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Solid-state quantum communication with Josephson arrays

Alessandro Romito and Rosario Fazio
NEST-INFM & Scuola Normale Superiore, Piazza dei Cavalieri 7, 56126 Pisa, Italy

C. Bruder
Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, 4056 Basel, Switzerland

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Josephson junction arrays can be used as quantum channels to transfer quantum information between distant sites. In this paper we discuss simple protocols to realize state transfer with high fidelity. The channels do not require complicated gating but use the natural dynamics of a properly designed array. We investigate the influence of static disorder both in the Josephson energies and in the coupling to the background gate charges, as well as the effect of dynamical noise. We also analyze the readout process, and its back action on the state transfer.

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The transmission of a quantum state through a channel between distant parties is an important issue in quantum communication. In optical systems photons can be transferred coherently over large distances.[1] However, it is also highly desirable to have similar protocols for quantum information transfer in solid-state environments. A possible solution would be to interface solid-state quantum hardware to optical systems.[2] Another possibility is to use flying qubits, i.e., to transfer the physical qubits along leads.[3] Inspired by the paper of Bose[4] we developed the idea to construct a genuine quantum transmission line using a Josephson junction array.

Recently, a spin chain with ferromagnetic Heisenberg interactions has been proposed for quantum communication.[4] It was shown that Heisenberg chains can be used to transfer unknown quantum states over appreciable distances ($\sim 10^2$ lattice sites) with high fidelity.[4–7] By preparing the state to be transferred at one end of the chain and waiting for a well-defined time interval, one can reconstruct the state at the other end of the chain. Even perfect transfer could be achieved over arbitrary distances in spin chains.[8] Quantum state transport through harmonic chains was considered in Ref. 9.

Josephson qubits are among the most promising candidates as building blocks of quantum information processors.[10,11] In this Rapid Communication, we extend their application range to quantum communication and show that a one-dimensional Josephson array is a natural transmission line for systems with superconducting charge qubits. We calculate the transmission fidelity and investigate the effect of static inhomogeneities and dynamical noise. We also analyze the readout process by a single-electron transistor (SET) at the end of the array. To our knowledge, this is the first realizable and concrete implementation of a solid-state quantum communication protocol following the idea of Bose.[4]

The model that we want to study is schematically illustrated in Fig. 1 and described by the Hamiltonian $H=H_{\mathrm{JJ}}+H_{\mathrm{qp}}+H_{\mathrm{coup}}$, where

$$H_{\mathrm{JJ}} = \frac{1}{2}\sum_{ij}^{L} (Q_i - Q_{xi})C_{ij}^{-1}(Q_j - Q_{xj}) - E_J\sum_{i}^{L-1} \cos \phi_{i,i+1} \quad (1)$$

is the Hamiltonian of a one-dimensional Josephson junction array[12] of length L, and $\phi_{i,i+1}=\phi_i-\phi_{i+1}$. The other terms of the Hamiltonian describe the measurement apparatus and will be discussed later. The charge Q_i and phase ϕ_i are canonically conjugated. The first term in Eq. (1) is the charging energy, C_{ij} is the capacitance matrix; the second is due to Josephson tunneling. An external gate voltage V_{xi} gives a contribution to the energy via the induced charges $Q_{xi}=2eq_{xi}=V_{xi}C_{ii}$. This external voltage can be either applied to the ground plane or unintentionally caused by trapped charges in the substrate (in this case Q_{xi} will be a random variable). We assume that each island is coupled to its nearest neighbors by junction capacitances C and to the ground by a capacitance C_0. In this case, the charging interaction has a range given by $\sqrt{C/C_0}$ in units of the lattice spacing of the array.[12] In the following we put $\hbar=k_B=1$.

In the charge regime $e^2C_{00}^{-1} \gg E_J$, the system is approximately described by only two charge states for each island.

