Coulomb blockade in quantum dots under ac pumping

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We study electron transport through a Coulomb blockaded quantum dot driven by an external periodic perturbation. The stationary state is determined by the balance between the heating of the dot electrons by the perturbation and cooling. We analyze two cooling mechanisms: electron exchange with the cold contacts and emission of phonons. Together with the usual linear Ohmic heating of the dot electrons we consider possible effects of dynamic localization. The combination of the abovementioned factors may result in a drastic change of the shape of the Coulomb blockade peak with respect to the usual equilibrium one.

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I. INTRODUCTION

At low temperatures electronic conduction through a quantum dot weakly coupled to the contacts is governed by the Coulomb blockade effect[1]—suppression of transport due to the energy cost of changing the number of electrons in the dot. Efficient conduction through such a dot is possible only when the electrostatic potential of the dot, controlled by external gates, is tuned to a special value where the Coulomb energies of the states with N and $N+1$ electrons in the dot are close for some N. As a result, the linear response conductance exhibits a sharp peak as a function of the gate voltages. The theory of the Coulomb blockade in equilibrium is well developed by now.[2]

In the last few years several experiments have been done on quantum dots under an external ac perturbation.[3] Under these nonequilibrium conditions the electronic temperature of the dot is determined by the balance between heating by the ac perturbation and cooling due to various mechanisms, rather than by the external cryostat. One of these mechanisms is simple exchange of electrons between the dot and the cold contacts, especially important at low temperatures. As the gate voltage is tuned away from the Coulomb blockade peak, the dot becomes effectively more closed, the cooling rate decreases, which affects the electronic temperature, thus changing the peak shape with respect to the equilibrium one. This simple qualitative consideration poses the problem, which is going to be studied in detail in the present work.

Another motivation to study these effects is the search for experimental signatures of dynamic localization (DL). Dynamic localization in the kicked quantum rotor[4] has been extensively studied theoretically,[5,6] and observed experimentally in trapped ultracold atoms in the field of a modulated laser standing wave.[7] We have shown recently[8,9] that an analogous suppression of the energy absorption may occur in a chaotic quantum dot under an ac excitation, e.g., similar to those studied in the experiments of Ref. 3. One of the best measured characteristics of a quantum dot is its conductance. To find a signature of the DL in the conductance, one should perform the measurements under the Coulomb blockade conditions (almost closed dot), since it is in this regime that the conduction is sensitive to the internal state of the dot. If the dot is well open, the effect of electron-electron interaction on the conduction is weak, and the conductance does not depend on the electron energy distribution in the dot.[10,11]

Typically, heating of a metallic dot by an ac perturbation is associated with classical Ohmic microwave absorption at a frequency ω by a small particle made of a metal with large conductivity $\sigma \gg \omega$. This picture is valid when the single-electron mean level spacing δ in the dot is small enough. From the quantum-mechanical point of view this situation is adequately described by the Fermi golden rule, which determines the probability of each single-electron transition per unit time Γ, proportional to the intensity of the perturbation.[12] The total energy E of electrons in the dot (as counted from the ground state) then grows linearly with time: $E(t)=\Gamma\omega^2 t/\delta \equiv W_0 t$. The criterion of validity of the Fermi golden rule is $\delta \ll \Gamma$, and $\Gamma \ll \omega$ is also assumed (here and below $\hbar=1$).

This picture (hereafter referred to as Ohmic absorption) is based on the assumption that each single-electron transition occurs independently of the previous ones. However, for a small but finite δ the effects of quantum interference accumulate after many transitions, leading to a decrease in the absorption,[8,9] and its complete suppression after a time $t_* \sim \Gamma/\delta^2$. This effect was named the dynamic localization in energy space. The corresponding "localization length" (the characteristic spread of the electron distribution function) $T_* \sim \Gamma\omega/\delta$ plays the role of the effective electronic temperature reached by the time t_*. Note that since the electron spectrum is unbounded, DL has nothing to do with the saturation of absorption by a pumped two-level system. DL is the consequence of level discreteness: at $\delta \to 0$ it takes longer time for the DL to develop, and for the continuous spectrum there is no DL. Since this effect drastically modifies the heating rate, the stationary state of the dot is strongly affected.

The random matrix theory approach, adopted in Refs. 8 and 9, correctly describes the single-particle properties of a chaotic dot in the energy range determined by the so-called Thouless energy E_{Th}. This energy is defined by the order of magnitude as the inverse of the ergodic time—the time required for an electron to travel across the dot, thus randomizing its motion due to scattering off impurities or the dot

boundaries. Another important energy scale is the Coulomb charging energy of the dot E_c: the Coulomb blockade effects are pronounced only if the effective temperature is smaller than E_c. Thus, in the following, the hierarchy of scales $\delta \lesssim \Gamma \lesssim \omega \lesssim T_* \lesssim E_{\mathrm{Th}}, E_c$ is assumed. From the experimental point of view it means that one should choose a dot satisfying the inequality $\delta \ll E_{\mathrm{Th}}, E_c$ as strongly as possible. Note that within validity range of the random matrix theory one can neglect multiphoton processes as they are of the order of the inverse matrix size.

