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Isotropic three-dimensional left-handed metamaterials

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We investigate three-dimensional left-handed and related metamaterials based on a fully symmetric multigap single-ring split-ring resonator (SRR) design and crossing continuous wires. We demonstrate isotropic transmission properties of a SRR-only metamaterial and the corresponding left-handed material that possesses a negative effective index of refraction due to simultaneously negative effective permeability and permittivity. Minor deviations from complete isotropy are due to the finite thickness of the metamaterial.

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The realization of a perfect lens[1] and other applications of negative refraction require the fabrication of three-dimensional, homogeneous, isotropic left-handed materials[2] (LHM) with simultaneously negative permittivity ε and magnetic permeability μ. So far, no such materials exist, either in nature or in the laboratory. Today's available LHM structures, based on the periodic arrangement of split-ring resonators[3] (SRR) and continuous metallic wires,[4] are only one dimensional[5–7] (1D), supporting left-handed properties only for propagation with fixed polarization in one direction, or two dimensional[8–10] (2D), where propagation in two directions with fixed polarization or one direction with arbitrary polarization is possible. Earlier attempts to design at least an isotropic SRR (Ref. 11) were lacking the symmetry of SRR and unit cell and required individual tuning of the parameters in the different spatial directions.

In this paper, we propose a three-dimensional (3D) isotropic LHM design that allows left-handed behavior for any direction of propagation and any polarization of the electromagnetic wave. Using numerical transfer matrix simulations, we verify the isotropic transmission properties of the proposed structures. Our data show excellent agreement with results expected for a homogeneous slab with the corresponding negative ε and μ.

Our metamaterials are defined as a 3D periodic continuation of a single rectangular unit cell, consisting of SRRs and continuous wires. The sample is a slab of metamaterial with a finite thickness of an integral number of unit cells and infinite extent in the perpendicular direction. The two surfaces of the slab are parallel to any face of the unit cell. An incident electromagnetic plane wave with wave vector **k** can be characterized by two angles: the incidence angle $\vartheta \in [0,\pi/2)$ between **k** and the surface normal **n** of the sample, and the angle $\phi \in (-\pi,\pi]$ between the projection of **k** into and some chosen edge of the unit cell inside the surface plane of the sample. The frequency of the incident wave is chosen such that the vacuum wavelength is approximately 10 times larger than the linear size of the unit cell and we expect effective medium behavior.

To achieve an isotropic metamaterial it is wise to start with a cubic unit cell. The SRR and the continuous wire provide a resonant negative μ and a negative plasmonic ε, respectively. This occurs only when there is a component of the magnetic field normal to the SRR plane and a component of the electric field parallel to the wire. Therefore we have to use one pair of SRR and wire for each of the three spatial directions. The idea is that if the interaction between the SRR and wire in one direction, and the mutual interaction in between the constituents in the different directions is negligible, we can, in the first order, control the three diagonal elements of the permittivity and permeability tensors independently. Previous results and extensive numerical simulations have further provided a set of design criteria to attain the best possible isotropy:

(i) To avoid the coupling of the electric field to the magnetic resonance[12,13] of the circular current in the SRR we have to provide mirror symmetry of the SRR plain with respect to the direction of the electric field. Otherwise the electric resonant response of the magnetic resonance or the emerging cross-polarization terms in more dimensional media may destroy the desired left-handed behavior. The problem can be avoided in 1D and certain 2D (1D propagation with arbitrary polarization, 2D propagation with fixed polarization) metamaterials. However, for an isotropic material with arbitrary direction and polarization of the incident wave this can only be guaranteed by an inversion symmetric SRR design.

(ii) From an isotropic, homogeneous effective medium we do not expect any cross-polarization scattering amplitudes. Suppression of the cross-polarization terms requires, besides an isotropic material discretization,[14] that the inversion symmetry of the unit cell is not simultaneously broken in both directions perpendicular to the direction of propagation. Therefore we have to center the SRRs on the faces of the unit cell and the wires with respect to the positions of the SRRs.

(iii) For the finite slab a description in terms of an homogeneous effective medium is only possible if the isotropy, i.e., the inversion symmetry of the slab, in the direction of the finite dimension is preserved.[15,16] As a consequence, we have to terminate the second surface of the slab by a repetition of the opposite surface. This is illustrated in Fig. 1 for a sample of one unit cell thickness.

(iv) To avoid effects of the periodicity it is imperative to have the magnetic resonance frequency well below the first periodicity band gap. But even in this case we observe artifacts[17] in the effective medium response, like resonance-

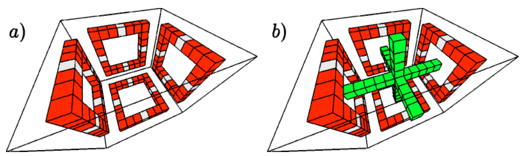

FIG. 1. (Color online) The design of a fully symmetric unit cell for a one-unit-cell thick slab of an isotropic SRR (a) and a left-handed (b) metamaterial. The interfaces are parallel to the left and right SRR. The metal of the four-gap SRR (red/dark gray) and the continuous wires (green/medium gray) is silver using a Drude-model permittivity around 1 THz. The SRR gaps are filled with a high-constant dielectric (light gray) with a relative permittivity $\varepsilon_{\text{gap}}=300$ to lower the magnetic-resonance frequency.

antiresonance coupling and negative imaginary parts. These issues have been addressed in detail in Ref. 15. Further we have to make sure that the magnetic resonance frequency is below the effective plasma frequency in the LHM, which is reduced compared to the plasma frequency of the wires by the additional electric (cut-wire) response of the SRR.[18,19]

(v) In the LHM, the best position for the wires has been found to be aligned with the middle of the SRR in the center between a SRR and its periodic continuation. This design minimizes the disturbance between SRRs and wires. Additionally, it is favorable not to put the edges of the SRRs too close to one another to avoid capacitive coupling across adjacent gaps.

To meet the symmetry requirements of the SRR we deploy a symmetric single-ring design with the gap distributed over all four sides as shown in Fig. 1. The four-gap single-ring SRR has been proven as a simpler, symmetric and working alternative to the conventional design using nested rings with a single gap each.[20] The gap width in the four-gap SRR has to be significantly reduced in comparison to a one-gap design since due to the receded capacitance of the serial capacitors the resonance frequency of the SRR would increase. Alternatively, we could increase the dielectric constant inside the gaps to move the resonance frequency down. In the simulations presented in this paper we chose the latter strategy.

For the numerical simulations we employ a lattice transfermatrix technique as described in Refs. 21 and 22 to calculate the complex scattering amplitudes of the sample. The finite discretization in conjunction with periodic boundary conditions does only allow a finite set of values for the component of the wave vector parallel to the surface, hence only a finite set of incidence directions. As a consequence, to calculate the scattering amplitudes for an incident plane wave under an arbitrary incidence angle ϑ we have to impose more general, quasiperiodic boundary conditions to the unit cell in both directions perpendicular to the stratification direction. This means the electromagnetic fields at either side of the unit cell are identified up to a phase e^{iQ_mL} where m enumerates the perpendicular directions, $Q_m \in [0,2\pi/L)$ are the Bloch momenta parallel to the surface, and L is the size of the unit cell. Since the parallel momentum is preserved across the surfaces, we can calculate the Q_m from the angles of incidence, $Q_1=(k_\parallel \bmod 2\pi/L)\cos\phi$, $Q_2=(k_\parallel \bmod 2\pi/L)\sin\phi$.

We calculate the transmission and reflection amplitudes for a slab of the isotropic SRR and the corresponding isotropic LHM metamaterial of one- and ten-unit cell thickness. The metals are implemented using a Drude-model permittivity for silver around 1 THz. The unit cell size is chosen accordingly to facilitate a resonance frequency in the same region. All results are given in terms of the dimensionless frequency $\omega=2\pi fL/c$ since up to the low terahertz region the scaling of the metamaterials resonance frequencies is linear and virtually independent on the permittivity of the metals.

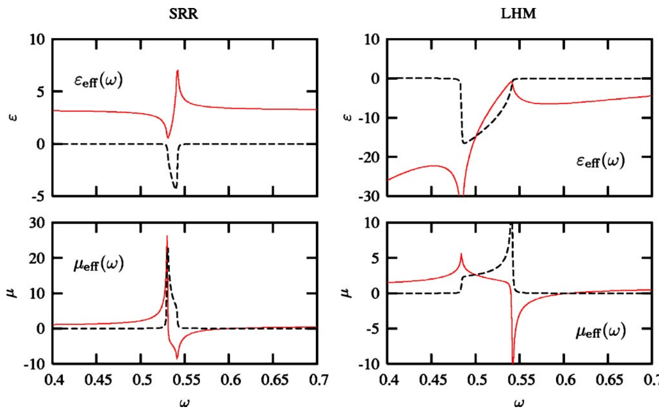

FIG. 2. (Color online) The retrieved real (solid line) and imaginary (dashed line) part of the effective $\varepsilon_{\text{eff}}(\omega)$ and $\mu_{\text{eff}}(\omega)$ of the homogeneous medium approximation for normal incidence ($\vartheta=0$) to a single unit cell layer of the SRR and LHM metamaterial as a function of the dimensionless frequency $\omega=2\pi Lf/c$.

Figure 2 shows the retrieved $\varepsilon(\omega)$ and $\mu(\omega)$ of the effective homogeneous medium approximation of the SRR and LHM slab of one-unit cell-thickness for normal incidence, proving the presence of the magnetic resonance featuring $\mu<0$ in the SRR and the left-handed bandpass with $\varepsilon<0,\mu<0$ in the LHM. The retrieved $\varepsilon(\omega)$ and $\mu(\omega)$ are, in good approximation, independent on the thickness of the slab. The clearly visible resonance-antiresonance coupling and the occurrence of negative imaginary parts are artifacts arising from the periodicity[15] that could be reduced by moving the magnetic resonance to sufficiently low frequency.

Figures 3 and 4 show the simulated transmission $|t(\omega)|^2$ around the magnetic resonance for the one- and ten-unit-cell thick slab, respectively, for different incident angles ϑ and polarizations. Because of the unit cell's symmetry it is sufficient to consider $\phi\in[0,\pi/4]$ for the TE and TM mode. Despite the square shape of the SRR, we find absolute independence of the scattering amplitudes (reflection not shown) on the orientation ϕ of the incidence plane for both, TE and TM mode, arbitrary ϑ, SRR and LHM metamaterial slabs of any thickness. This finding is important for experimental realizations as it demonstrates that a circular shape of the SRRs is not required for isotropic results. As expected for a homogeneous slab, the transmission spectra depend on the incidence angle ϑ, and, for non-normal incidence, also differ for the TE and TM polarization. In accordance with the retrieved ε and μ, the SRR spectra show a transmission dip at the magnetic resonance frequency ≈ 0.54, which widens into a stop band for the longer slab, while the LHM spectra show the corresponding transmission peak, developing into a well-

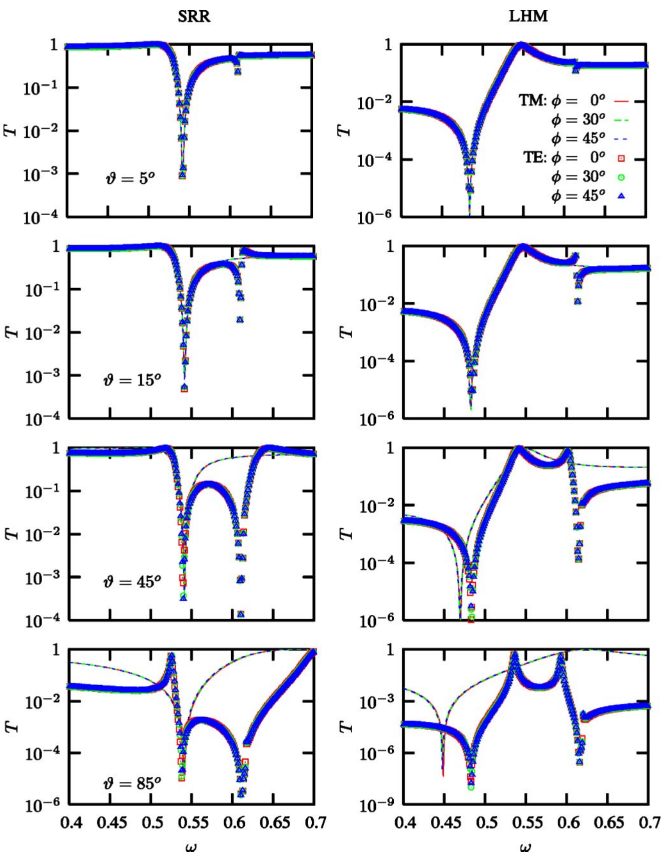

FIG. 3. (Color online) Transmission spectra $T=|t(\omega)|^2$ for a single layer of unit cells of the SRR and the LHM metamaterial for various angles of incidence (ϑ) and polarizations (ϕ, TE, and TM mode).

