Nonlocal effects in superfluid turbulence: Application to the low-density- to high-density-state transition and to vortex decay

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We discuss a phenomenological equation for the evolution of vortex tangle in counterflow superfluid turbulence, which takes into account the influence of the nonlocal effects, introducing into the original equation of Vinen two simple corrective terms dependent on a nonvanishing ratio between the average separation between vortex lines and the diameter of the channel. The equation allows one to describe, in relatively good agreement with experimental results, the two turbulent regimes present in counterflow superfluid turbulence and the transition between them. The decay rate of the vortex line density L, when the heat flux is suddenly turned off, is also investigated; due to the simplicity of the model, which does not take into account the coupling of the line density L with the superfluid velocity, this decay agrees with experiments in the initial and intermediate stages, but does not describe the full slow-down observed at very long times.

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I. INTRODUCTION

Nonlocal terms are receiving much attention in current transport theory due to the recent stimulus of research on nanoscale systems, where the size of the system becomes comparable to the mean free path (mfp) of particles. In fact, this situation is found not only in small systems, but also in macroscopic systems, when the mfp becomes sufficiently long, as in the analysis of short-wavelength perturbations, where the wavelength becomes comparable to the mfp. This is, for instance, the case of low-density gases, which has been studied in the framework of kinetic theory, or of neutron-scattering experiments in liquids, described by molecular hydrodynamics.

We want to stress here some questions on a physical situation that is an interesting candidate to be considered from this perspective. We refer to superfluid turbulence in narrow channels,[1–4] a situation that may be of practical interest in cryogenic applications to keep small systems at low temperatures by removing heat through the flow of superfluid helium along thin capillaries.[2]

Superfluid turbulence in counterflow has been the subject of many experiments and theories; the usual descriptions consider a vortex tangle with vortex line density L, giving the total length of vortex lines per unit volume. The analysis of L, for different values of the counterflow velocity, has been undertaken mainly from phenomenological perspective,[5–7] but very interesting microscopic models have also been developed.[8]

From the phenomenological point of view, the most well-known equation describing the evolution of L under a counterflow characterized by the intensity V of the relative velocity $\mathbf{V}$ between normal and superfluid components is the Vinen's equation,[5] which is

$$\frac{dL}{dt} = \alpha V L^{3/2} - \beta\kappa L^2, \tag{1.1}$$

where $\kappa = h/m$ is the quantum of vorticity (m is the mass of the ^{4}He atoms and h Planck's constant; thus, $\kappa = 9.97\times 10^{-4}$ cm^2 s^{-1}), and α and β are phenomenological dimensionless parameters, which may depend on temperature.

In experiments where turbulence is generated by thermal counterflow in a tube of circular cross section, the vortex line density is observed to develop from a low-density state (TI) to a higher-density state (TII) that can be associated with the homogeneous state.[9,10] Vinen's equation describes satisfactorily only the fully developed second turbulent regime, but not the TI turbulent low-density state and, therefore, it does not account for the transition from TI to TII regimes.

To derive his equation, Vinen assumes that the time derivative of L is composed of two opposite contributions

$$\frac{dL}{dt} = \left[\frac{dL}{dt}\right]_f - \left[\frac{dL}{dt}\right]_d, \tag{1.2}$$

where subscripts f and d denote formation and destruction of vortices per unit of time and volume, respectively. In wide channels, the growth of the line density is due to the mutual friction force between superfluid and normal components and the decay is originated by a cascade-like process of vortex breakup, due to the vortex reconnection, but in narrow channels the walls also play an important role.

Vinen assumes that the term $[dL/dt]_f$ depends on the quantum of circulation κ, the local and instantaneous value of L, and the force $\mathbf{f}$ between the vortex line and the normal component, which is linked to the intensity V of the counterflow velocity; dimensional analysis leads to the equation[5,6]

$$\left[\frac{dL}{dt}\right]_f = VL^{3/2}\phi_f\left[\frac{V}{\kappa L^{1/2}}\right], \tag{1.3}$$

where ϕ_f is some dimensionless function of its dimensionless argument. By analogy with the growth of a vortex ring, Vinen assumed that the dimensionless function ϕ_f is constant, obtaining

$$\left[\frac{dL}{dt}\right]_f = \alpha VL^{3/2}, \tag{1.4}$$

with α a dimensionless constant.