Possible cooling mechanisms for electrons in the dot are (i) electron exchange with the contacts and (ii) energy exchange with the phonon subsystem. Both electrons in the contacts and phonons in the dot are assumed to be maintained at a constant temperature T_0 determined by the cryostat. In the following we analyze the interplay of the above-mentioned effects in heating and cooling, and see how they affect the shape of the Coulomb blockade peak. In a short preliminary version of this study we have considered only the first cooling mechanism.[13] Here we include cooling by phonon emission which, to the best of our knowledge, has been little studied for a quantum dot.

The paper is organized as follows. In Sec. II we analyze the heating and discuss how it is affected by dynamic localization. In Secs. III and IV we give a detailed analysis of the two cooling mechanisms. In Sec. V we consider the resulting stationary state and the Coulomb blockade peak shape. Section VI contains remarks about another possible way of detecting the dynamic localization via the measurement of the nonlinear absorption, which does not require connecting the dot to the contacts. Finally, in Sec. VII we summarize the main results.

II. HEATING BY ac PERTURBATION

In the Ohmic regime the energy absorption by electrons is linear in the field intensity and given by $W_0 = \Gamma\omega^2/\delta$ (we remind the reader that Γ is a measure of the microwave field intensity, equal to the probability per unit time of a single one-photon transition). The same expression can be obtained from simple classical arguments considering a small particle made of a metal with a large finite conductivity $\sigma \gg \omega$.

In the regime of the strong dynamic localization the absorption is no longer given by the simple Ohmic expression. For noninteracting electrons in a closed dot the absorption becomes completely suppressed by interference corrections that develop in a characteristic time $t_* \sim \Gamma/\delta^2$, and the effective temperature of the electrons, reached by that time, is $T_* \sim \Gamma\omega/\delta$. Absorption is possible only due to inelastic processes which destroy the quantum-mechanical phase and thus all interference effects. If the dephasing rate $\gamma_\phi \ll 1/t_*$, the residual absorption rate is given by

$$W_{\mathrm{in}} \sim W_0 \gamma_\phi t_* = T_*^2 \frac{\gamma_\phi}{\delta}. \tag{1}$$

If $\gamma_\phi \gtrsim 1/t_*$, the strong dynamic localization is destroyed by dephasing, and the absorption is given by the Ohmic expression with small weak localization corrections considered in Refs. 8 and 9.

Generally speaking, the dephasing rate γ_ϕ by itself is not a well-defined quantity, unless the mechanism of the dephasing is specified. Equation (1) was justified in Ref. 14, where electron-electron collisions in the dot were considered. In fact, it has a wider range of applicability: the main condition is that dephasing should be caused by a sequence of distinct events with average frequency γ_ϕ, each of them destroying the electron phase completely, rather than phase diffusion. This means that γ_ϕ roughly coincides with the quasiparticle relaxation rate. This is certainly the case for electron escape to the contacts, since then the escaping electron is effectively replaced by another one with an absolutely random phase. This is also true for electron-electron and electron-phonon collisions, since the typical energy transferred in a collision is of the order of the (effective) electronic temperature in the dot, which is large: $T \gg 1/t_*, \gamma_\phi$ (this inequality follows from $T_* \gg 1/t_*$ due to $\Gamma, \omega \gg \delta$, and from $T \gtrsim T_*$).

In all these cases the following arguments can be applied. As $\gamma_\phi t_* \ll 1$, the collisions are rare, so the electrons spend most of the time in the states localized in energy, and have definite phase relationships. If the phase of some electron is destroyed in a collision, its wave packet starts spreading in energy. It localizes again after the time $\sim t_*$, having spread by $\sim T_*$. Thus, the ac driven dynamics following the collision leads to a change of the total electronic energy of $\sim T_*$ per collision. The sign of this change is, however, arbitrary, because a periodic perturbation can equally cause transitions up and down the spectrum. Only the presence of the filled Fermi sea below (i.e., an energy gradient of the electronic distribution function) makes absorption the preferred direction, which means that if the electronic temperature $T \gg T_*$, the energy absorbed per collision is on the average $\sim T_*^2/T$ rather than T_*. The effective number of electrons that can participate in a collision is $\sim T/\delta$ (due to the degenerate Fermi statistics). During the time interval $\sim 1/\gamma_\phi$ each of these electrons participates in one collision, so the total number of collisions per unit time is $\sim (T/\delta)\gamma_\phi$. This gives the energy absorption rate $W_{\mathrm{in}} \sim (T_*^2/T)(T/\delta)\gamma_\phi$, which is exactly Eq. (1).

Equation (1) allows for another simple interpretation. Each electron spends a time $\sim 1/\gamma_\phi$ in a localized state without absorbtion. After each collision during the time $\sim t_*$ it absorbs the energy from the microwave field, then it localizes again and waits for the next event (provided that $t_* \ll 1/\gamma_\phi$). The absorption rate of the whole system is then given by the simple weighted average $W_{\mathrm{in}} \sim W_0 \gamma_\phi t_*$, which is again Eq. (1).

An important point is that dephasing rate, generally speaking, depends on the electronic temperature, which results in a temperature-dependent absorption rate in the DL regime. The temperature, in turn, is determined by the balance between energy absorption and cooling. This feedback leads to a nontrivial dependence of the characteristics of the stationary state on the control parameters, which will manifest itself in a change of the Coulomb blockade peak shape, as will be shown below. The absorption itself becomes nonlinear with the field intensity through the dependence of γ_ϕ on W_0.

III. COOLING DUE TO ELECTRON ESCAPE

We characterize the coupling of the dot to the two contacts by single-particle escape rates γ_1 and γ_2. When they are much smaller than the mean single-particle level spacing δ in the dot, the fluctuations of the total charge on the dot are small. If the dot is coupled to several gates through capacitances C_i and voltages V_i are applied to the gates, the electrostatic energy of the dot with N electrons on it is given by

$$E(N) = \frac{e^2N^2}{2C} + \sum_i \frac{C_iV_i}{C} eN, \quad C \equiv \sum_i C_i, \tag{2}$$

where $e^2/(2C) \equiv E_c$ is the charging energy. The energy cost of adding an electron is

$$U \equiv E(N+1) - E(N) = \frac{e^2}{C}\left[N + \frac{1}{2} + \sum_i \frac{C_iV_i}{e}\right]. \tag{3}$$

If all gates have the same voltage, then (up to a constant) U is given by this voltage. Generally, we will call *U the reduced gate voltage*; it is a natural control parameter for the system. If the expression in the brackets is of the order of unity and the temperature $T \ll E_c$, the conductance through the dot is suppressed due to the Coulomb blockade. If the gate voltages are tuned so that the expression in the brackets is small for some particular N, the dot conductance $G(U)$ exibits a peak for these values of U. The width of the peak $\Delta U \sim T$, which can be used to measure the temperature of the system.

A. Sequential tunneling

When U is tuned to the peak, the main contribution to the conductance comes from the leading order of the perturbation theory in the dot-contact coupling. For characteristic temperatures $T \gg \delta$ one can describe the system by rate equations of Kulik and Shekhter.[15] We consider these equations for the case when the electron energy distribution function in the dot f_ϵ is nonequilibrium. Let the distribution in the αth contact be $f_\epsilon^{(\alpha)}$. Assuming the dot to have either N or $N+1$ electrons with the probabilities p_N, p_{N+1} to have N or $N+1$ electrons on the dot (all others are neglected, so $p_N + p_{N+1} = 1$), we can write the rate equation as

$$\frac{dp_N}{dt} = 2p_{N+1}\sum_{\alpha=1,2}\gamma_\alpha \int f_\epsilon(1 - f_{\epsilon+U}^{(\alpha)})\frac{d\epsilon}{\delta} - 2p_N \sum_{\alpha=1,2}\gamma_\alpha \int (1 - f_\epsilon) f_{\epsilon+U}^{(\alpha)} \frac{d\epsilon}{\delta}, \tag{4}$$

where the factor of 2 comes from the spin degeneracy. The distributions in the contacts are assumed to be Fermi-Dirac ones with the temperature T_0:

$$f_\epsilon^{(\alpha)} = f_\epsilon^{T_0} \equiv \frac{1}{e^{\epsilon/T_0} + 1}. \tag{5}$$

As usual, we require p_N and p_{N+1} to be stationary. Shifting the distribution in one of the contacts by an infinitesimal voltage, one obtains the linear response conductance G:

$$G(U) = \frac{2e^2}{\delta}\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} \times \frac{F_{\rm in}^2(U)F'_{\rm out}(U) - F'_{\rm in}(U)F_{\rm out}^2(U) + F_{\rm in}(U)F_{\rm out}(U)}{[F_{\rm in}(U) + F_{\rm out}(U)]^2}, \tag{6}$$

$$F_{\rm in}(U) \equiv \int (1 - f_\epsilon) f_{\epsilon+U}^{T_0} d\epsilon, \quad F_{\rm out}(U) \equiv \int f_\epsilon (1 - f_{\epsilon+U}^{T_0}) d\epsilon. \tag{7}$$

In the equilibrium case, when $f_\epsilon = f_\epsilon^{T_0}$ as well, the last fraction in the right-hand side of Eq. (6) reduces to the familiar expression $(1/2)(U/T)/\sinh(U/T)$.

Tunneling events lead to the change in the distribution function in the dot. The kinetic equation describing this process can be obtained straightforwardly from equations of Ref. 15 and reads

$$\frac{\partial f_\epsilon}{\partial t} = (\gamma_1 + \gamma_2)\frac{(1 - f_\epsilon) f_{\epsilon+U}^{T_0} F_{\rm out}(U) - f_\epsilon(1 - f_{\epsilon+U}^{T_0})F_{\rm in}(U)}{F_{\rm in}(U) + F_{\rm out}(U)}. \tag{8}$$

If we introduce the functions

$$\mathcal{E}_{\rm in}(U) \equiv \int (1 - f_\epsilon) f_{\epsilon+U}^{T_0} \epsilon d\epsilon, \tag{9}$$

$$\mathcal{E}_{\rm out}(U) \equiv \int f_\epsilon (1 - f_{\epsilon+U}^{T_0}) \epsilon d\epsilon, \tag{10}$$

and denote by $\gamma \equiv \gamma_1 + \gamma_2$ the total single-electron broadening, the cooling rate for the dot electrons (the total energy loss per unit time) can be written as

$$W_{\rm out}(U) = \frac{\gamma}{\delta}\frac{\mathcal{E}_{\rm out}(U)F_{\rm in}(U) - \mathcal{E}_{\rm in}(U)F_{\rm out}(U)}{F_{\rm in}(U) + F_{\rm out}(U)}. \tag{11}$$

From the kinetic equation (8) one can also extract the single-particle escape rate for a particle with the energy ϵ:

$$\gamma_{\rm esc} = \gamma(1 - f_{\epsilon+U}^{T_0})\frac{F_{\rm in}(U)}{F_{\rm in}(U) + F_{\rm out}(U)}. \tag{12}$$

In the following we will use the expression for $\gamma_{\rm esc}$ at $\epsilon = 0$ as an estimate. We will also use the Fermi-Dirac form for the electronic distribution function with some temperature T. This is true only if electron-electron collisions restore the Fermi-Dirac shape much faster than it is modified by other processes. If this is not the case, T still gives the characteristic width of the distribution function. It is determined by the balance between heating by the ac field and cooling considered in the previous section.

We also assume the electronic temperature in the dot to be much higher than the temperature of the contacts [the latter can be made as low as $\sim$10 mK (Ref. 16)] which is true if the pumping power is high enough. Then we can set the temperature of the contacts to be zero, which allows an explicit calculation in Eqs. (6)–(11) [we denote $x \equiv U/(2T)$, $G_0 \equiv G(U=0)$]:

$$F_{\rm in}(U) = T\ln[1+e^{-2x}], \tag{13}$$

$$F_{\rm out}(U) = F_{\rm in}(-U), \tag{14}$$

$$\frac{G(U)}{G_0} = 1 - \frac{x\tanh x}{\ln(2\cosh x)}, \tag{15}$$

$$\mathcal{E}_{\rm in}(U) = -2T^2\int_x^\infty (1-\tanh y)y\,dy, \tag{16}$$

$$\mathcal{E}_{\rm out}(U) = -\mathcal{E}_{\rm in}(-U), \tag{17}$$

$$\frac{W_{\rm out}(U)}{(\gamma/\delta)T^2} = \frac{\pi^2}{12} - x^2 + \frac{2x}{\ln(2\cosh x)}\int_0^x y\tanh y\,dy, \tag{18}$$

$$\frac{\gamma_{\rm esc}(U)}{\gamma} = \frac{1}{2} - \frac{|x|}{2\ln(2\cosh x)}. \tag{19}$$

B. Inelastic cotunneling

At large $U \gg T$ sequential tunneling becomes suppressed exponentially. In this situation both the conduction and cooling become dominated by cotunneling—a second-order process whose probability contains an additional small factor $\gamma\delta/U^2$. Obviously, only inelastic cotunneling[17] can contribute to cooling. Elastic cotunneling,[18] which does not change the electronic state of the dot, contributes to conduction at temperatures $T < \sqrt{E_c\delta}$. We will be interested in higher temperatures and do not consider this contribution.

A straightforward generalization of the considerations of Ref. 17 to the nonequilibrium case leads to the following expression for the conductance in terms of the electronic distribution functions and the kinetic equation for the distribution in the dot:

$$G(U) = \frac{4e^2\gamma_1\gamma_2}{\pi U^2\delta^2}\int f_{\epsilon-\Omega}(1-f_\epsilon)(1-f_{\epsilon'}^{T_0})\left(-\frac{\partial f_{\epsilon'+\Omega}^{T_0}}{\partial\epsilon'}\right)d\epsilon d\epsilon' d\Omega, \tag{20}$$

$$\frac{\partial f_\epsilon}{\partial t} = \frac{\gamma^2}{\pi U^2\delta}\int [(1-f_\epsilon)f_{\epsilon-\Omega}(1-f_{\epsilon'}^{T_0})f_{\epsilon'+\Omega}^{T_0} - f_\epsilon(1-f_{\epsilon-\Omega})f_{\epsilon'}^{T_0}(1-f_{\epsilon'+\Omega}^{T_0})]d\epsilon' d\Omega. \tag{21}$$

For a Fermi-Dirac distribution $f_\epsilon = f_\epsilon^T$ the integrals can be calculated explicitly for any temperatures T, T_0:

$$G(U) = \frac{2\pi e^2\gamma_1\gamma_2}{3U^2\delta^2}(T^2+T_0^2), \tag{22}$$

$$W_{\rm out}(U) = \frac{2(\gamma_1+\gamma_2)^2}{15\pi\delta^2U^2}(T^4-T_0^4). \tag{23}$$

The electron escape rate at $\epsilon = 0$ can be extracted from the kinetic equation (21):

$$\gamma_{\rm esc} = \frac{\pi}{6}\frac{\gamma^2}{U^2\delta}(T^2+2T_0^2). \tag{24}$$

If we set, as before, $T_0 = 0$, we obtain the following explicit expressions:

$$\frac{W_{\rm out}}{(\gamma/\delta)T^2} = \frac{\pi^3}{30}\frac{\gamma/\delta}{x^2}, \quad \frac{G}{G_0} = \frac{\pi}{6}\frac{\gamma/\delta}{x^2}, \quad \frac{\gamma_{\rm esc}}{\gamma} = \frac{\pi}{24}\frac{\gamma/\delta}{x^2}. \tag{25}$$

C. Photon-assisted tunneling

So far the only effect of the ac perturbation we were interested in was to cause transitions between single-particle states in the dot. The perturbation, however, may possess a component $V\cos\omega t$, proportional to the unit matrix in the dot single-particle Hilbert space. In a closed dot this component does not cause any transitions and can be gauged out completely, so it does not affect any observables, either single-particle or many-particle ones (in particular, it does not affect electron-electron collisions).

However, when the dot is connected to contacts, this is no longer the case, as the diagonal component is responsible for the photon-assisted tunneling.[19] This effect can be taken into account by replacing the dot electron distribution function f_ϵ in the above formulas by

$$f_\epsilon \to \sum_{n=-\infty}^{\infty} J_n^2(V/\omega) f_{\epsilon+n\omega}, \tag{26}$$

where J_n is the Bessel function. Photon-assisted tunneling will not be important for our considerations if the smearing of the distribution function given by Eq. (26) is much smaller than the thermal smearing. Using the asymptotic expansion of $J_n(z)$ at large n, this condition can be written as

$$J_n(z) \sim \frac{1}{\sqrt{2\pi n}}\left(\frac{ez}{2n}\right)^n \Rightarrow \max\{V,\omega\} \ll T. \tag{27}$$

The condition $\omega \ll T$ is authomatically fulfilled if $\Gamma \gg \delta$ and $T > T_* = \Gamma\omega/\delta$. As for the condition $V \ll T$ we note that within the $N\times N$ random matrix approximation, adopted in Ref. 13, we have $\langle V^2\rangle = (1/N)\Gamma\delta$ so that $V \to 0$ as $N \to \infty$.

In addition to the random component with zero mean included in the random-matrix treatment, V can have a deterministic part. It is given by the spatial average of the perturbation potential over the dot volume, and enters our model as an *independent* parameter. Thus in order to fulfill the condition $V \ll T$ a special experimental care should be taken.

IV. COOLING DUE TO PHONON EMISSION

A. General expressions

Another important mechanism of electronic energy relaxation is emission of phonons. For mesoscopic metallic rings with diffusive electronic motion this problem was addressed in Ref. 20 For quantum dots energy relaxation at frequencies smaller than the mean level spacing has been considered;[21,22] here we are interested in the opposite limiting case, δ being

the smallest energy scale. Below we estimate the corresponding cooling rate for clean (ballistic) quantum dots made out of 2D electron gas (2DEG) in a GaAs/AlGaAs heterostructure[3] and bulk 3D phonons.

For ballistic dots (whose size L is smaller than the elastic mean free path ℓ) one does not need to take into account phonon-induced impurity displacements,[23] so the phonon-induced potential felt by the electrons can be written in the form

$$\hat{V}(\mathbf{r}) = \int \frac{d^3\mathbf{q}}{(2\pi)^3}\hat{V}(\mathbf{q})e^{i\mathbf{q}\cdot\mathbf{r}} = \sum_{\mathbf{q},\lambda} V_{\mathbf{q},\lambda}\hat{b}_{\mathbf{q},\lambda}e^{i\mathbf{q}\cdot\mathbf{r}} + \text{H.c.}, \quad (28)$$

where $\hat{b}_{\mathbf{q},\lambda}$ is the annihilation operator for a phonon mode λ with the wave vector $\mathbf{q}$. The detailed form of the coupling $V_{\mathbf{q},\lambda}$ depends on the specific coupling mechanism to be specified below.

The probability of the electronic transition from an initial single-particle state s with the energy ϵ_s and the wave function $\psi_s(\mathbf{r})$ to the final state s' with the energy $\epsilon_{s'}$ and the wave function $\psi_{s'}(\mathbf{r})$, accompanied by absorption or emission of one phonon, is given by the Fermi golden rule

$$w_{s\to s'}^{\text{abs(em)}} = 2\pi\sum_{\mathbf{q},\lambda}\left|\int \psi_{s'}^*(\mathbf{r})e^{\pm i\mathbf{q}\cdot\mathbf{r}}\psi_s(\mathbf{r})d^d\mathbf{r}\right|^2 |V_{\mathbf{q},\lambda}|^2 \times\left(N_{\mathbf{q},\lambda}+\frac{1}{2}\mp\frac{1}{2}\right)\delta(\epsilon_{s'}-\epsilon_s\mp\omega_{\mathbf{q},\lambda}), \quad (29)$$

where $N_{\mathbf{q},\lambda}$ is the phonon occupation number before the transition and $\omega_{\mathbf{q},\lambda}$ is the phonon frequency; the upper sign corresponds to the phonon absorption, the lower one to emission. Introducing the transition rate

$$w(\epsilon,\epsilon') = \delta^2\sum_{s,s'}(w_{s\to s'}^{\text{abs}} + w_{s\to s'}^{\text{em}})\delta(\epsilon-\epsilon_s)\delta(\epsilon'-\epsilon_{s'}), \quad (30)$$

averaged over the random dot realizations, we can write the kinetic equation for the electronic distribution function f_ϵ:

$$\frac{\partial f_\epsilon}{\partial t} = \int [w(\epsilon',\epsilon)(1-f_\epsilon)f_{\epsilon'} - w(\epsilon,\epsilon')f_\epsilon(1-f_{\epsilon'})]\frac{d\epsilon'}{\delta}. \quad (31)$$

The average rate (30) is determined by the electronic wave function correlations in the dot:

$$\Pi_{\epsilon,\epsilon'}(\mathbf{r},\mathbf{r}') \equiv \sum_{s,s'}\psi_s(\mathbf{r})\psi_s^*(\mathbf{r}')\psi_{s'}(\mathbf{r}')\psi_{s'}^*(\mathbf{r}) \times\delta(\epsilon-\epsilon_s)\delta(\epsilon'-\epsilon_{s'}), \quad (32)$$

averaged over the dot realizations. Then we can write the average transition rate as

$$w(\epsilon,\epsilon') = 2\pi\delta^2\sum_{\mathbf{q},\lambda}\Pi_{\epsilon,\epsilon'}(\mathbf{q},\mathbf{q})|V_{\mathbf{q},\lambda}|^2 \times[N_{\mathbf{q},\lambda}\delta(\epsilon'-\epsilon-\omega_{\mathbf{q},\lambda}) + (N_{\mathbf{q},\lambda}+1)\delta(\epsilon'-\epsilon+\omega_{\mathbf{q},\lambda})], \quad (33)$$

with the Fourier transform defined as

$$\Pi_{\epsilon,\epsilon'}(\mathbf{q},\mathbf{q}') \equiv \int \Pi_{\epsilon,\epsilon'}(\mathbf{r},\mathbf{r}')e^{-i\mathbf{q}\cdot\mathbf{r}+i\mathbf{q}'\cdot\mathbf{r}'}d^3\mathbf{r}d^3\mathbf{r}'. \quad (34)$$

Statistical properties of ballistic dots have been extensively studied (in Refs. 24 and 25, for a review see Refs. 2 and 26). For $|\epsilon-\epsilon'|$ smaller than the Thouless energy E_{Th} one can use the following estimate:

$$\Pi_{\epsilon,\epsilon'}(\mathbf{q},\mathbf{q}) \sim \frac{1}{E_{\text{Th}}\delta}\min\{1,(q_\parallel L)^2\}, \quad (35)$$

where the factor $(q_\parallel L)^2$ appears when $q_\parallel L \ll 1$ ($\mathbf{q}_\parallel$ is the component of the wave vector parallel to the plane of the 2DEG). As a result, the transition rate $w(\epsilon,\epsilon')$ depends only on the transferred energy $\omega \equiv \epsilon-\epsilon'$.

We assume the electronic temperature (determined by the balance between heating and cooling) to be much higher than the lattice temperature (determined by the external cryostat). In this case one can neglect any phonon population present, $N_{\mathbf{q},\lambda}=0$, so only emission of phonons can occur, and $w(\omega) \propto \theta(\omega)$. For a power-law dependence, $w(\omega)\propto\omega^\alpha\theta(\omega)$, and Fermi-Dirac electron distribution in the dot (5) the cooling rate is given by

$$W_{\text{out}} = \int \frac{d\epsilon}{\delta}\frac{d\omega}{\delta}\omega w(\omega)f_\epsilon(1-f_{\epsilon-\omega}) \propto \frac{T^{\alpha+3}}{\delta^2}. \quad (36)$$

Obviously, such a power-law dependence can be parametrized by a single parameter T_{ph} and written as $W_{\text{out}} = T^{\alpha+3}/T_{\text{ph}}^{\alpha+1}$. From the kinetic equation (31) one can also extract the single-particle relaxation rate γ_{ph}. For electrons with the typical energy $\epsilon\sim T$ and $w(\omega)\propto\omega^\alpha\theta(\omega)$ we obtain $\gamma_{\text{ph}}(T)\sim\delta(T/T_{\text{ph}})^{\alpha+1}$.

B. Specific mechanisms

To consider specific electron-phonon coupling mechanisms, we describe phonons in terms of the lattice displacement operator for each normal phonon mode λ:

$$\hat{\mathbf{u}}_\lambda(\mathbf{r}) = \sum_{\mathbf{q}}\sqrt{\frac{1}{2\mathcal{V}\rho_\lambda\omega_{\mathbf{q},\lambda}}}\mathbf{e}_{\mathbf{q},\lambda}\left[\hat{b}_{\mathbf{q},\lambda}e^{i\mathbf{q}\cdot\mathbf{r}} + \hat{b}_{\mathbf{q},\lambda}^\dagger e^{-i\mathbf{q}\cdot\mathbf{r}}\right]. \quad (37)$$

The displacement of each mode is directed along the unit vector $\mathbf{e}_{\mathbf{q},\lambda}$. To each mode corresponds some mass which is the total mass of the unit cell for acoustic phonons or the reduced mass for optical phonons; dividing it by the unit cell volume one obtains the corresponding density ρ_λ. Finally, $\mathcal{V}$ is the 3D quantization volume. At low temperatures we are interested in, only acoustic phonons can be emitted. We approximate their dispersion by $\omega_q = v_s q$, with v_s being the sound velocity, while the density ρ_λ coincides with the density of the crystal ρ_0.

Deformational coupling to the acoustic phonons is due to the local change of the electronic energy bands under strain:

$$\hat{V}^{\text{def}}(\mathbf{q}) = \Xi_{jl}iq_j\hat{u}_l(\mathbf{q}), \quad (38)$$

where Ξ_{jl} is the deformational coupling tensor. In a bulk crystal it, generally speaking, depends on the electronic wave vector $\mathbf{k}$ (Ref. 27). In doped GaAs, when typical electronic

wave vectors are close to the Brillouin zone center and one approximates the periodic part of the Bloch function by that for $\mathbf{k}=0$, this dependence vanishes and $\Xi_{jl}=\Xi\delta_{jl}$. The leading anisotropic (i.e., dependent on the direction of $\mathbf{k}$) correction at small finite $\mathbf{k}$ should be smaller by a factor of $(ka)^2$, where a is the lattice constant (the first order in $\mathbf{k}$ correction should vanish due to the time-reversal symmetry). Hence we can estimate its magnitude as $\sim(k_Fa)^2\Xi\sim na^2\Xi$, where k_F is the electronic Fermi wave vector and n is the 2D electron density.

The isotropic (independent of the direction of $\mathbf{k}$) part of the deformation potential is subject to screening.[27] The electrons inside the dot can screen the fields with wave vectors $q_\parallel$ down to $\sim 1/L$. We assume that the Fourier components with $q_\parallel \ll 1/L$ are also screened, either by the 2DEG outside the dot, or by the metallic gate. Thus, by the order of magnitude, we can use the expression for the static (due to $v_s \ll v_F$) screening by an infinite 2DEG, which results in the renormalization:

$$\Xi \to \frac{\Xi}{1+1/(q_\parallel a_s)} \approx q_\parallel a_s \Xi, \tag{39}$$

where a_s is the 2D screening length (equal to half the electronic Bohr radius), and we consider $q_\parallel a_s \ll 1$. Thus, the effective deformation potential is suppressed by a small factor: either by $q_\parallel a_s$ for the isotropic part of the potential, or by na^2 for the anisotropic part.

Piezoelectric coupling to acoustic phonons is due to the longitudinal electric field induced by the strain. We express the potential in terms of the electromechanical tensor e^{em}_{ijl}, which relates the induced polarization to the strain tensor

$$\hat{V}^{\mathrm{piezo}}(\mathbf{q}) = -\frac{4\pi e e^{\mathrm{em}}_{ijl}}{\varepsilon}\frac{q_i q_j}{q^2}\hat{u}_l(\mathbf{q}). \tag{40}$$

Here ε is the background dielectric constant of the material. The in-plane piezoelectric field is also subject to screening, which brings a factor of $q_\parallel a_s$.

The component of the piezoelectric field perpendicular to the dot plane is not screened by the electrons. Instead, it affects the confinement and shifts the subbands, which can be viewed as a Stark effect. If $q_\parallel \neq 0$, the shift of the subband depends on the in-plane coordinate and represents an additional effective potential felt by the electrons. If we assume the confinement of the electrons by an asymmetric triangular potential well formed by the constant force F on one side and a hard wall on the other, the confinement energy $\epsilon_z \sim (\hbar^2F^2/m)^{1/3}$, while $\partial\epsilon_z/\partial F \equiv a_z$ is of the order of the extent of the confined state in the z direction. Thus, we can estimate

$$\hat{V}^{\mathrm{Stark}}(\mathbf{q}) \sim q_z a_z \hat{V}^{\mathrm{piezo}}(\mathbf{q}). \tag{41}$$

This effective in-plane potential is also subject to screening, which brings an additional factor of $q_\parallel a_s$. As a result, we can generally write

$$V_{\mathbf{q}} \sim A\sqrt{\frac{q}{\mathcal{V}\rho_0 v_s}}, \tag{42}$$

with A given by

$$qa_s\Xi, \quad na^2\Xi, \quad \frac{4\pi e e^{\mathrm{em}} a_s}{\varepsilon}, \quad q_z a_z \frac{4\pi e e^{\mathrm{em}} a_s}{\varepsilon},$$

for the screened isotropic deformation potential, anisotropic deformation potential, screened in-plane piezoelectric field, and the perpendicular piezoelectric field, respectively.

Let us estimate the relative importance of these mechanisms, using the numbers for GaAs from Ref. 28. The bare deformation potential $\Xi\sim 10$ eV, the screening length $a_s \approx 50$ Å, and the lattice constant $a\approx 5$ Å. We will be interested in temperatures $T\sim 0.1-1$ K, so we indeed have $qa_s \ll 1$, and we are in the regime $qL \gg 1$ (for $L\sim 1$ μm). For $v_s q = 1$ K we have $q\approx 3\times 10^{-3}$ Å^{-1} ($v_s\approx 5\times 10^5$ cm/s), so $qa_s \approx 0.15$. For $n=10^{12}$ cm^{-2} $na^2\approx 2.5\times 10^{-3}$, so the screened isotropic part is more important than the anisotropic one. The only independent component of the electromechanical tensor in GaAs $e^{\mathrm{em}}_{14}\approx 1.4\times 10^7$ V/cm, the dielectric constant $\varepsilon\approx 13$, so $4\pi e e^{\mathrm{em}}_{14} a_s/\varepsilon \approx 7$ eV, which is of the order of the unscreened deformation potential. For the screened potential due to the perpendicular piezoelectric field, as typically $a_z\sim 100$ Å (Ref. 29), we have a smallness of $q_z a_z$. In conclusion, contrary to the estimates of Ref. 22, we obtain that the in-plane piezoelectric coupling is more important than the deformational one.

As a result, we arrive at the estimate

$$W_{\mathrm{out}}(T) \sim \frac{(4\pi e e^{\mathrm{em}} a_s/\varepsilon)^2}{\rho_0 v_s^5 E_{Th}\delta} T^6 \equiv \frac{T^6}{T_{\mathrm{ph}}^4}. \tag{43}$$

For GaAs $\rho_0 v_s^5 \approx (0.074\ \mathrm{eV})$,[4] (the density $\rho_0\approx 5.3$ g/cm^3), for a typical dot[3] $\delta\sim 1$ μeV, $E_{\mathrm{Th}}\sim 100$ μeV, so we obtain $T_{\mathrm{ph}}\sim 0.1$ meV ~ 1 K.

V. STATIONARY STATE

A. