Reduced visibility of Rabi oscillations in superconducting qubits

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Coherent Rabi oscillations between quantum states of superconducting microcircuits have been observed in a number of experiments, albeit with a visibility that is typically much smaller than unity. Here, we show theoretically that the coherent coupling of superconducting phase qubits to background charge fluctuators [R. W. Simmonds *et al.*, Phys. Rev. Lett., **93**, 77003 (2004)] leads to a significantly reduced visibility if the Rabi frequency is comparable to the coupling energy of microcircuit and fluctuator. For larger Rabi frequencies, transitions to the second excited state of the superconducting microcircuit become dominant in suppressing the Rabi oscillation visibility.

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I. INTRODUCTION

Possible applications in quantum information processing have renewed the interest in quantum coherent phenomena of micrometer-scale Josephson junction (JJ) circuits.[1] During recent years, several experiments have demonstrated Rabi oscillations of a macroscopic variable including charge, phase, combinations of both, and flux,[2–6] which persist up to several microseconds.[4] However, in many of the reported experiments, the Rabi oscillation visibility is significantly smaller than unity even at times short compared to the decoherence time. Characteristic values are as small as 10% (Ref. 4) and 50%,[3,7] respectively, which indicates either substantial leakage out of the computational basis or a pronounced randomization of the state occupation over short time scales. In the following, we discuss three possible mechanisms that result in a reduced visibility, namely, (i) coherent coupling to a background fluctuator which induces transitions between the eigenstates of the superconducting microcircuit;[7] (ii) population of higher excited states caused by a strong driving field, i.e., the ac current or voltage applied to drive Rabi oscillations; and (iii) the excitation of Bogoliubov quasiparticles.

All three mechanisms are well-known sources of decoherence in superconducting qubits (squbits) and their contributions to the decoherence time have been quantified.[8–15] The purpose of this paper is to establish to what extent these mechanisms contribute to the reduced visibility of squbit Rabi oscillations. The identification of the dominant mechanism responsible for the small visibility is of central importance for large-scale quantum algorithms, where fidelities approaching unity are required for an individual gate. The main result of this paper is that fluctuators are the dominant source of visibility reduction for phase squbits at Rabi frequencies small compared to the squbit-fluctuator coupling strength, while leakage becomes increasingly important for large Rabi frequencies. Although our model is technically related to systems analyzed in the context of decoherence, there are important differences. Decoherence models consider fluctuators with pronounced incoherent dynamics coupled to the squbit by an Ising-like interaction.[9–11] By contrast, motivated by recent experiments,[7,16] we show that the *coherent* dynamics of a squbit-fluctuator system with a *transverse* exchange coupling leads to a reduced visibility and beating in the Rabi oscillation signal of the squbit. Technically, our system is most closely related to models for two spins interacting with a bath analyzed in the context of decoherence recently.[17,18] For large Rabi oscillation frequencies, we identify virtual transitions to the second excited squbit state as dominant source of reduced visibility. This mechanism persists for adiabatic switching, in stark contrast to the leakage scenarios discussed previously.[12,13]

II. COHERENT SQUBIT-FLUCTUATOR COUPLING

The anticrossings in the microwave spectra of a phase squbit[7] and the observation of real-time oscillations[16] provide strong evidence that some fluctuators are coupled coherently to a squbit. Because the squbit-fluctuator coupling has a transverse component,[7] fluctuators induce *transitions* between different squbit basis states, randomizing the occupation probabilities. We introduce a pseudospin notation and define $\hat{s}_z=(|1\rangle\langle 1|-|0\rangle\langle 0|)/2$ and $\hat{s}_x=(|1\rangle\langle 0|+|0\rangle\langle 1|)/2$ in terms of the ground (first excited) state $|0\rangle$ ($|1\rangle$) of the squbit, such that $|s_z=\downarrow\rangle$ represents the squbit ground state. Similarly, $|I_z=\downarrow\rangle$ and $|I_z=\uparrow\rangle$ denote the ground and first excited states of the fluctuator, respectively. Squbit Rabi oscillations are driven by an ac current resonant with the squbit level splitting $\hbar\omega_{10}$. In our pseudospin notation, the ac current acts as transverse magnetic field b_x and, in the corotating frame,

$$\hat{H} = \delta\hat{I}_z + J(\hat{s}_x\hat{I}_x + \hat{s}_y\hat{I}_y) + b_x\hat{s}_x, \tag{1}$$

where $\delta=\hbar(\omega_{eg}-\omega_{10})$ is the detuning of the fluctuator level splitting $\hbar\omega_{eg}$ relative to the squbit. The ansatz for the transverse squbit-fluctuator exchange coupling with coupling strength J is microscopically motivated from the observation that the critical current of a JJ may depend on the position of a fluctuator in its bistable potential.[7] As shown below, a fluctuator influences the squbit dynamics strongly if $|\delta|<J$ or $|\delta\pm b_x|<J$. For the JJ in Ref. 7, $J/h\simeq 25$ MHz while the typical level spacing between different fluctuator resonances is of order 60 MHz. This allows us to restrict our analysis to

one fluctuator with minimum $|\delta|$ or $|\delta \pm b_x|$ first. For the experimental temperature $T \simeq 20$ mK$\ll \hbar\omega_{10}/k_B$, $\hbar\omega_{eg}/k_B$, at the beginning of the Rabi pulse, $t=0$, both squbit and fluctuator are in their ground states and $|\psi(0)\rangle = |\downarrow;\downarrow\rangle$ in the product basis $|s_z;I_z\rangle$. The experimentally accessible quantity is the probability $p_1(t)$ for the squbit to occupy its first excited state as a function of Rabi pulse duration, $p_1(t)=\Sigma_{I_z=\uparrow,\downarrow}|\langle\uparrow;I_z|\psi(t)\rangle|^2$. While the energy level splitting $\hbar\omega_{eg}$ of a fluctuator is fixed, the squbit level splitting $\hbar\omega_{10}$ can be tuned via the dc bias current through the JJ, which allows one to measure squbit Rabi oscillations for varying δ and b_x.

We now calculate the dynamics of $|\psi(0)\rangle = |\downarrow;\downarrow\rangle$ and $p_1(t)$ as a function of δ and b_x. The squbit-fluctuator exchange coupling gives rise to a linear dependence of the eigenenergies on J for $|\delta| \lesssim J$ and $|\delta \pm b_x| \lesssim J$, where cross-relaxation processes between squbit and fluctuator reduce the visibility of the squbit Rabi oscillations.[19] While $p_1(t)$ is readily obtained from integration of the Schrödinger equation for arbitrary b_x, δ, and J, here we focus on the two cases $\delta=0$ and $\delta \pm b_x=0$, where the cross relaxation between squbit and fluctuator is most efficient. For $\delta=0$, we find

$$p_1(t)|_{\delta=0} = \frac{1}{2}\left[1-\cos\left(\frac{Jt}{2\hbar}\right)\cos\left(\frac{\sqrt{J^2+4b_x^2}t}{2\hbar}\right) - \frac{\sin(Jt/2)\sin[(\sqrt{J^2+4b_x^2}t)/2\hbar]}{\sqrt{1+(2b_x/J)^2}}\right] \xrightarrow{|b_x|/J\gg 1} \frac{1}{2} \times\left[1-\cos\left(\frac{Jt}{2\hbar}\right)\cos\left(\frac{b_x t}{\hbar}\right)\right]. \tag{2}$$

For $\delta \pm b_x=0$, the exact expression for $p_1(t)$ is too long to be presented here, but for $|b_x|/J \gtrsim 2$ it is well approximated by

$$p_1(t)|_{\delta\pm b_x=0} \simeq \frac{1}{2}\left[1-\cos\left(\frac{Jt}{4\hbar}\right)\cos\left(\frac{b_x t}{\hbar}\right)\right]. \tag{3}$$