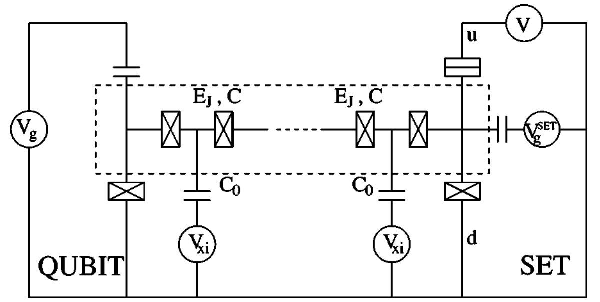

FIG. 1. Dashed box: one-dimensional Josephson array proposed for the transmission of quantum states. The crossed rectangles denote the Josephson junctions between the islands. The state prepared on the left-most island is transfered to the right-most island by the time evolution generated by the Hamiltonian. Left: the Cooper-pair box (charge qubit) used to prepare the state. Right: the SET transistor used as measurement device.

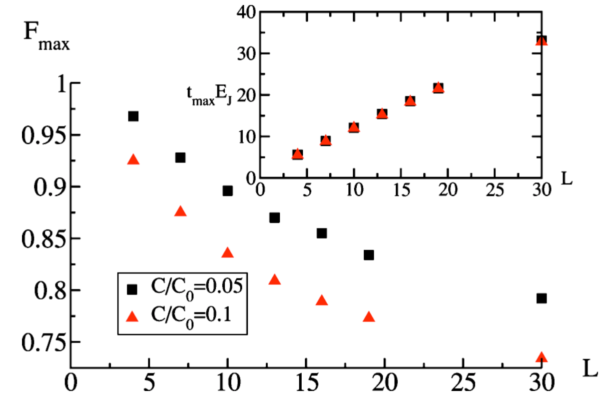

FIG. 2. (Color online) Maximum value of the fidelity as a function of the length of the chain for two different values of $C/C_0 \ll 1$ and $(2e)^2/(E_J C_0)=10$. Inset: The time at which the maximum is reached.

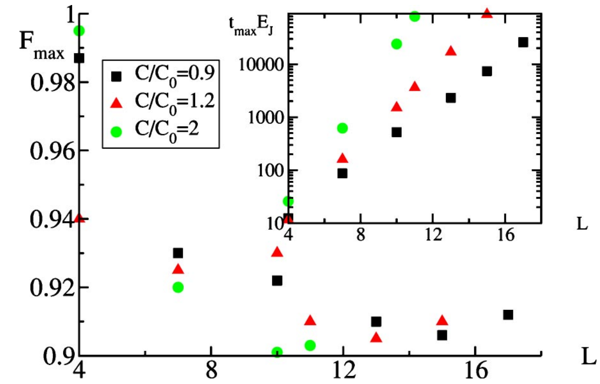

FIG. 3. (Color online) Maximum value of the fidelity as a function of the length of the chain for three different values of $C/C_0 \gtrsim 1$ and $(2e)^2/(E_J C_0)=10$. Inset: The time at which the maximum is reached.

The chain Hamiltonian H_{JJ} is equivalent to an anisotropic XXZ spin-1/2 Heisenberg model,[13,14] the Josephson chain is thus different from the XY and Heisenberg cases.[4] It is characterized by a strong anisotropy between the z direction and the xy plane. Moreover the z coupling has a range that depends on the electrostatic energy and can extend over several lattice constants.

At $t=0$, the chain is initialized in the state $|\boldsymbol{\psi_0}\rangle = |\psi,000\ldots0\rangle$, where $|0\rangle$ ($|2\rangle$) denotes the state of an island without (with) an excess Cooper pair, and $|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|2\rangle$ is the state that has been prepared in the left-most island. This initial state is not an eigenstate of the Hamiltonian; it will evolve as a function of time. In fact, as the total charge $Q=\Sigma_i Q_i$ is a conserved quantity, the dynamics is restricted to the $L+1$-dimensional space $\mathcal{H}=\mathcal{H}_0 \oplus \mathcal{H}_2$ of total charge zero, $\mathcal{H}_0$, and charge two, $\mathcal{H}_2 = \text{span}\{|j\rangle\}$, where $|j\rangle$, $1 \leq j \leq L$ is the state with an excess Cooper pair on the jth site. In this basis the Hamiltonian reads

$$H_{\text{JJ}}|j\rangle = 2e^2\left(C_{jj}^{-1} - 2\sum_{i=1}^{L} C_{ij}^{-1} q_{xi}\right)|j\rangle - \frac{E_J}{2}[(1-\delta_{jL})|j+1\rangle + (1-\delta_{j1})|j-1\rangle]. \tag{2}$$

We first calculate the fidelity of transmission and the time required for the transfer of information as a function of the coupling constants of the Josephson chain. The quality of the transmission is quantified by the fidelity of the (mixed) state ρ_L of the right-most island (site L) to the initial state

$$F_L(t) = \frac{1}{4\pi}\int \langle\psi|\rho_L(t)|\psi\rangle d\Omega. \tag{3}$$

This definition gives the fidelity averaged over all possible initial states on the Bloch sphere, $1/2 \leq F_L \leq 1$.

The fidelity is a strongly oscillating function of time. Only at well-defined times the state is transferred faithfully through the chain. This does not necessarily correspond to the time in which a Cooper pair has been transferred, since also the relative phases of the state have to be reconstructed. In Fig. 2 we show the value of the first fidelity maximum and the time at which it is reached as a function of the length L of the array and for different values of the ratio C/C_0. For the parameters considered, the fidelity is never smaller than 75%. For longer chains, or if the condition $C_0 \gg C$ is released, the first maximum of the fidelity is considerably reduced. Another option is to fix a threshold for the fidelity of transmission and seek for the first local maximum above the threshold. The time at which these maxima occur increases exponentially with the chain length. The value of the fidelity does not necessarily decrease on increasing L, and for larger arrays a higher fidelity can be achieved (although at larger times), see Fig. 3. The results of Figs. 2 and 3 are encouraging since they indicate that faithful state transmission using Josephson chains is already possible with present-day technology.

Since experimental arrays are never completely homogeneous, we now consider the case in which a small amount of static disorder is present. In general, imperfections will reduce the fidelity. In Fig. 4 we show both the effects of bond disorder (Josephson couplings distributed around an average value) and site disorder (mimicking the effect of static back-

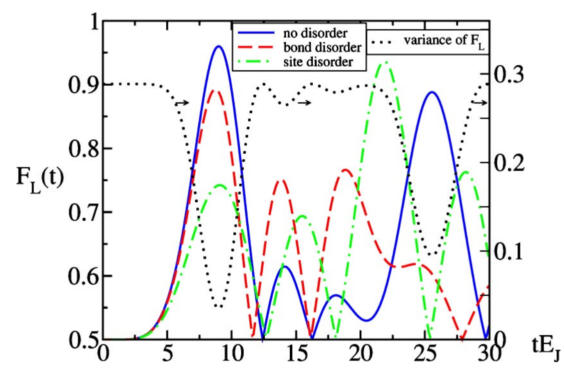

FIG. 4. (Color online) Fidelity as a function of time for an array of length $L=7$, $(2e)^2/(E_J C_0)=10$ and $C=0$, i.e., a junction capacitance much smaller than the ground capacitance. Disorder parameters: relative variance $\Delta E_J/E_J=0.1$ for bond disorder; absolute variance $\Delta Q_x/2e=0.025$ for site disorder. Dotted line (right axis): variance of the fidelity for the case without disorder.

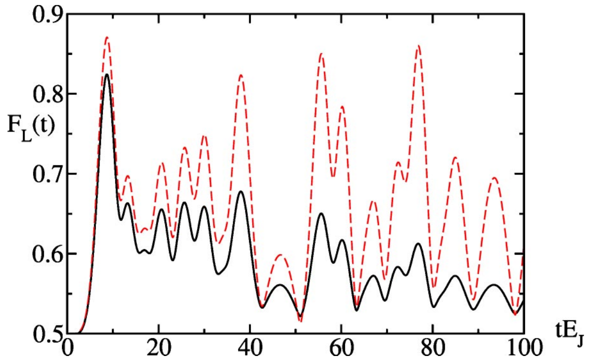

FIG. 5. (Color online) Fidelity vs time in the presence of gate voltage fluctuations (full line). The dashed line is the noiseless case. $L=7$, $e^2/(E_J C_0)=10$, $C/C_0=0.1$, $\gamma=0.01$.

ground charges and/or different capacitances) and compare them to the case without disorder. The effect of charge disorder appears to be more disruptive. This is because additional frequencies enter the dynamical evolution making the reconstruction of the additional wave function more difficult. The dotted line in Fig. 4 shows the variance of the fidelity for the case without disorder. Around the maxima of the fidelity, the variance is small, i.e., the transmission quality for any given state is high.

Dynamical fluctuations play a different role. They arise from gate-voltage fluctuations and are described by adding stochastic terms to the gate voltages, $q_{xi} \to q_{xi} + \xi_i(t)$ in the Hamiltonian in Eq. (1). Here we choose a very simple model and assume the $\xi_i(t)$ to be independently Gaussian distributed: $\langle \xi_i(t) \rangle = 0$, $\langle \xi_i(t) \xi_j(t') \rangle = \gamma \delta_{ij} \delta(t-t')$. Nevertheless, due to capacitive coupling between separated sites, such stochastic factors result in correlated stochastic terms in the effective Hamiltonian Eq. (2), $H_{\text{noise}}|j\rangle = H_{\text{JJ}}|j\rangle - 2\Xi_j(t)|j\rangle$, where the zero-averaging Gaussian distributed functions $\Xi_i(t)$, are uniquely fixed by $\langle \Xi_i(t) \Xi_j(t') \rangle = \gamma [(C^{-1})^2]_{ij} \delta(t-t')$. Averaging out the stochastic terms leads to the master equation for the density matrix of the chain in the space $\mathcal{H}$,

$$\dot{\rho} = -i[H_{\text{JJ}}, \rho] - \sum_{i,j=1}^{L} \frac{\gamma [(C^{-1})^2]_{ij}}{8e^2} (Q_i Q_j \rho - 2 Q_i \rho Q_j + \rho Q_i Q_j), \tag{4}$$

where the operators Q_i projected on the space $\mathcal{H}$ are $Q_i = 2e|i\rangle\langle i|$. The state of the system develops into a completely incoherent mixture in which any charged state is equally probable, $\rho_{ii}(t \to \infty) = (1/L)\rho_{11}(t=0)$. The average fidelity as defined in Eq. (3) is reduced to $F_\infty = 1/2 + 1/(6L)$, corresponding to an almost unfaithful transmission. The time dependence of the fidelity in the noisy system is presented in Fig. 5 where it is compared with the fidelity in the absence of noise. The peaks of the fidelity are not smeared out by noise. The dominant effect of the coupling to the environment is the relaxation of the fidelity amplitude towards the stationary value (independent on the initial state). Numerically, such relaxation takes place on a characteristic time scale $\sim 1/(L\gamma)$. Thus, to observe high values of the fidelity it is important to have a maximum at a short time. As a consequence, $C/C_0 \ll 1$ is preferable, and the condition $\gamma \ll E_J/L^2$ is required to have a high value for the first maximum of the fidelity.

Finally we discuss how the fidelity can be measured in a practical setup. To do this, we assume that the right-most island (site L) is part of a SET transistor. We therefore specify the effective coupling Hamiltonian between the right-most island and the leads,[15]

$$H_{\text{qp}} = \sum_{b=u,d,L} \sum_{k,\sigma} \epsilon_b(k) \gamma^{\dagger}_{k\sigma b} \gamma_{k\sigma b}, \tag{5}$$

$$H_{\text{coup}} = \sum_{b=u,d,L} [e^{-i(\phi_L - \varphi_b)/2} X_b + \text{H.c.}] + - \sum_{b=u,d} J_b \cos(\phi_L - \varphi_b) - \sum_{b=u,d} V_b Q_b, \tag{6}$$

where γ, $\gamma^\dagger$ are annihilation and creation operators of quasiparticles in the grain L and in the leads, u and d (see Fig. 1). The operator $X_b = \Sigma_{k,q,\sigma} T_{qk} \gamma^{\dagger}_{k\sigma b} \gamma_{q\sigma L}$ describes quasiparticles tunneling into the grain with an associated charge increasing $e^{-i(\phi_L - \varphi_b)/2}$. Q_b is the total charge entering the chain from the up or down reservoirs. We assume nonvanishing quasiparticle tunneling only across the upper junction $(eV_d = 0, eV_u = -eV \approx -2\Delta)$ and conversely we allow coherent Cooper pairs tunneling only in the lower junction $(J_u = 0, J_d = J \ll E_J)$. Gate voltages are chosen so that the SET is off resonance and we can therefore neglect the stationary current through the SET due to the Cooper-pair quasiparticle cycle.[15]

The measurement device modifies the dynamics of Cooper pairs on the chain and requires taking into account quasiparticle excitations on the Lth site of the chain. By neglecting quasiparticle tunneling we would have a coherent dynamics for the charges in the chain described by the Hamiltonian $H_0 = H(T_{qk} \to 0)$. Tracing out the quasiparticle degrees of freedom results, instead, in an incoherent dynamics described by a master equation for the reduced density matrix $\tilde{\rho}$ of charges in the chain.[16] In the basis of eigenstates of H_0, $H_0|\alpha\rangle = E_\alpha|\alpha\rangle$,[17] the master equation reads[16]

$$\dot{\tilde{\rho}}_{\alpha\beta}(t) = -i\langle \alpha | [H_0, \tilde{\rho}] | \beta \rangle - \sum_{\mu\nu}{}' R_{\alpha\beta\mu\nu} \tilde{\rho}_{\mu\nu}. \tag{7}$$

The prime indicates that the sum has to be performed over states with energies such that $|E_\alpha - E_\beta - E_\mu + E_\nu| \ll 1/\Delta t$, Δt being the time over which the coarse-graining implicit in Eq. (7) takes place.

As we are interested in the time evolution over short times, let us discuss in some detail our approximations. We first assume that $J \ll E_J$, so that, in evaluating the kernel R, we neglect the Josephson coupling to the leads.[18] In this case the spectrum of H_0 is $\{E_0, E_{1/2}, E_{\bar{M}}^-\}$, $\bar{M} = 1, \dots, L$. Due to the energy scale separation $E_{\bar{M}}^- - E_{\bar{N}}^- \lesssim E_J \ll 1/\Delta t \lesssim E_{\bar{M}}^- - E_{1/2} \sim E_{\bar{M}}^- - E_0 \ll eV$, the sum in Eq. (7) mixes population and coherences of the density matrix only in the subspace of $\bar{M}$ states. In this case the coarse-grained dynamics of Eq. (7) can resolve the time scales of order $1/E_J$ that we are interested in. In this approximation, the only nonvanishing terms of the kernel R are found to be $R_{00(1/2)(1/2)} \simeq -R_{(1/2)(1/2)(1/2)(1/2)} \simeq -\Gamma$, $\mathrm{Re}\{R_{(1/2)(1/2)\bar{M}\bar{N}}\} \simeq -\lambda_{\bar{M}\bar{N}}$, $\mathrm{Re}\{R_{\bar{M}0\bar{N}0}\} \simeq \frac{1}{2}\lambda_{\bar{M}\bar{N}}$, $\mathrm{Re}\{R_{\bar{M}\bar{N}\bar{P}\bar{Q}}\} \simeq \frac{1}{2}[\delta_{\bar{M},\bar{P}}\lambda_{\bar{N}\bar{Q}} + \delta_{\bar{N},\bar{Q}}\lambda_{\bar{M}\bar{P}}]$, where $\lambda_{AB} = \Gamma\langle A|L\rangle\langle L|B\rangle$. In these expressions we approximated $\Gamma \simeq \int_0^\infty ds \exp(ieVs)\langle X_u(s)X_u^\dagger(0)\rangle \simeq \int_0^\infty ds \exp\{i[eV \pm (E_{\bar{M}} - E_{1/2})]s\}\langle X_u(s)X_u^\dagger(0)\rangle$ as a consequence of the separation of energy scales discussed above. We also neglected all other exponentially small $(\sim e^{-eV/T})$ rates.

Finally let us address the proposed measurement protocol. It consists in disconnecting the right-most site from the rest of the chain at a time $t^\star \ll 1/\Gamma$ and in measuring the time-integrated current through the SET $I = e/T\int_0^\infty \tilde{I}(\tau)d\tau = e/T\int_{t^\star}^\infty \tilde{I}(\tau)d\tau + \mathcal{O}(\Gamma t^\star)$ where T is the time between two pulses; it is the largest time scale in the system. The instantaneous particle current is $\tilde{I}(t) = \Gamma[\tilde{\rho}_{LL}(t) + \tilde{\rho}_{(1/2)(1/2)}(t)]$. The last term $\mathcal{O}(\Gamma t^\star)$ in the current corresponds to $\int_0^{t^\star}\tilde{I}(\tau)d\tau = \int_0^{t^\star} dt[\tilde{\rho}_{LL}(t) + \tilde{\rho}_{(1/2)(1/2)}(t)] < \Gamma t^\star$. As at time $t > t^\star$ the SET is disconnected from the rest of the chain and is out of resonance, the measured current is $\sim e/T[2\tilde{\rho}_{LL}(t^\star) + \tilde{\rho}_{(1/2)(1/2)}(t^\star)]$. This measurement scheme does not provide a tomography for the state of the right-most site: the measured current does not depend on the coherences of $\rho_L(t^\star)$, to which the fidelity is sensitive. Nevertheless, the peaks in the current correspond exactly to the maxima of the fidelity as shown in Fig. 6. The current decay in time due to quasiparticle tunneling happens on a time scale $\sim 1/\Gamma$ irrespective of the length of the chain. Therefore, our measurement scheme can be used also for long chains, the main constraint are disorder and gate voltage fluctuations. In this sense the current measurement allows to check the theoretical prediction for the fidelity of state transfer.

In conclusion, we have proposed to use a Josephson junction chain as a solid-state quantum communication channel. We have analyzed the state preparation, its propagation (with a model appropriate for Josephson nanocircuits), the role of the measuring apparatus and the effect of noise and imperfections. This, we believe, is an important and necessary step towards the experimental realization of quantum communication in solid-state systems. Present-day technology should allow the realization of quantum channels of the type described here.

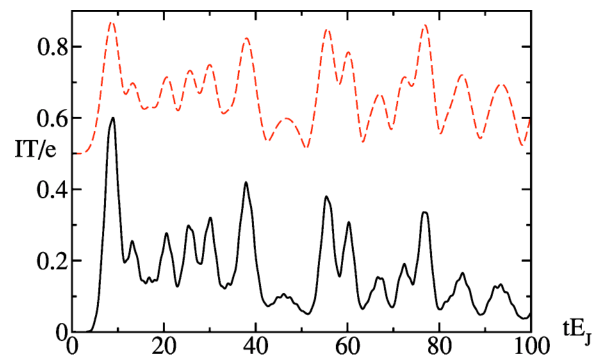

FIG. 6. (Color online) Full line: time dependence of the current (in units of e/T) through the SET. Dashed line: fidelity of the isolated chain. $\Gamma = 0.05$; all other parameters as in Fig. 5.

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