The form of the $[dL/dt]_d$ destruction term was determined in analogy with classical turbulence, obtaining

$$\left[\frac{dL}{dt}\right]_d = -\beta\kappa L^2. \tag{1.5}$$

A theoretical value for β, proposed in Ref. 5, is $\beta=\chi/2\pi$, where χ is a constant of the order of unity, which depends on temperature.

Equation (1.1) has been given a physical microscopic basis by Schwarz, starting from statistical considerations on vortex-line dynamics.[8] In the microscopic model by Schwarz, the vortex lines are represented in the parametric form $\mathbf{s}(\xi,t)$, ξ being the length along the line. The equation of motion of the line depends on $\mathbf{s}'$, $\mathbf{s}''$ and on the higher-order derivatives $\mathbf{s}'''$, $\mathbf{s}''''$ and so on, which follow a hierarchy of evolution equations. To truncate this hierarchy, it is assumed that the derivatives become uncorrelated in a distance of order of the average vortex separation $L^{-1/2}$. Starting from this model, Schwarz is able to derive Vinen's equation.

An open question is what would be the evolution equation for the tangle in situations in which the vortex separation $L^{-1/2}$, becomes comparable to the diameter d of the channel. In fact, Eq. (1.1) is valid when the average separation between vortex lines is much smaller than the diameter of the channel d, in which case the evolution of L does depend on the local values of V and L. However, experiments have been carried out in channels in wide range of diameters d, or in a wide range of speeds V, in such a way that the value of $L^{-1/2}/d$ may become comparable to 1; this happens, for instance, for low values of V, either in situations in which V is increasing to reach turbulence,[11] or in situations in which V or L are decreasing, as in turbulence decay.[12] In fully developed turbulence, L is high and $L^{-1/2}/d$ is likely to be small even for narrow channels. Problems arise, however, when $L^{-1/2}/d$ becomes comparable or higher than 1. These situations are found (a) in narrow channels, (b) in the transition from TI to TII turbulent regimes, (c) in the late stages of the decay of turbulence after the counterflow velocity has been set to zero, and (d) in the transport of short-wave second-sound across a vortex tangle (in this case, the relevant ratio is $L^{-1/2}/\lambda$, with λ the wave-length, rather $L^{-1/2}/d$). These situations are of special interest for researchers in nonconventional transport theory, because they provide examples of situations in which the size of the system or the typical wavelength of perturbation is comparable to the mfp.

In the present paper, we discuss a phenomenological generalization of Vinen's equation, including nonlocal contributions related to a nonvanishing value $L^{-1/2}/d$, and we apply them to the study of the transition from TI to TII regimes and of some aspects of the decay of vortices in counterflow superfluid turbulence, with the aim of contributing to describe such phenomena and to illustrate the relevance of nonlocal effects.

In Sec. II, we discuss our proposed phenomenological generalization of Eq. (1.1), taking into account these kinds of effects. As an illustration of the possible physical interest of the generalized equation, in Secs. III and IV we apply it to the study of two situations in which the presence of nonlocal terms is important: the transition from TI to TII turbulent states and the decay of vorticity in counterflow superfluid turbulence. The results show that the proposed generalized equation for the evolution of vortex line density L is able to explain the transition from TI to TII turbulent regimes and yields a slower decay of L, at intermediate times. We do not claim that these terms are the only possible explanation of the observed effects, but they are shown to have a non-negligible influence, in such a way that they should be taken into account in future analyses of these problems.

II. PHENOMENOLOGICAL GENERALIZATION OF VINEN'S EQUATION

Our aim is to incorporate to the usual hydrodynamical description of superfluid turbulence based on Eq. (1.1) nonlocal effects, arising when the ratio $L^{-1/2}/d$ is not negligible, and therefore allowing one to incorporate into the equation the effects of the walls.