Equations (2) and (3) describe the Rabi oscillations of a squbit in the presence of a fluctuator. The transverse coupling to a fluctuator introduces an additional Fourier component which leads to beating of the Rabi-oscillation signal, in agreement with experimental results, where typically $|b_x|/J \geqslant 2$. For short time scales, the J-dependent factor in Eqs. (2) and (3) leads to a decrease in Rabi oscillation visibility. In particular, the first maximum in $p_1(t)$ is reduced relative to unity. For $|b_x|/J \gtrsim 2$, we obtain $p_1(t=\pi\hbar/b_x) \simeq 1-(\pi J/4b_x)^2$ and $p_1(t=\pi\hbar/b_x) \simeq 1-(\pi J/8b_x)^2$ for $\delta=0$ and $\delta \pm b_x=0$, respectively.[20] While a resonant fluctuator reduces $p_1(t=\pi\hbar/b_x)$ to 0.85 for $|b_x|/J \simeq 2$, Rabi oscillations with amplitude 1 are predicted to emerge for $|b_x|/J \gg 1$, in stark contrast to experimental results where the first local maximum of $p_1(t)$ is of order 0.5 even for large b_x.[7]

In a realistic system, decoherence of the squbit and fluctuator dynamics will damp out both the Rabi oscillations and the beating predicted for $p_1(t)$ in Eqs. (2) and (3). In order to discuss reduced visibility in the presence of decoherence, we focus on a simple model system in which the fluctuator has a finite decoherence rate $\gamma/\hbar$ while its relaxation rate vanishes ("diagonal coupling" of a bath and fluctuator). The dynamics of the system is determined by the master equation for the density matrix,[21]

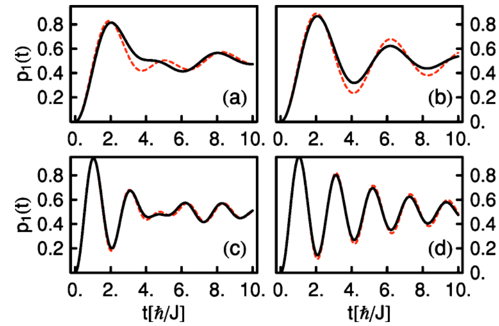

FIG. 1. (Color online) Rabi oscillations for a squbit-fluctuator system. The probability $p_1(t)$ to find the squbit in state $|1\rangle$ is obtained from numerical integration of Eq. (4) (solid line) and the analytical solution Eq. (5) (dashed line) which is valid for $b_x/J \gg 1$. For weak decoherence of the fluctuator, $\gamma/J<1$ [(a) and (c)], $p_1(t)$ shows damped beating. For $\gamma/J>1$, damped single-frequency oscillations are restored. The fluctuator leads to a reduction of the first maximum in $p_1(t)$ to ~ 0.8 [(a) and (b)] and ~ 0.9 [(c) and (d)], respectively. The parameters are (a) $b_x/J=1.5$, $\gamma/J=0.5$; (b) $b_x/J=1.5$, $\gamma/J=1.5$; (c) $b_x/J=3$, $\gamma/J=0.5$; (d) $b_x/J=3$, $\gamma/J=1.5$.

$$\dot{\hat{\rho}}(t) = -\frac{i}{\hbar}[\hat{H},\hat{\rho}(t)] + (\gamma/2\hbar)[4\hat{I}_z\hat{\rho}(t)\hat{I}_z - \hat{\rho}(t)]. \tag{4}$$

A similar system was analyzed in Ref. 17 using different numerical and approximate analytical approaches. Here, we provide an analytical solution of Eq. (4) for $|b_x|/J \gg 1$ where, to leading order in J, $\hat{H} \simeq \delta\hat{I}_z + J\hat{s}_x\hat{I}_x + b_x\hat{s}_x$. Then, the 16 coupled differential equations in Eq. (4) decouple into sets of four differential equations. We calculate $p_1(t)=\Sigma_{I_z=\uparrow,\downarrow}\langle\uparrow;I_z|\hat{\rho}(t)|\uparrow;I_z\rangle$ from the explicit solution of Eq. (4). For $\delta=0$ and $\hat{\rho}(0)=|\downarrow;\downarrow\rangle\langle\downarrow;\downarrow|$,

$$p_1(t) \simeq \frac{1}{2} - \frac{1}{4}\,\mathrm{Re}\left\{\left[\left(1-\frac{\gamma}{\sqrt{\gamma^2-J^2}}\right)e^{-(\gamma+\sqrt{\gamma^2-J^2})t/2\hbar} + \left(1+\frac{\gamma}{\sqrt{\gamma^2-J^2}}\right)e^{-(\gamma-\sqrt{\gamma^2-J^2})t/2\hbar}\right]e^{-ib_x t/\hbar}\right\}. \tag{5}$$

