Dominance of the electron-phonon interaction with forward scattering peak in high-T_c superconductors: Theoretical explanation of the ARPES kink

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The angle-resolved photoelectron spectroscopy (ARPES) spectra in high-T_c superconductors show four distinctive features in the quasiparticle self-energy $\Sigma(\mathbf{k},\omega)$. They can be explained consistently by the phenomenological microscopic theory in which the electron-phonon interaction with the forward-scattering peak dominates over the Coulomb scattering. This theory explains why there is *no shift* of the nodal kink at 70 meV in the superconducting state, contrary to the observed *shift* of the antinodal singularity at 40 meV. The theory predicts a *kneelike* structure of $|\mathrm{Im}\,\Sigma(\omega)|=|\mathrm{Im}\,\Sigma_{ph}(\omega)+\mathrm{Im}\,\Sigma^{C}(\omega)|$, which is phonon dominated, $|\mathrm{Im}\,\Sigma(\omega_{ph})|\approx|\mathrm{Im}\,\Sigma_{ph}(\omega_{ph})|\sim\pi\lambda_{ph}\omega_{ph}/2$, for $\omega\approx\omega_{ph}^{(70)}$, and for $\omega>\omega_{ph}^{(70)}$ shows linear behavior $|\mathrm{Im}\,\Sigma(\omega)|\approx|\mathrm{Im}\,\Sigma_{ph}(\omega_{ph})|+\pi\lambda_{C,\varphi}\omega/2$, due to the Coulomb scattering. ARPES spectra give $\lambda_{ph}>1$—which is obtained from $\mathrm{Re}\,\Sigma$, and $\lambda_C<0.4$—obtained from $\mathrm{Im}\,\Sigma$, i.e., $\lambda_{ph}\gg\lambda_C$. The *dip-hump* structure in the spectral function $A(\mathbf{k}_F,\omega)$ comes out naturally from the proposed theory.

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The pairing mechanism in high-temperature superconductors (HTSC) is under intensive debate.[1,2] In that respect angle-resolved photoelectron spectroscopy (ARPES) experiments play a central role in the theory, since they give information on the quasiparticle spectrum, lifetime effects, and indirectly on the pairing potential. Recent ARPES experiments on various HTSC families, such as $La_{2-x}Sr_xCuO_4$ and $Bi_2Sr_2CaCu_2O_{8+\delta}$ (BISCO),[3–6] show at least four distinctive features in the quasiparticle self-energy $\Sigma(\mathbf{k},\omega)$: (i) There is a *kink* in the normal-state quasiparticle spectrum, $\omega(\xi_{\mathbf{k}})$, in the *nodal direction* $(0,0)-(\pi,\pi)$ at the energy $\omega_{kink}^{(70)}\lesssim 70$ meV, which is a characteristic oxygen vibration energy $\omega_{ph}^{(70)}$. However, the *kink is not shifted* in the superconducting state, contrary to the prediction of the *standard Eliashberg theory*.[7] The latter contains integration over the whole Fermi surface and over the energy, giving that singularities in $\omega(\xi_{\mathbf{k}})$ (along all directions) must be shifted in the superconducting state by the maximal gap value Δ_0: (ii) In the *antinodal region*, near $(\pi,0)$ [or $(0,\pi)$], there is a singularity in $\omega(\xi_{\mathbf{k}})$ in the normal state at $\omega_{sing}^{(40)}\approx 40$ meV—which is also a characteristic oxygen vibration energy $\omega_{ph}^{(40)}$. This singularity is *shifted* in the superconducting state (at $T\ll T_c$) to $\omega\approx 60$ meV ($=\omega_{ph}^{(40)}+\Delta_0$), where Δ_0 (≈ 20 meV) is the maximal superconducting gap at the antinodal point. The experimental slopes of $\mathrm{Re}\,\Sigma(\mathbf{k},\omega)$ at the kink (and singularity) give the electron-phonon interaction (EPI) coupling constant $\lambda_{ph}>1$. The different shifts of $\omega_{kink}^{(70)}$ and $\omega_{sing}^{(40)}$ occur in the superconducting state we call the *ARPES nonshift puzzle*. (iii) There is a *kneelike* structure of $|\mathrm{Im}\,\Sigma(\omega)|=|\mathrm{Im}\,\Sigma_{ph}(\omega)+\mathrm{Im}\,\Sigma^{C}(\omega)|$, which is for $\omega\approx\omega_{ph}^{(70)}$, phonon dominated $|\mathrm{Im}\,\Sigma(\omega)|\approx|\mathrm{Im}\,\Sigma_{ph}(\omega_{ph})|\sim\pi\lambda_{ph}\omega_{ph}/2$ with $\lambda_{ph}>1$ (obtained from $\mathrm{Re}\,\Sigma$), and for $\omega>\omega_{ph}^{(70)}$ there is a linear behavior of $|\mathrm{Im}\,\Sigma(\omega)|\approx|\mathrm{Im}\,\Sigma_{ph}(\omega_{ph})|+\pi\lambda_{C,\varphi}\omega/2$ with $\lambda_C<0.4\ll\lambda_{ph}$, which is due to the Coulomb scattering. (iv) There is a *dip-hump* structure in the spectral function $A(\mathbf{k}_F,\omega)$ with the quasiparticle peak sharpening in the superconducting state near the antinodal point.

These distinctive features in the ARPES spectra *cannot be explained* by the theory of the spin-fluctuation interaction (SFI) due to the following reasons: (i) in the SFI there are no phonons and characteristic energy at 70 meV. (ii) The intensity of the SFI spectrum [$\sim\mathrm{Im}\,\chi(\mathbf{Q},\omega)$—the spin susceptibility at $\mathbf{Q}=(\pi,\pi)$], is pronounced in slightly underdoped materials but strongly suppressed in the normal state of the optimally doped HTSC oxides.[8] At the same time their critical temperatures differ only slightly ($\delta T_c\sim 1$ K). Such a huge reconstruction of the SFI spectrum, but with a small effect on T_c, gives strong evidence against the SFI mechanism of pairing (see more in Ref. 1). (iii) The SFI theory[9] assumes unrealistically large coupling energy $g_{sf}\approx 0.65$ eV (with the coupling constant $\lambda_{sf}(\sim g_{sf}^2)\approx 2.5$), while the ARPES (Refs. 3 and 4, and 6) resistivity[1] and magnetic[10] measurements give much smaller $g_{sf}\lesssim 0.1$ eV, i.e., $\lambda_{sf}<0.2<\lambda_C\lesssim 0.4$, giving small T_c. (iv) If the kink at 70 meV is due to the magnetic spectrum, this would be strongly rearranged in the superconducting state, contrary to the ARPES results. On the other hand, the phonon energies are only slightly ($\leqslant 5\%$) changed in the superconducting state. (v) The magnetic-resonance mode at 41 meV, which appears only in the superconducting state,[9] cannot cause the kink, since the latter is observed also in the normal state of almost all hole-doped HTSC. Moreover, the kink is observed in $La_{2-x}Sr_xCuO_4$, where there is no magnetic resonance mode.[3] We show in the paper that the four distinctive features in the ARPES spectra can be explained by the phenomenological microscopic theory in which the electron-phonon interaction (EPI) with the forward scattering peak (FSP) dominates over the Coulomb interaction—we call it the *EPI-FSP model*.

The central question for EPI theory is the following: Why is the antinodal singularity $\omega_{sing}^{(40)}$ shifted in the superconducting state, but the nodal kink $\omega_{kink}^{(70)}$ is not? This result cannot be explained by the standard Eliashberg theory for the isotropic EPI,[7] which predicts that $\omega_{sing}^{(40)}$ and $\omega_{kink}^{(70)}$ should be shifted in the superconducting state to $\omega_{sing}^{(40)} \to \omega_{ph}^{(40)} + \Delta_0$ and $\omega_{kink}^{(70)} \to \omega_{ph}^{(70)} + \Delta_0$, respectively. However, it can be explained by the EPI-FSP model, which contains the following basic ingredients: (i) The *EPI is dominant* in HTSC and its spectral function $\alpha^2 F(\mathbf{k},\mathbf{k}',\Omega)$ [see Eq. (4)], has a pronounced FSP at $\mathbf{k}-\mathbf{k}'=0$, with the narrow width $|\mathbf{k}-\mathbf{k}'|_c \ll k_F$. This *assumption* is supported by the theory of the EPI in strongly correlated systems described by the *t-J* model.[11,1] Near the Fermi surface one expects that $\alpha_{ph}^2 F(\mathbf{k},\mathbf{k}',\Omega) \approx \alpha_{ph}^2 F(\varphi,\varphi',\Omega)$,[13] and in the *t-J* model one has $\alpha_{ph}^2 F(\varphi,\varphi',\Omega) \sim \gamma_c^2(\varphi-\varphi')$, where the charge vertex $\gamma_c(\varphi-\varphi')$ is strongly peaked at $\varphi-\varphi'=0$ with the width $\delta\varphi_w \ll \pi$—the forward-scattering peak (FSP).[11,1] Thereby, in leading order $\alpha_{ph}^2 F(\varphi,\varphi',\Omega) \approx \alpha_{ph}^2 F(\varphi,\Omega)\delta(\varphi-\varphi')$, which picks up the main physics whenever $\delta\varphi_w \ll \pi$.[1] The calculations in the *t-J* model with the EPI (Refs. 11 and 1) predict the following important results: (i) the strength of pairing is due to the EPI, while the residual Coulomb interaction (including spin fluctuations) triggers the pairing to the *d* wave one; (ii) the transport coupling constant λ_{tr} entering the resistivity $\varrho \sim \lambda_{tr} T$ is much smaller than the pairing one λ_{ph}, i.e., $\lambda_{tr} < \lambda_{ph}/3$. We stress that in the *t-J* model the FSP in the EPI is a general effect *by affecting electronic coupling to all phonons*. This is an important result, since for some phonons (for instance the half-breathing modes of O ions) the bare coupling constant $g_0^2(q)$ is peaked at large $q \sim 2k_F$, and therefore it is detrimental for *d*-wave pairing. It is renormalized by strong correlations and $g_{ren}^2(q) = g_0^2(q)\gamma_c^2(q)$ is peaked at much smaller q, thus contributing constructively to *d*-wave pairing. Recent Monte Carlo calculations[12] in the finite *U* Hubbard model with the EPI confirm the existence of the FSP in the EPI found in Ref. 11. (ii) The dynamical part (beyond the Hartree-Fock) of the Coulomb interaction is characterized by the spectral function $S_C(\mathbf{k},\mathbf{k}',\Omega)$, which is at present difficult to calculate. However, the ARPES nonshift puzzle implies that S_C is either peaked at small transfer momenta $|\mathbf{k}-\mathbf{k}'| \ll k_F$ or it is so small that the shift is weakly affected and below the experimental resolution. Since the ARPES data give small $\lambda_C < 0.4$, i.e., $\lambda_C \ll \lambda_{ph} > 1$, the kink position is practically insensitive to the $\mathbf{k}$ dependence of S_C. For simplicity we assume that the former case is realized [see the discussion after Eq. (4)]. (iii) The scattering potential due to nonmagnetic impurities has a pronounced forward-scattering peak. This assumption is also supported by the *t-J* model.[11,1] Moreover, the latter property makes *d*-wave pairing robust in the presence of impurities.[1]

The Matsubara Green's function is defined by $[k=(\mathbf{k},\omega_n)]$,

$$G_k = \frac{1}{i\omega_k - \xi_{\mathbf{k}} - \sum_k(\omega)} = -\frac{i\tilde{\omega}_k + \xi_{\mathbf{k}}}{\tilde{\omega}_k^2 + \xi_{\mathbf{k}}^2 + \tilde{\Delta}_k^2}, \tag{1}$$

where $\xi_{\mathbf{k}}$, $\tilde{\omega}_k$, and $\tilde{\Delta}_k$ are the bare quasiparticle energy, renormalized frequency, and gap, respectively.[13] The two-dimensional (2D) Fermi surface of HTSC is parametrized by $\mathbf{k}=(k_F+k_\perp, k_F\varphi)$, where $k_F(\varphi)$ is the Fermi momentum and $k_F\varphi$ is the tangent on the Fermi surface.[13] In that case $\xi_{\mathbf{k}} \approx v_{F,\varphi}k_\perp$ and $\int d^2k[\cdots] \approx \int\int d\xi d\varphi k_{F,\varphi}/v_F(\varphi) = \int\int N_{\varphi,\xi} d\xi d\varphi$.