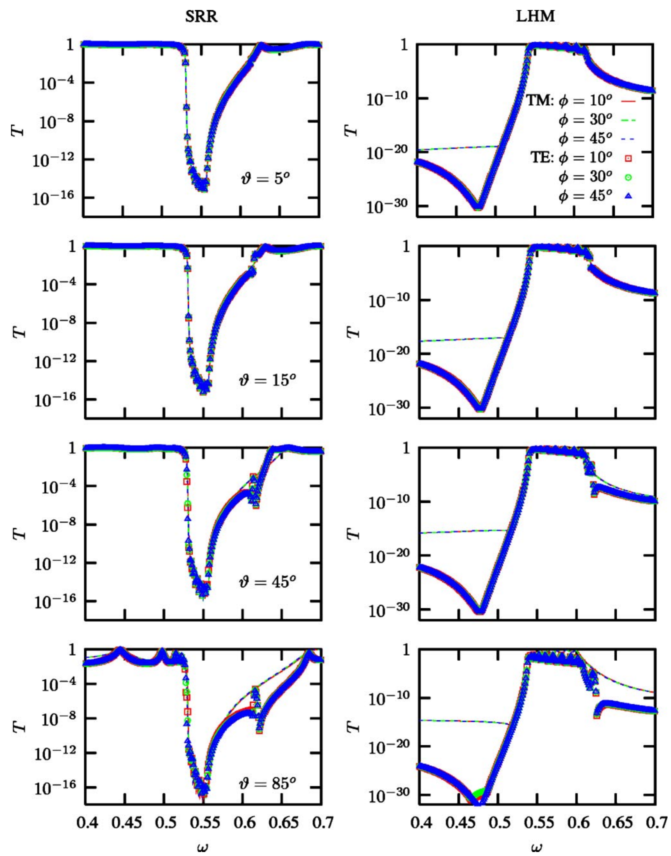

FIG. 4. (Color online) Transmission spectra $T=|t(\omega)|^2$ for a 10-unit-cell-thick slab of the SRR and the LHM metamaterial for various angles of incidence (ϑ) and polarizations (ϕ, TE, and TM mode).

defined LH passband for the longer system. This behavior prevails from normal to nearly parallel incidence. Surprisingly, for off-normal incidence a second spectral feature emerges for both SRR and LHM metamaterial at around $\omega \approx 0.61$ but only in the TE polarization, which gets less accentuated for the longer slab. However, this turns out to be an expected feature for an isotropic, homogeneous slab and has a simple explanation: The analytic transmission amplitude has the form $t^{-1}=\cos qd-(i/2)(\zeta+1/\zeta)\sin qd$, where q is the component of the wave vector perpendicular to the surface inside the slab, k the corresponding component in vacuum, and $\zeta=\mu k/q$ for the TE and $\zeta=\varepsilon k/q$ for the TM mode, what becomes the impedance of the slab for the case of normal incidence. q is given by the dispersion relation $q^2+k_\parallel^2-\mu\varepsilon\ \omega^2/c^2=0$. Whenever $\mu(\omega)$ for the TE mode or $\varepsilon(\omega)$ for the TM mode becomes zero, the prefactor of the sine in t^{-1} diverges. For normal incidence, ie., $k_\parallel=0$, this divergence is compensated by the vanishing $q\propto(\mu\varepsilon)^{1/2}\rightarrow 0$. For $\vartheta>0$, however, $k_\parallel\neq 0$ renders the sine nonzero such that the divergence of the prefactor leads to a dip in the transmission t. The observed dip for the TE mode in the simulations indeed clearly corresponds to the zero of the permeability present in Fig. 2, further supporting the isotropic behavior of the metamaterial slabs.