Note the intriguing dependence of $p_1(t)$ on the fluctuator decoherence rate $\gamma/\hbar$ and the coupling strength J (Fig. 1). For $\gamma/J \ll 1$,

$$p_1(t) \simeq \frac{1}{2}\left[1-e^{-\gamma t/2\hbar}\cos\left(\frac{Jt}{2\hbar}\right)\cos\left(\frac{b_x t}{\hbar}\right)\right] \tag{6}$$

shows the beating already derived in Eq. (2), but with a finite damping rate $\gamma/2\hbar$. In contrast, for $\gamma/J \gg 1$,

$$p_1(t) \simeq \frac{1}{2}\left[1-e^{-J^2 t/4\hbar\gamma}\cos\left(\frac{b_x t}{\hbar}\right)\right] \tag{7}$$

exhibits single-frequency oscillations with a damping rate $J^2/4\hbar\gamma$ (see also Ref. 17). This can be understood from the observation that, for $\gamma \gg J$, the phase of the fluctuator is randomized on a time scale short compared to h/J and the mean field $J\langle\hat{I}_x\rangle$ acting on the squbit is averaged out to leading order, which restores the single-frequency oscillations in Eq. (7).

We discuss next to what extent far-off-resonant fluctuators with $|\delta| \gg |b_x| \gg J$ reduce the Rabi-oscillation visibility. Because of the perturbative parameter $J/|\delta| \ll 1$, the visibility reduction in $p_1(t) \simeq (1 - J^2/4\delta^2)\sin^2(b_x t/2\hbar)$ is small for a single off-resonant fluctuator. More generally, a large number of off-resonant fluctuators interacts with the squbit, and the problem is closely related to the dynamics of an electron spin coupled to a bath of nuclear spins by the hyperfine contact interaction.[22] To leading order in the coupling constants, a set $\{i\}$ of off-resonant fluctuators with coupling constants J_i and detunings δ_i reduces the Rabi-oscillation visibility by $\Sigma_i J_i^2/4\delta_i^2$. Introducing the distribution function $P(E)$ of the fluctuator level splittings, for constant $J_i = J$,

$$\sum_i J_i^2/4\delta_i^2 = \frac{J^2}{4}\int_{|E-\hbar\omega_{10}|>|b_x|;E>E_c} \frac{dEP(E)}{(\hbar\omega_{10} - E)^2}, \tag{8}$$

where the integral is evaluated for all energies with $|E - \hbar\omega_{10}| > |b_x|$ (off-resonance condition) larger than a lower energy cutoff E_c given by the fluctuator decoherence rate. Evaluating the integral for the distribution function $P(E) \propto 1/E$ characteristic for fluctuator level spacings ($1/f$ noise), we find $\Sigma_i J^2/4\delta_i^2 \simeq 2J^2 P(\hbar\omega_{10})/4|b_x| = 0.2J/|b_x|$ for the experimental parameters of Ref. 7. If cross correlations between fluctuators can be neglected, the reduction of the first maximum in $p_1(t)$ is obtained by summing the contributions from resonant fluctuators with $\delta \simeq 0$ and $\delta \pm b_x \simeq 0$ [Eqs. (2) and (3)] and off-resonant fluctuators [Eq. (8)]. An upper bound is given by $1 - p_1(t = \pi\hbar/b_x) \leqslant 2J^2 P(\hbar\omega_{10})/4|b_x| + (3/32)(\pi J/b_x)^2$, which decreases with increasing $|b_x|$. This shows that, even when both resonant and off-resonant fluctuators are taken into account, the small visibility $p_1(t = \pi\hbar/b_x) \leqslant 0.5$ observed in the limit of large Rabi frequencies, $|b_x|/h \gg J/h$ in Ref. 7, cannot be effected by fluctuators.