After the ξ-integration the Eliashberg equations in the FSP model read

$$\tilde{\omega}_{n,\varphi} = \omega_n + \pi T \sum_m \frac{\lambda_{1,\varphi}(n-m)\tilde{\omega}_{m,\varphi}}{\sqrt{\tilde{\omega}_{m,\varphi}^2 + \tilde{\Delta}_{m,\varphi}^2}} + \sum\nolimits_{n,\varphi}^{C}, \tag{2}$$

$$\tilde{\Delta}_{n,\varphi} = \pi T \sum_m \frac{\lambda_{2,\varphi}(n-m)\tilde{\Delta}_{m,\varphi}}{\sqrt{\tilde{\omega}_{m,\varphi}^2 + \tilde{\Delta}_{m,\varphi}^2}} + \tilde{\Delta}_{n,\varphi}^{C}, \tag{3}$$

where $\lambda_{1(2),\varphi}(n-m) = \lambda_{ph,\varphi}(n-m) + \delta_{mn}\gamma_{1(2),\varphi}$ with the electron-phonon coupling function $\lambda_{ph,\varphi}(n)$,

$$\lambda_{ph,\varphi}(n) = 2\int_0^\infty d\Omega \frac{\alpha_{ph,\varphi}^2 F_\varphi(\Omega)\Omega}{\Omega^2 + \omega_n^2}. \tag{4}$$

Note that Eqs. (2) and (3) have a *local form* as a function of the angle φ, and $\tilde{\omega}_{n,\varphi}$ and $\tilde{\Delta}_{n,\varphi}$ at different points on the Fermi surface are *decoupled*. Just this (decoupling) property of the Eliashberg equations in the EPI-FSP model is crucial for solving the ARPES nonshift puzzle. $\Sigma_{n,\varphi}^C$ is due to the dynamical Coulomb effects and its calculation is the most difficult part of the problem. Since $\Sigma_{n,\varphi}^C \sim \gamma_c(\varphi-\varphi')$ we assume that it is also "local" on the Fermi surface, although this assumption is not crucial at all, because $\lambda_C \ll \lambda_{ph}$. After the ξ integration it reaches the same form as the second term in Eq. (2), where $\lambda_{1,\varphi}(n-m)$ is replaced by the Coulomb coupling function $\lambda_{C,\varphi}(n-m)$. The latter has the same form as Eq. (4) but $\alpha_{ph,\varphi}^2 F_\varphi(\Omega)$ is replaced by $S_{C,\varphi}(\Omega)$. ARPES spectra give evidence that $\mathrm{Im}\,\Sigma_\varphi^C(\omega) \approx -\pi\lambda_{C,\varphi}\omega/2$ at $T < \omega < \Omega_C$, which we reproduce by the *phenomenological expression* for $S_{C,\varphi}(\omega) = A_{C,\varphi}\Theta(|\omega|-T)\Theta(\Omega_C-|\omega|)$. $A_{C,\varphi}$ is normalized to obtain $\lambda_{C,\varphi} \lesssim 0.4$. $\tilde{\Delta}_{n,\varphi}^C$ in Eq. (2) is due to the Coulomb interaction and includes the following: (i) the Hartree-Fock pseudopotential, which maximizes T_c when $\langle\tilde{\Delta}_{n,\varphi}\rangle_F = 0$ and favors unconventional (*d*-wave) pairing; (ii) the dynamical part of the Coulomb interaction is unknown and therefore $\tilde{\Delta}^C$ must be approximated. The SFI theory assumes that $\tilde{\Delta}^C(\mathbf{k},\omega_n)$ depends on the dynamical spin susceptibility χ_s. Since $\mathrm{Im}\,\chi_s(\mathbf{q},\omega)$ is peaked at $\mathbf{Q}=(\pi,\pi)$, this term is repulsive and favors *d*-wave pairing. Although $\tilde{\Delta}_{n,\varphi}^C$ contributes little to the strength of $\tilde{\Delta}_{n,\varphi}$, it is important to trigger superconductivity from *s*-wave to *d*-wave pairing.[1,11]

In Eqs. (1) and (2) nonmagnetic impurities are included also. The theory of the *t-J* model predicts that strong correlations induce the FSP in the impurity-scattering matrix, being $t(\varphi,\varphi',\omega) \sim \gamma_c^2(\varphi-\varphi')$.[1] In leading order one has $t(\varphi,\varphi',\omega) \sim \delta(\varphi-\varphi')$, thereby not affecting any pairing. In reality, impurities are pair breaking for *d*-wave pairing and the next-to-leading term is necessary. This term is controlled by two scattering rates, $\gamma_{1,\varphi}$ and $\gamma_{2,\varphi}$, where $\gamma_{1,\varphi} - \gamma_{2,\varphi} \geq 0$. The case $\gamma_{1,\varphi} = \gamma_{2,\varphi}$ mimics the extreme forward-scattering peak, not affecting T_c, while $\gamma_{2,\varphi} = 0$ means an isotropic and strong pair-breaking scattering.[1]

