accounts for the observed maximum in the mass $M_v^{cc}(T)$.

The results shown in Fig. 3(b) can be nicely understood thanks to the approximate identity which states $|\langle\alpha|\nabla_v H_0|\beta\rangle| \approx |E_\alpha - E_\beta|$ when both states belong to the core levels.[8] Numerical calculation confirms this relation with excellent accuracy for all temperatures, except near T_c. Then, we can approximate [see Eq. (13)]

$$M_v^{cc,n}(T) \approx 2m\xi_0 \left| \frac{f(\epsilon_{n-1/2}(T)) - f(\epsilon_{n+1/2}(T))}{\epsilon_{n-1/2}(T) - \epsilon_{n+1/2}(T)} \right|, \quad (14)$$

where $\epsilon_{n\pm 1/2}$ are the core-level energies for $\mu = n \pm 1/2$. In particular, for $n=0$ we have the mass $M_v^{cc,0}(T) = 2m\xi_0 \tanh[\beta\epsilon_{1/2}(T)]/2\epsilon_{1/2}(T)$. At zero temperature we get $M_v^{cc,0}(0) = 2m\xi_0/2\epsilon_{1/2}(0) \sim m\xi_0^2$ since $\Delta(0)/\epsilon_{1/2}(0) \sim \xi_0$. This is precisely the mass of the electrons occupying the area $\sim \xi_0^2$.

On the other hand, $M_v^{cc,n\neq 0}(T) \approx 2m\xi_0\beta/[4\cosh^2(\beta\epsilon_n(T)/2)]$, $\epsilon_n(T) \equiv [\epsilon_{n+1/2}(T) + \epsilon_{n-1/2}(T)]/2$, provided the temperature is much larger than the typical core energy spacing, $T \gg |\epsilon_{n+1/2}(T) - \epsilon_{n-1/2}(T)|$. As the typical core energy spacings are a few percent of the energy gap $\Delta(T)$, this is not a very restrictive condition except near $T=0$. Then one can easily check that this approximate form for $M_v^{cc,n}(T)$ rises to a maximum value for T equal to some fraction of the gap $\Delta(0)$, in agreement with the observed maximum in the mass shown in Figs. 3(a) and 3(b).

One finds that Eq. (14) is nonvanishing as $T \to T_c$, and $\epsilon_{n\pm 1/2}(T) \to 0$. The core-to-core mass seemingly survives at the transition temperature. A similar behavior, in fact a divergence of the core-to-core mass at T_c, was predicted in the WKB calculation of HAZ. However, both Eqs. (14) and (10) of HAZ are predicated on the existence of well-resolved core states. Instead, all the core levels collapse into the continuum eventually and Eq. (14) loses its meaning altogether.

Since M_v^{ee} makes a negligible contribution to mass at all temperatures its behavior is not analyzed in detail.

IV. SUMMARY AND OUTLOOK

Summarizing our findings, (i) At zero temperature the effective mass $M_v(T=0)$ is the mass of electrons occupying a cylinder of radius ξ_0, in agreement with earlier calculations.[8] It increases upon higher temperature, reaching a maximum at $T \approx 0.5T_c$, and vanishes at T_c. (ii) Transitions between localized eigenstates forming the core spectrum are mainly responsible for the effective mass. Hence, $M_v(T) \approx M_v^{cc}(T)$. On the other hand, dissipation experienced by a moving vortex is due to the extended states as it requires transition between states of the same energy.

Suppose now that the core levels got smeared due to the impurities. If all higher angular momentum core levels except $n=0$ disappeared, we would expect $M_v^{cc,n\neq 0}(T)$ to be effectively zero, and the mass behaves as $M_v(T) \approx M_v^{cc,0}(T)$ showing the monotonic behavior of Fig. 3(b). If all core levels disappeared, we will be left with the extended states whose mass contribution is shown to be quite small. It is conceivable that in this case the mass becomes small, in agreement with predictions of Ref. 7.

To fully describe the vortex dynamics, one must of course know η in Eq. (1) with equal accuracy as the mass itself. If the motion is overdamped, an object's mass is no longer the governing factor in the dynamics. Since the damping rate rises monotonically with temperature, it is conceivable that

above some crossover temperature T^*, the vortex motion gets overdamped.

How do we calculate η within the same framework of numerical diagonalization proposed in this paper? Dissipation due to transitions among equal-energy states is only well defined in the thermodynamic limit, $R\to\infty$. To analyze the $R=\infty$ behavior, one should presumably rely on a finite-size scaling analysis of various transition amplitudes. Study of size dependence may be necessary to get a better estimate of the extended-states mass, too. A standard treatment of quantum dissipative dynamics would predict that if the environment is ohmic, i.e., $J(\omega)\sim\eta\omega$ at low frequency, the effective mass is linearly divergent. A WKB-based calculation indeed gives a linear spectral density for the vortex,[5] and extended-to-extended mass employing similar approximation is also predicted to behave as $M_v^{ee}\sim m\times\Delta(0)/\omega_c$ for some cutoff ω_c.[6] The finite-size R employed in the numerical calculation can serve as the effective cutoff.

The choice $R/\xi_0=5$ used in our work is, we believe, large enough to conclude that the core-to-core mass obtained in this paper is that of the thermodynamic limit, $M_v^{cc}(R/\xi_0=5)\approx M_v^{cc}(R/\xi_0=\infty)$, since all core levels are localized over a few coherence lengths. On the other hand, extended states do not meet this requirement, at least for ultraclean systems with no impurity scattering to provide the cutoff. Although $M_v^{ee}(R/\xi_0=5)$ turned out to be negligible in comparison to the core-to-core mass, one cannot completely rule out the R-dependent growth of M_v^{ee} as predicted by quantum environmental dynamics. We will return to these issues in a future publication.

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