III. ENERGY SHIFTS INDUCED BY ac DRIVING FIELD

Exploring the visibility reduction at a time scale of 10 ns requires Rabi frequencies $|b_x|/h \gtrsim 100$ MHz. We show next that, in this regime, transitions to the second excited squbit state lead to an oscillatory behavior in $p_1(t)$ with a visibility smaller than 0.7. For characteristic parameters of a phase squbit, the second excited state $|2\rangle$ is energetically separated from $|1\rangle$ by $\omega_{21} = 0.97\omega_{10}$.[10] Similarly to $|0\rangle$ and $|1\rangle$, the state $|2\rangle$ is localized around the local energy minimum in Fig. 2(a). For adiabatic switching of the ac current, transitions to $|2\rangle$ can be neglected as long as $|b_x| \ll \hbar\Delta\omega = \hbar(\omega_{10} - \omega_{21}) \simeq 0.03\hbar\omega_{10}$. However, for b_x comparable to $\hbar\Delta\omega$, the applied ac current strongly couples $|1\rangle$ and $|2\rangle$ because $\langle 2|\hat{\phi}|1\rangle \neq 0$, where $\hat{\phi}$ is the phase operator. For typical parameters, $b_x/\hbar\Delta\omega$ ranges from 0.05 to 1, depending on the irradiated power.[7,10] Taking into account the second excited state of

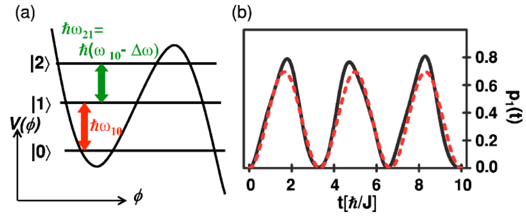

FIG. 2. (Color online) (a) Level scheme for the squbit in Refs. 7 and 10. (b) $p_1(t)$ obtained from numerical integration of the Schrödinger equation (solid line) in comparison with the analytical result in Eq. (10)(dashed line) for $b_x/\hbar\Delta\omega = 1/3$.

the phase squbit, the squbit Hamiltonian in the rotating frame is[13]

$$\hat{H} = -\hbar\Delta\omega|2\rangle\langle 2| + b_x(|0\rangle\langle 1| + \sqrt{2}|1\rangle\langle 2| + \text{H.c.})/2. \tag{9}$$

In the following, we neglect fluctuators, which is valid for the short-time dynamics if $|b_x| \gg J$.

The time evolution of $|\psi(0)\rangle = |0\rangle$ is readily calculated by integration of the Schrödinger equation. Expanding $|\psi(t)\rangle$ in $(b_x/2\hbar\Delta\omega)$, we find that

$$p_1(t) = \left[1 - \frac{3}{2}\left(\frac{b_x}{2\hbar\Delta\omega}\right)^2\right]^2 \sin^2 \frac{b_x[1 - (b_x/2\hbar\Delta\omega)^2]t}{\hbar} \tag{10}$$

exhibits single-frequency oscillations with reduced visibility [Fig. 2(b)]. Part of the visibility reduction can be traced back to leakage into state $|2\rangle$. More subtly, off-resonant transitions from $|1\rangle$ to $|2\rangle$ induced by the driving field lead to an energy shift of $|1\rangle$, such that the transition from $|0\rangle$ to $|1\rangle$ is no longer resonant with the driving field, which also reduces the visibility. For $b_x/2\hbar\Delta\omega = 1/3$, corresponding to $b_x/h = 150$ MHz in Ref. 7, Eq. (10) predicts a visibility of 0.7. Note that the off-resonant transition to $|2\rangle$ discussed here is induced by a large *amplitude* of the driving field and not by additional *frequency* components that result, e.g., from nonadiabatic switching of the driving field[12] or current noise in the microcircuit.[23] Rather, the visibility reduction in Eq. (10) corresponds to a steady-state solution of the Schrödinger equation and persists even for adiabatic switching.[24] Whether such off-resonant transitions are responsible for the reduced Rabi oscillation visibility can be tested experimentally from the dependence of the Rabi frequency and amplitude on b_x. Equation (10) predicts a nonlinear dependence of the Rabi frequency on b_x, while the visibility is predicted to decrease quadratically with b_x. Decreasing visibility with increasing driving field has indeed been observed in some Rabi oscillation experiments,[25] but further quantitative analysis is required to determine the explicit functional dependence.