First of all, we briefly comment that Vinen himself proposed to take into account the effects of the walls, introducing in the production term of Eq. (1.1) a term proportional to $L^{-1/2}/d$:[6]

$$\frac{dL}{dt} = -\beta\kappa L^2 + \alpha VL^{3/2}\left[1-\omega\frac{L^{-1/2}}{d}\right], \tag{2.1}$$

with ω a phenomenological positive parameter. The intuitive interpretation of Eq. (2.1) is that the vortex generation mechanism is inactive within a characteristic distance $L^{-1/2}$ from the wall. Since this idea is related to the generation mechanism, it is logical that the decay term is left unchanged (see Refs. 7–10).

However, according to Schwarz,[8,12,13] in narrow channels the walls play an important role in vortex dynamics, both in generation and decay, as fluctuations are assumed to grow and annihilate on them, whereas for wider channels the onset of turbulence is more similar to that in classical fluids. Therefore, these arguments, as well as experimental results[12,14] on the decay of turbulence, suggest the need to modify as well the destruction term, not only the generation term. Consequently, in this work, we propose to modify Vinen's equation (1.1) in the simple form

$$\frac{dL}{dt} = \frac{\alpha V L^{3/2}}{1+\alpha'\left(\frac{L^{-1/2}}{d}\right)^2} - \frac{\beta\kappa L^2}{1+\beta'\left(\frac{L^{-1/2}}{d}\right)^2}. \tag{2.2}$$

Here α' and β' are coefficients that could depend on temperature. It is obvious that when $L^{-1/2}/d$ is very small, one recovers Vinen's equation.

Now we examine the physical motivations of the correction factors, beyond the simple consideration that the corresponding generalized terms should be always positive. These corrections indicate a reduction both of the rate of formation and of the rate of decay of the vortices when $L^{-1/2}$ is not negligible as compared to d. Physical reasons for this reduction may be attributed to the pinning of vortices on small irregularities of the walls. This may have two different opposite contributions: due to the tendency to remain pinned on the walls instead to going fully to the bulk flow, the walls would reduce the rate of formation of the vortices in the flow, as compared with the same volume of the fluid in the absence of the wall. On the other hand, the fact that, once pinned on the walls, the vortices become more resistant to elimination would imply a reduction in the rate of destruction. It is not clear *a priori* which of these reductions should be predominant over the other one. Our analysis will allow us to compare the consequences of both of them with experimental observations related to the transition from TI turbulence to TII turbulence.

Beyond these physical motivations to expect a reduction of the formation and the destruction rates, we briefly mention that in other different contexts (kinetic theories of gases,[15,16] generalized hydrodynamics,[17] and extended thermodynamics[18,19]) nonlocal corrective terms have been proposed that have the phenomenological form adopted in Eq. (2.2). Indeed, in the presence of a perturbation with wave vector k, the classical transport coefficients (thermal conductivity λ and shear viscosity η) are modified in the form

$$\lambda_{Grad}(k) = \frac{\lambda_0}{1+l^2k^2}, \quad \eta(k) = \frac{\eta_0}{1+l^2k^2}, \tag{2.3}$$

l being the mfp of the particles in the gas. Thus, lacking, for the moment, detailed microscopic models to describe the expected reduction of the rate of formation and destruction of the vortices, we propose in Eq. (2.2) to describe them in a way analogous to Eq. (2.3), i.e., by introducing a second-order polynomial in $L^{-1/2}/d$ in the denominator, which is a form simple enough to allow for sufficiently detailed analysis of its consequences.

III. THE TRANSITION FROM TI TO TII TURBULENT REGIMES

According to Vinen's equation (1.1), the stationary regime of counterflow superfluid turbulence is

$$L^{1/2} = \frac{\alpha}{\beta}\frac{V}{\kappa}. \tag{3.1}$$

This is in agreement with experiments at low and at high values of V, but with different values of the proportionality coefficients (the so-called TI and TII regimes) separated by a transition region. Vinen's equation (1.1) does not describe the two different values of the proportionality coefficient between $L^{1/2}$ and V, nor the transition from the one to the other. In contrast, our proposed generalization (2.2) describes these features in a natural, though relatively simplified way, as is shown in this paragraph.