Figures 5 and 6 show the transmission spectra analytically calculated for homogeneous slabs of the corresponding lengths, assuming homogeneous $\varepsilon(\omega)$, $\mu(\omega)$ taken from the results shown in Fig. 2. The similarity to the simulation results, particularly for the SRR case, is striking, again confirming the isotropic behavior of the metamaterial slabs. Slight deviation of the simulated transmission for both SRR and LHM can be attributed to two principal deficiencies of our estimation of $\varepsilon(\omega)$ and $\mu(\omega)$: First, the artifacts arising from the periodicity will introduce an explicit dependence of the incident angle ϑ since the unit cell is rectangular (and we expect the band structure to look different for various directions). Second, due to the symmetric surface termination the

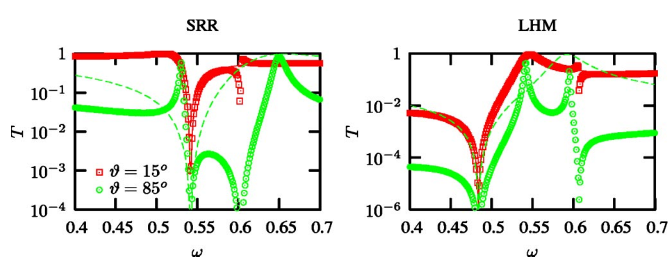

FIG. 5. (Color online) Analytic transmission spectra $T=|t(\omega)|^2$ of the TE and TM mode for a single layer of *homogeneous* unit cells of the SRR and LHM material for two angles of incidence. For $\mu(\omega)$ and $\varepsilon(\omega)$ we used the retrieved values for normal incidence (Fig. 2).

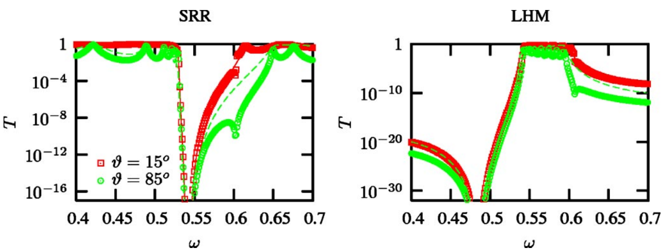

FIG. 6. (Color online) Same as Fig. 5 but for 10 unit cell thickness.

relevant periodicity for the retrieval shown in Fig. 2 is one mesh step longer than the period in the perpendicular directions. These effects will become negligible if we move the magnetic resonance to sufficiently low frequency while maintaining the size of the unit cell.

The LHM transmission spectra reveal an additional, more serious problem. The finite thickness of the slab leads to an essential asymmetry in the electric response between the directions parallel and normal to the surface. The electric field in the normal direction sees a finite, instead of a continuous wire, which changes the electric response, hence $\varepsilon(\omega)$ in that direction, leading to an deformed spectrum for large ϑ. This problem only affects the TM mode for oblique incidence. As is apparent from Fig. 4, within the LH passband we can expect this issue to become less important with the increasing thickness of the slab. However, in the transmission gaps the near unity reflection is dominated by the few surface layers of the slab, thus experiencing the termination of the finite wires for any system length.

We are aware that the geometry of the SRR with dielectric-filled gaps shown in Fig. 1 is not very convenient for experimental realizations. However, the issues discussed in this paper are generic and do not require this particular realization of the SRRs. Experimentally more realistic SRRs that can operate at very low frequency, not sacrificing the symmetry, can be designed based on multilayer four-gap structures or two-layer opposing conventional SRRs.[13] For low frequencies also the wires may be thinner, such that the crossing can easily replaced by centered wires which just touch each other in the center of the unit cell.

In conclusion, we presented a design for a LHM and SRR-only metamaterial based on a fully symmetric unit cell. Transfermatrix simulations of finite thickness slabs for oblique incident plane waves revealed almost isotropic transmission spectra. Further, the simultaneously negative ε and μ has been confirmed for the LHM slab. We emphasized the importance of symmetry issues in the design of isotropic metamaterials. The asymmetry introduced by the gaps in the conventional SRR design has to be especially avoided. Additionally, the magnetic resonance frequency has to be chosen much lower than the first band gap arising from the metamaterials intrinsic periodicity to avoid interference with the band structure and the breakdown of the effective medium behavior. Minor deviations from the isotropic behavior, which occur particularly in the LHM slab, have been attributed to the finite thickness of the metamaterial that modifies the electric response in the short direction. Most of the imperfections become unimportant for sufficiently long systems.

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