IV. EXCITATION OF QUASIPARTICLES

Transient high-frequency components in the switching pulses induce leakage to states outside the computational basis. Bogoliubov quasiparticle excitations represent a large class of excited states which is often ignored for the discussion of leakage because the excitation gap is large. For both charge-based and phase-based squbits, the excitation of quasiparticles during the measurement process has been quantified and is known to limit the decoherence time.[2,14,15] In phase squbits, quasiparticles trigger leakage to finite-voltage states or to the excited squbit states $|2\rangle$ or $|3\rangle$ by incoherent tunneling across the JJ at a rate determined by the single-particle tunneling rate and the total number of quasiparticles.[15]

While the number of quasiparticles created during readout can be decreased by increasing experimental waiting times,[15] Bogoliubov quasiparticles are also excited *during* an experimental cycle by high-frequency Fourier components of an external current or voltage pulse with $\omega > 2\Delta/\hbar$, where Δ is the energy gap of the superconductor. Microscopic mechanisms for quasiparticle excitation include, e.g., dissipative tunneling through the JJ. We calculate the excitation rate Γ of quasiparticle pairs for an oscillating current component $\delta I_\omega \cos(\omega t)$ with $\omega > 2\Delta/\hbar$. Using a semiclassical approximation, the high-frequency current induces oscillations $\delta\phi(t)$ of the superconducting phase around its equilibrium value $\phi_0 = \arcsin(I/I_c) \simeq \pi/2$, where I and I_c are the dc bias current and critical current of the JJ, respectively. The amplitude and frequency of $\delta\phi(t)$ are determined by the externally imposed current, $\delta\phi(t) = -[(e\delta I_\omega)/(\hbar\omega^2 C)]\cos\omega t$, where C is the JJ capacitance. Dissipative tunneling through the JJ—processes in which a quasiparticle pair is excited under absorption of energy $\hbar\omega$ from the bias current—is described by both the normal current and the Josephson cosine term.[26] To lowest nonvanishing order in $(e\delta I_\omega)/(\hbar\omega^2 C)$, the time-averaged power dissipation $\overline{P}$ at the JJ is[26]

$$\begin{aligned}\overline{P} &= \frac{1}{2}\left(\frac{\delta I_\omega}{C\omega}\right)^2 \frac{e}{\hbar\omega}[I_n(\hbar\omega) + I_2(\hbar\omega)\cos\phi_0] \\ &\simeq \frac{1}{2}\left(\frac{\delta I_\omega}{C\omega}\right)^2 \frac{e}{\hbar\omega} I_n(\hbar\omega), \end{aligned} \tag{11}$$

with the standard expressions for the normal current I_n and the Josephson cosine term I_2. For $\hbar\omega \to 2\Delta^+$, I_n is determined by the normal-state resistance R_n of the JJ,[27] $I_n(2\Delta^+) = (\pi/4R_n)(2\Delta/e)$. The second line of Eq. (11) is valid for squbits biased close to the critical current, where $\phi_0 \simeq \pi/2$. The quasiparticle excitation rate is obtained from $\overline{P}$ via[28]

$$\Gamma = \frac{\overline{P}}{\hbar\omega} = \frac{\pi\delta I_\omega^2}{8\hbar C^2 R_n \omega^3}. \tag{12}$$

Γ is negligibly small unless $C \leqslant 1$ fF and $\delta I_\omega \geqslant 1$ pA. With typical parameters $R_n = 29\ \Omega$, $C = 1$ fF, and $\omega = 4\ \mathrm{K}k_B/\hbar$, we obtain $\Gamma = 1\ \mathrm{ms}^{-1} \times [\delta I_\omega\ (\mathrm{pA})]^2$. For instantaneous switching, a current pulse with amplitude $I_{ac} = 10$ nA and duration $T = 10$ ns, $\delta I_\omega \simeq I_{ac}/\omega T = 1$ pA for $\omega = 4\ \mathrm{K}k_B/\hbar$. These values show that quasiparticle excitation by dissipative tunneling cannot explain the substantial visibility reduction evidenced in current Rabi-oscillation experiments for phase squbits. However, the excitation of Bogoliubov quasiparticles is relevant in view of the ultimate goal of reducing quantum gate errors to less than 10^{-4}.

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