Indeed, for L different from zero, the stationary solution of Eq. (2.2) can be written as

$$V = \kappa\frac{\beta}{\alpha}L^{1/2}\frac{Ld^2+\alpha'}{Ld^2+\beta'}. \tag{3.2}$$

Using suitable values of the coefficients, which will be discussed below, this equation exhibits a crossover from the region of TI turbulence, where L is small, dominated by nonlocal effects and by the influence of the walls, to a TII turbulent region, where L is high and does not depend on the size of the channel, corresponding to Vinen's equation. In fact, for very small values of L, Eq. (3.2) can be approximated as

$$V \simeq \kappa\frac{\beta}{\alpha}\frac{\alpha'}{\beta'}L^{1/2} \quad \text{or} \quad L^{1/2} \simeq \frac{\alpha}{\beta}\frac{\beta'}{\alpha'}\frac{V}{\kappa} = h_1\frac{V}{\kappa}, \tag{3.3}$$

while, for high values of $L^{1/2}$, it results from Eq. (3.2) that

$$V \simeq \kappa\frac{\beta}{\alpha}L^{1/2} \quad \text{or} \quad L^{1/2} \simeq \frac{\alpha}{\beta}\frac{V}{\kappa} = h_2\frac{V}{\kappa}. \tag{3.4}$$

We recall now that in the TII turbulent regime (high values of $L^{1/2}$), the line density L is well described by Vinen solution (3.1). As a consequence, the ratio α/β furnishes an approximate value of the coefficient h_2:

$$h_2 = \frac{\alpha}{\beta}. \tag{3.5}$$

From Eq. (3.4) we also obtain the ratio of the coefficients α' and β' as

$$\frac{\alpha'}{\beta'} = \frac{h_2}{h_1}. \tag{3.6}$$

Observe that the transition from TI to TII turbulent regimes happens when the counterflow velocity V reaches a critical value V_{c2} and the quantity $y=L^{1/2}d$ a value y_{c2}. In correspondence to this transition, the slope of the stationary solution (3.2) undergoes a rapid change and the curvature becomes equal to zero. Therefore, at the point V_{c2} Eq. (3.2) must present an inflection point. The coordinate of this flex point can be easily expressed as function of α' and β' in the following way:

$$V_{c2} = \frac{\kappa}{d}\frac{1}{4}\frac{\beta}{\alpha}\sqrt{3\beta'}\left(3+\frac{\alpha'}{\beta'}\right), \tag{3.7a}$$

TABLE I. Values of the parameters appearing in Eq. (3.2) for two different values of the temperature. The values for χ are from Ref. 13.

T (K)	h_1	h_2	χ	β'	α'	α'/β'
1.5	0.0205	0.127	0.78	40	247	6.19
1.7	0.0243	0.166	1.3	40	274	6.86

$$L_{c2}^{1/2} = \frac{1}{d}\sqrt{3\beta'}. \tag{3.7b}$$

As one sees, we obtain for V_{c2} and $L_{c2}^{1/2}$ a $1/d$ dependence, in agreement with experiments.[14] Thus the corrective coefficients α' and β' characterize the transitions from TI to TII turbulent regimes: indeed, Eq. (3.7b) allows us to determine, using the experimental value of L at the transition TI-TII, the coefficient β'. Equation (3.7a), taking into account Eq. (3.5), which furnishes an approximate value of β/α, allows us to determine the ratio α'/β' (and therefore, the value of α'). The very rapid change of L with V near V_{c2} furnishes only a restricted interval of possible values for L_{c2}; therefore, in the following, comparing with experimental data, we choose $y_{c2}=11$ for both temperatures (1.5 and 1.7 K, which will be examined below), which corresponds to $\beta'=40$. The values of h_1, h_2, α', and β' obtained are reported in Table I.

In Fig. 1, the plots of the stationary solutions (3.2), at temperatures $T=1.5$ K and $T=1.7$ K, are shown. We must observe that fixing the parameters h_1, h_2 in Eqs. (3.3) and (3.4) to obtain the desired asymptotic behavior and β' in Eq. (3.7b) to describe the observed value of L_{c2}, leads to a slight underestimation of the critical velocity V_{c2} in Eq. (3.7). This could indicate that the nonlocal terms may have a more complicated form than the one assumed in Eq. (2.2); for instance, the exponent in the term $L^{-1/2}/d$ could be different from 2, or higher-order terms could be considered, maybe an infinite number of terms through a continued-fraction expansion. However, since for the moment we do not have a sufficiently clear microscopic motivation for such a change, we will keep the simple form proposed in Eq. (2.2), which clearly shows that nonlocal effects cannot be ignored *a priori*, but may play an important role in the transition. Indeed, the present model allows us to describe in a simple way the existence of such a transition from the TI to the TII turbulent regimes, and to obtain it with relatively good agreement with experimental results, especially for the lowest value of the temperature.