The quasiparticle energy $\omega(\xi_{\mathbf{k}})$ is the pole of the retarded Green's function. For numerics we take for simplicity the Lorentzian shape for $\alpha_{ph,\varphi}^2 F_\varphi(\Omega)$ centered at ω_{ph}. For a quali-

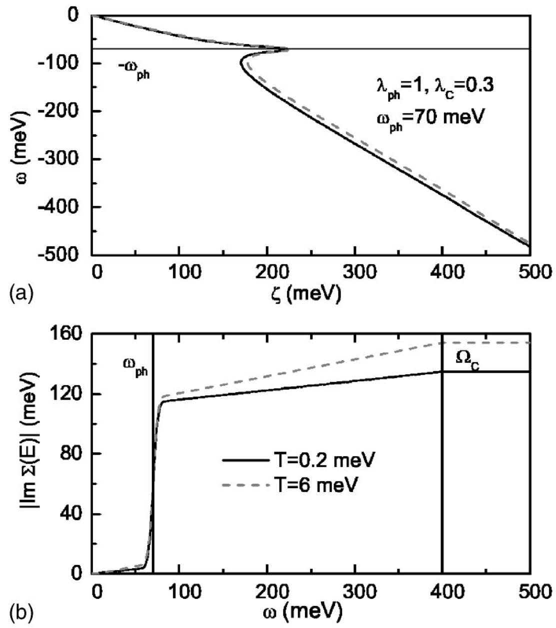

FIG. 1. (a) The quasiparticle-spectrum $\omega(\xi_{\mathbf{k}})$ and (b) the imaginary self-energy $\mathrm{Im}\,\Sigma(\xi=0,\omega)$ in the nodal direction ($\varphi=\pi/4$) in the superconducting ($T=0.2$ meV) and normal ($T=6$ meV) state. $\Omega_C=400$ meV is the cutoff in S_C.

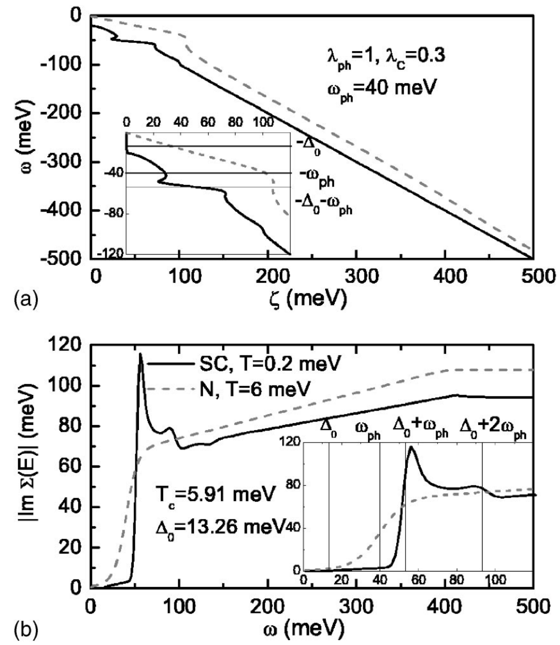

FIG. 2. (a) The quasiparticle-spectrum $\omega(\xi_{\mathbf{k}})$ and (b) the imaginary self-energy $\mathrm{Im}\,\Sigma(\xi=0,\omega)$ in the antinodal direction ($\varphi=0;\pi/2$) in the superconducting ($T=0.2$ meV) and normal ($T=6$ meV) state.

tative explanation of the ARPES nonshift puzzle we take moderate values for $\lambda_{ph,\varphi}\approx\lambda_{ph}=1$, $\lambda_C=0.3$ in both the nodal and antinodal directions. They can take larger values especially in the antinodal region. It is apparent from Eqs. (1) and (2) that the quasiparticle renormalization is local (angle decoupled) on the Fermi surface. This behavior is expected to be realized in a more realistic model with the finite width $\delta\varphi_w$, but with $\delta\varphi_w \ll \pi$.[1]