A description of the microscopic phenomenon underlying the TI-TII transition, which agrees with Eq. (2.2) can be the following: the TI turbulent regime is an inhomogeneous and locally polarized state dominated by the influence of the walls. When the counterflow reaches the critical value V_{c2}, this state becomes unstable and the flow undergoes a transition to the fully developed turbulent regime TII, which is homogeneous and independent on the walls.

It is seen that the reduction in the rate of formation (described by α') is much higher than the reduction in the rate of decay (given by β'). This could be interpreted in terms of a higher tendency of vortices to remain pinned on the walls than to pin on them, when they are in motion; indeed, this means higher difficulty that the vortices unpin and go free to the bulk flow (high reduction of formation, due to the presence of the wall) than that they pin to the irregularities of the wall (lower influence of the walls on the decay, which, however, it is also reduced). This strong reduction of the formation term in comparison with the destruction one agrees qualitatively with Vinen's proposal (2.1), in which only the formation term was reduced. However, the reduction in the decay may also be experimentally observed, in contrast with Eq. (2.1), which does not modify the corresponding term (and does not describe the TI-TII transition).

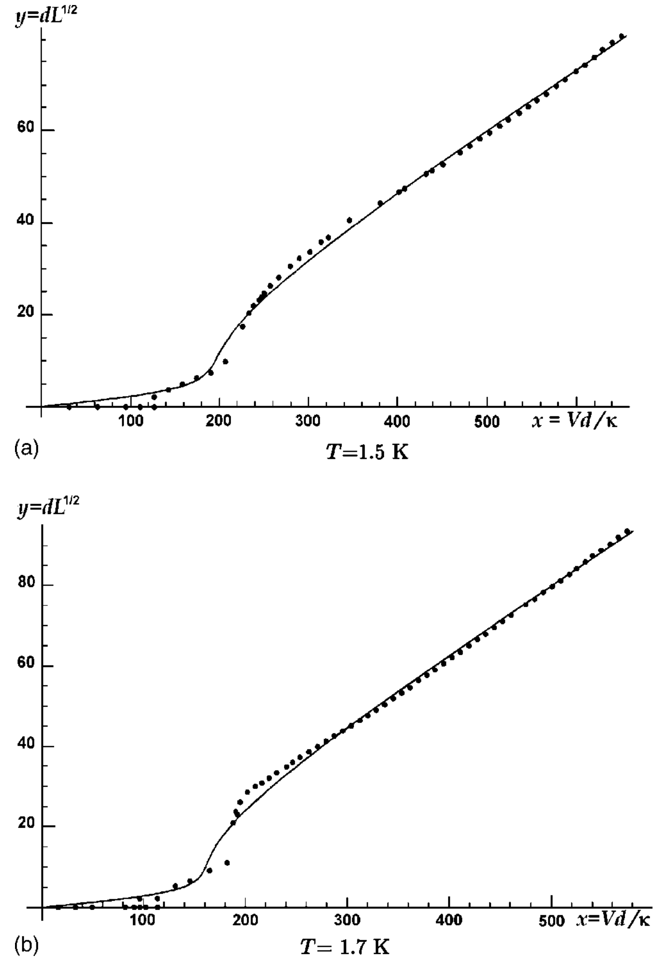

FIG. 1. Plots of $y=L^{1/2}d$ via $x=Vd/\kappa$ as result from Eq. (3.2): (a) at $T=1.5$ K and (b) at $T=1.7$ K, choosing $\alpha/\beta=h_2$, $\beta'=40$ and α' obtained from Eq. (3.6) and given in Table I. Points are experimental values of Martin and Tough (Ref. 11).

Our model does not strictly require that the normal fluid becomes turbulent, as the explanation of the TI-TII transition proposed by Melotte and Barenghi[20] requires; what it shown is a drastic increase in the line density L, which practically eliminates the relatively stabilizing influence of the walls; the corresponding decrease in the average separation between vortex lines ($L^{-1/2}$) would favor vortex reconnections in the bulk of the superfluid, thus contributing to reinforce the increase of L, then yielding a relatively abrupt transition. However, the possibility that the normal fluid becomes turbulent cannot be excluded.