(i) *Kink in the spectrum in the nodal direction* at $\omega_{kink}^{(70)}\approx 70$ meV in $\omega(\xi_{\mathbf{k}})$ means that the quasiparticles moving along the nodal direction ($\varphi=\pi/4$) interact with phonons with frequencies up to 70 meV,[14] i.e., $\alpha^2_{ph,\pi/4}F_{\pi/4}(\Omega)\neq 0$ for $0<\Omega\lesssim 70$ meV. Since $\Delta_{\pi/4}(\omega)=0$ the local form of Eq. (2) implies that $\omega(\xi_{\mathbf{k}})$ is *not shifted* in the superconducting state. Numerical calculations in Fig. 1(a) confirm this result that is in agreement with ARPES results.[3] For a realistic phonon spectrum the theoretical singularity in $\omega(\xi_{\mathbf{k}})$ [shown in Fig. 1(a)] is expected to be smeared, having also an additional structure due to other phonons which contribute to $\alpha^2F(\omega)$.

(ii) *The singularity in the antinodal direction* (not the kink) in $\omega(\xi_{\mathbf{k}})$ at $\omega_{sing}^{(40)}$ in the antinodal direction ($\varphi\approx\pi/2$) is observed in ARPES in the normal and superconducting state of $La_{2-x}Sr_xCuO_4$ and BISCO.[6] This means that the quasiparticles moving in the antinodal direction interact with a narrower phonon spectrum centered around $\omega_{ph}^{(40)}\approx 40$ meV. Since $|\Delta_{\pi/2}(\omega)|=\Delta_0$, then Eq. (1) gives that in the normal state $\omega(\xi_{\mathbf{k}})$ is singular at $\omega_{sing}=\pm\omega_{ph}^{(40)}$, while in the superconducting state *the singularity is shifted* to $\omega_{sing}^{(40)}=\pm[\omega_{ph}^{(40)}+\Delta_0]$. This is confirmed by numerical calculations in Fig. 2(a) for $\omega(\xi_{\mathbf{k}})$, and in Fig. 2(b) for $\mathrm{Im}\,\Sigma(\varphi,\omega)$, for $\lambda_{ph}=1$ and $\lambda_C=0.3$. ARPES experiments give $\lambda_{ph}>1$ in the antinodal region.[6] Note that the theoretical singularity in Fig. 1(a) is stronger than in Fig. 2(a), because the calculations are performed for the same temperature, and since $\omega_{ph}^{(70)}>\omega_{ph}^{(40)}$ the latter singularity is smeared by temperature effects more than the former. The real shape of these singularities depends on microscopic details—the presence of the van Hove singularity in the antinodal region, etc.

(iii) *The kneelike shape of* $\mathrm{Im}\,\Sigma(\xi=0,\omega)$ is shown in Fig. 1(b) for the nodal kink (at $\omega_{ph}=70$ meV) and in Fig. 2(b) for the antinodal singularity (at $\omega_{ph}=40$ meV). In both cases there is a clear kneelike structure for ω near ω_{ph}, which is in accordance with the recent ARPES results in various HTSC families.[3–6] From Fig. 1(b) it is also seen that for $\omega_{ph}^{(70)}<\omega<\Omega_C$ the linear term is discernable in $|\mathrm{Im}\,\Sigma| \sim |\mathrm{Im}\,\Sigma_{ph}(\omega_{ph})|+\pi\lambda_{C,\varphi}\omega/2$, while for $\omega\approx\omega_{ph}^{(70)}$ the slope of $|\mathrm{Im}\,\Sigma(\xi=0,\omega)|$ is steeper, since for $\lambda_{ph}(=1)\gg\lambda_C(=0.3)$ the term $|\mathrm{Im}\,\Sigma_{ph}(\omega_{ph})|$ $[\gg|\mathrm{Im}\,\Sigma^C(\omega_{ph})|]$ dominates. The kneelike shape of $\mathrm{Im}\,\Sigma(\xi=0,\omega)$, as well as the nonshift effect of the kink at 70 meV, are "smoking gun" results for HTSC theories that favor the EPI as the pairing interaction. At present only the EPI-FSP model (theory) is able to explain the four distinctive features in ARPES spectra in a consistent way. The kneelike structure in the normal state was also obtained in Ref. 15, where the EPI and Coulomb interactions are treated phenomenologically. The EPI-FSP model predicts also the kneelike structure in the antinodal region. However, in this case the closeness of the antinodal point to the van Hove singularity may influence $\Sigma_{ph}(\xi=0,\omega)$ significantly and change its shape too.

(iv) *ARPES dip-hump structure*. The EPI-FSP model explains qualitatively the dip-hump structure in $A(\varphi,\omega)=-\mathrm{Im}\,G(\varphi,\omega)/\pi$ which was observed recently in ARPES.[4] In Fig. 3(a) it is seen that the dip-hump structure is realized in

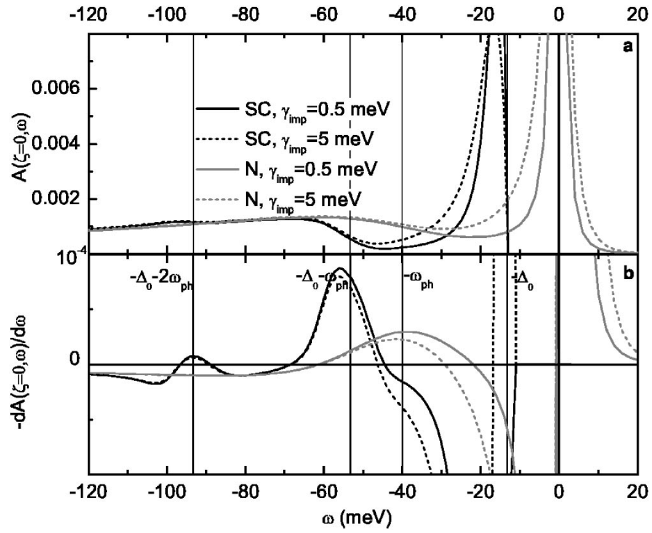

FIG. 3. (a) The spectral function $A(\xi=0,\omega)$ and (b) $-dA(\xi=0,\omega)/d\omega$ in the antinodal direction in the superconducting ($T=0.2$ meV) and normal ($T=6$ meV) state for various impurity-scattering rates γ_1 and $\gamma_2=0$; $\lambda_{ph}=1$, $\lambda_C=0.3$.

the normal state already for a moderate value of $\lambda_{ph}=1$. The dip is more pronounced in the superconducting state where the peak in $A(\omega)$ is appreciably narrowed, which is in accordance with ARPES experiments.[4] Contrary to expectations, the dip energy does not coincide with the (shifted) phonon energy at $\omega_{ph}=40$ meV. However, the positions of the maxima of $-dA/d\omega$ appear near the energies $(-\Delta_0-n\omega_{ph})$ as it is seen in Fig. 3(b). The calculations give also a dip in both the antinodal and nodal density of states $N(\omega)$ (not shown) already for $\lambda_{ph}=1$, which is more pronounced for larger $\lambda_{ph}(>1)$.

We stress some important points: (i) in Eqs. (1)–(4) the Migdal vertex corrections due to the electron-phonon interaction are neglected. It is shown in Ref. 16 that these corrections may increase T_c significantly, by decreasing at the same time the isotope effect, even for $\lambda_{ph}<1$. Concerning the local structure of the self-energy (on the Fermi surface) there is a hope that our qualitative explanation of the nonshift puzzle will survive also in this case. (ii) At present it is unclear if the ARPES kink at 70 meV is due to a single phonon mode at this frequency or if it simply characterizes the end of the broad-phonon spectrum. The latter case is clearly observed in tunneling experiments.[1] Some recent ARPES spectra in $La_{2-x}Sr_xCu_4$ points also to at least four phonon modes interacting with electrons.[5]

In conclusion, the four distinctive features in the quasiparticle self-energy, obtained from the ARPES spectra in HTSC materials, are explained consistently by the phenomenological microscopic theory of electron-phonon interaction (EPI) with the forward-scattering peak (FSP), which dominates over the Coulomb scattering. This theory (supported also by the calculations in the *t-J* model) explains why there is *no shift* of the nodal kink at 70 meV in the superconducting state, contrary to the observed *shift* (~20 meV) of the antinodal singularity at 40 meV. The nonshift puzzle is a direct consequence of the existence of the FSP in the EPI, i.e., due to the long-range character of the electron-phonon interaction in HTSC oxides.[17] The existence of the FSP is supported at least by two specific (for HTSC) interactions: (i) by *strong correlations* and (ii) by the pronounced *long-range Madelung EPI*, due to the layered ionic-metallic structure of HTSC oxides.[1,18] However, for the quantitative theory the EPI-FSP model must be refined to include realistic phonon and band structures of HTSC oxides.

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