Note also that transition TI-TII is observed in channels whose transverse sections have a low aspect ratio (i.e., circle, square) but not in those with high aspect ratio (i.e., elongated ellipses, elongated rectangles),[1,2] where only one turbulent state, similar to state TII, is observed. From the perspective of the present paper, this fact is not surprising; in a cylindrical channel the TI-TII transition is observed when $L^{-1/2}/d$ is of the order 10^{-1}, whereas in a rectangular channel the transition from TI to TII would be expected to take place for $L^{-1/2}/d_{\max} \simeq 10^{-1}$, but if d_{max} is large, such a transition would take place for small values of L and therefore the TI regime would be reduced to a very narrow range of flows, and it would be practically unobservable.

IV. VORTEX DECAY TOWARD A QUIESCENT STATE

In the preceding section it was seen that the wall effects on the reduction of vortex formation rate were much higher than on the reduction of the decay rate. However, the latter one also deserves attention, due to its role on the decay of vorticity in counterflow superfluid turbulence, after the heat flux, which is proportional to V, is suddenly set to zero. This is a somewhat artificial situation, as V will decay not suddenly, but follows a dynamical equation; however, it will be enough to explore some new features of Eq. (2.2) in a problem independent of the previous one, and to illustrate the convenience to modify not only the generation term, but also the decay term.

According to Vinen's equation (1.1), the decay of L after V is set to zero is described by

$$\frac{dL}{dt} = -\beta\kappa L^2, \tag{4.1}$$

thus leading to

$$\frac{1}{L(t)} = \frac{1}{L_0} + \beta\kappa t. \tag{4.2}$$

This solution corresponds to the decay of a homogeneous vortex tangle, which occurs when L is high. However, comparison with experimental data[12,14] indicates that the decay of L is slower than his prediction. We will study here how nonlocal terms in Eq. (2.2), increasingly important as L is lowered, may contribute to the mentioned slowing down of the decay. It must be noted, in fact, that the results for the steady state situation described in Sec. III do depend on the ratio of the rate-reducing terms, rather than on their absolute values, i.e., they stay the same if the whole right-hand term in Eq. (2.2) is multiplied by $1+\beta'/Ld^2$. This would leave invariant the results of Sec. III, but will modify the unsteady result of the present section. However, we will stick the reasoning leading to Eq. (2.2), which outlines the actual role of each rate-reducing factor rather than only to the ratio of the factors.

With this aim, we now analyze the decay process using Eq. (2.2); namely,

$$\frac{dL}{dt} = -\frac{\beta\kappa L^2}{1+\beta'\dfrac{L^{-1}}{d^2}}. \tag{4.3}$$

The solution of Eq. (4.3) is

$$t = \frac{1}{\beta\kappa}\left[\frac{1}{L} - \frac{1}{L_0} + \frac{\beta'}{2d^2}\left(\frac{1}{L^2} - \frac{1}{L_0^2}\right)\right]. \tag{4.4}$$

First, we observe that, for high values of L, the terms depending on the dimension d of the channel become negligible, and one recovers Vinen's decay (4.2). For decreasing L, the influence of nonlocal terms will become more important and the decay will be slower. In fact, when L is small enough in such a way that the nonlocal terms become dominating in the denominator of Eq. (4.3), one is lead to a decay of the kind

$$\frac{1}{L^2(t)} \approx 2\kappa d^2 \frac{\beta}{\beta'} t, \tag{4.5}$$

instead of Eq. (4.2). Thus, Eq. (4.3) exhibits a crossover from the initial behavior (4.2), corresponding to Vinen's equation, independent on the size of the channels, to evolution (4.5), dominated by nonlocal effects, and by the influence of the wall. Indeed, expansion (4.5) suggests that for small L, L decays as $L \sim t^{-1/2}$.

In Fig. 2 we have plotted the solution (4.4) of Eq. (4.3) at $T=1.5$ K and $T=1.7$ K, choosing $\beta=\chi/2\pi$ and $\beta'=40$ (this value for β' is the one found in the previous section to describe the TI-TII transition). We have chosen as initial values for L the highest values used in the experiments described in Ref. 11.

We have also compared our models (maintaining the same value of the corrective coefficient β') with experimental data of Schwarz and Rozen.[12] The counterflow channel of Schwarz and Rozen was a large rectangular waveguide tube 1.00 by 2.35 cm in cross section, the temperature of the liquid was $T=1.9$ K.

Figures 17 and 18 of Schwarz and Rozen[12] show the presence of two distinct decay regimes. Introducing the quantity $w=1/\beta\kappa L$, the slope of the experimental curve, very near to $t=0$ (Fig. 18 of Ref. 12) is about 1 and remains constant for the first few seconds (2 or 3 s); this regime corresponds to the rapid decay characteristic of the homogeneous state (as predicted by Vinen). The vortex tangle then switches to a regime of much slower decline, where the slope of the experimental curve is about 0.09.

In order to compare the results of our models with the experimental values of Ref. 12, in Fig. 3 we have plotted the values of $1/\beta\kappa L$, obtained using our model, as function of t. As one sees, the terms dependent on $L^{-1/2}/d$ (which are very small, being large the diameter d of the channel) cannot explain the anomalous slow-down, but also in the large channel used by Schwarz and Rozen, they are not negligible. In particular, Fig. 3 shows that the switch to the regime of slower decline happens in correspondence to the

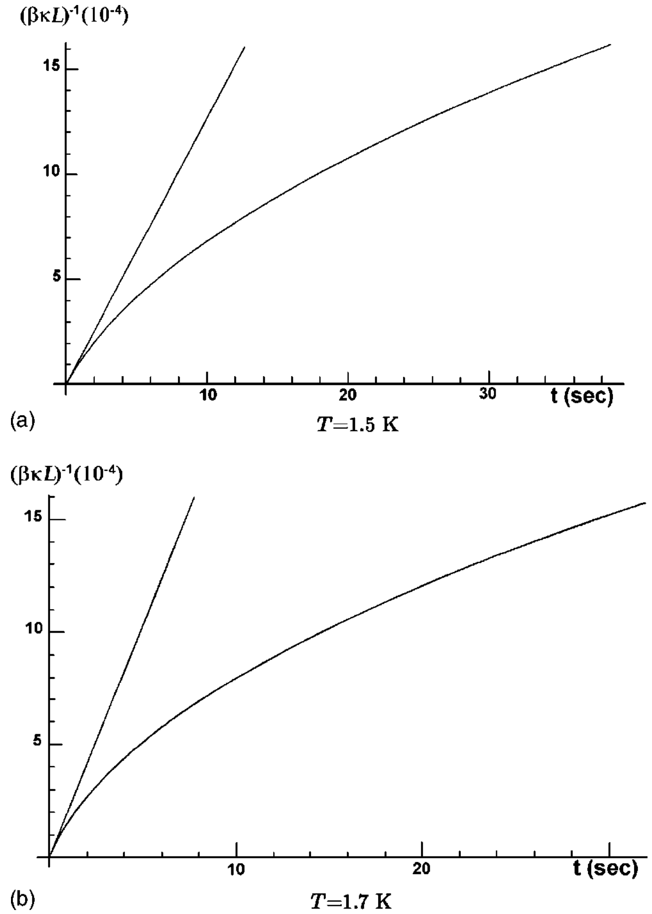

FIG. 2. Plots of the curve of Eq. (4.4): (a) at $T=1.5$ K and (b) at $T=1.7$ K, choosing $\beta=\chi/2\pi$ and $\beta'=40$. The initial values are $1/L_0=1.52\times10^{-6}$ cm^2 at $T=1.5$ K and $1/L_0=1.13\times10^{-6}$ cm^2 at $T=1.7$ K. The straight line is solution (4.2).

values of t and L, where the influence of the walls becomes appreciable.

There are, however, several proposals to ascribe the anomalous slowing down of the vortex decay with time (see, for example, Refs. 12 and 21). Most of these explanations are based on macroscopic hydrodynamic ideas (our explanation, too, is based on molecular hydrodynamics ideas, in some sense). Thus, the fast decay is associated with the homogeneous high-density tangle, whereas the slow one is related to the influence of spatial inhomogeneities of the system.[12,22] Thus, according to Schwarz and Rozen[12] and to Geurst and van Beelen,[21] this inhomogeneity in the tangle would imply inhomogeneity in the flow, and viscous effects related to the normal fluid would come into play and become dominant, in such a way that, during its deceleration, not immediately followed by the superfluid, it would produce a certain small difference between $\mathbf{v}_n$ and $\mathbf{v}_s$ capable of sustaining the vortex tangle for a long time. These authors use Vinen's equation (1.1), but do not take into account nonlocal terms as those included in Eq. (4.3). We think that a consistent analysis of the decay should include them besides taking into account hydrodynamic effects related to the evolution equation of V. Indeed, they are numerically important, at least, in an intermediate stage of the evolution, and there is no reason to exclude them *a priori* in a detailed analysis of this phenomenon.

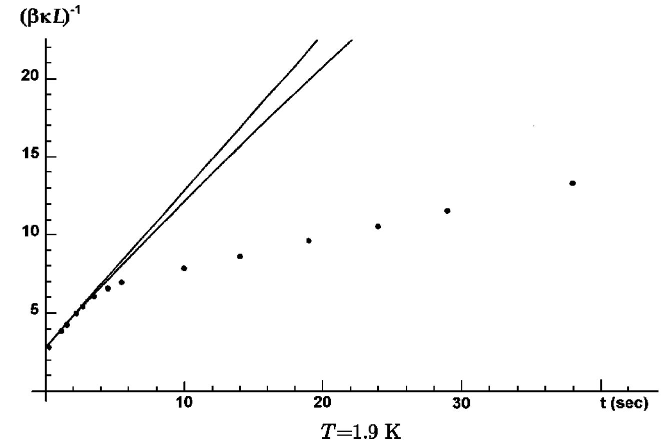

FIG. 3. (a) Plot of the curve of equation (4.4) at $T=1.9$ K, choosing $\beta=\chi/2\pi$ $(\chi=1.55)$ and $\beta'=40$. The initial value is $1/\beta\kappa L_0=3.854$ (i.e., $1/L_0=9.48\times10^{-4}$ cm^2). The straight line is solution (4.2). Points are experimental values of Schwarz and Rozen (Ref. 12).

V. DISCUSSION

We have stressed that superfluid turbulence may be exciting not only from the already known perspectives, but also from the point of view of systems in which the mfp becomes comparable to the size of the system. Our proposal is a phenomenological statement analogous to the modifications of transport coefficients in kinetic theory of gases and in generalized hydrodynamics and aims to stimulate this kind of research; i.e., accounting for wall effects reducing the rate of vortex formation and vortex decay. We have described these effects, due to the pinning of vortices on the walls, by means of Eq. (2.2), using a simple mathematical form. It turns out that the new terms are able to describe in a simple way the transition from TI turbulence to TII turbulence. In this model, TI turbulence (low values of L) is dominated by wall effects; namely, by vortex lines pinned to the walls. When L becomes high enough in order that $L^{-1/2}$, and the average separation of vortex lines becomes lower than d, there is a fast reconnection of vortex lines in the bulk of the fluid, and the effect of pinned vortices becomes negligible.

On the other hand, when applied to the analysis of vortex decay after setting to zero the heat flux in counterflow turbulence, the correction terms introduced in Eq. (2.2) yield a decay of L as $L\sim t^{-1/2}$ instead of Vinen's result $L\sim t^{-1}$, which would be valid for short times, when $L^{-1/2}$ is sufficiently smaller than the diameter of the channel. Our equations show that, in contrast with Vinen's result, which does not depend on the diameter, in the other situations the decay depends on d as $L\sim d^{-1}t^{-1/2}$.

Notwithstanding this relative slowing down at intermediate times, the full slow-down of the decay observed at very long times, is still much slower than the decay at intermediate times. The most plausible explanation for such long time slowing down is that proposed by Schwarz and Rozen[12] according to which the viscosity of the normal component plays a dominant role. For narrow channels,the viscous deceleration of the normal fluid would be much smaller than in wide channels, and it could be short enough to leave observable the slow decay due to the nonlocal terms.

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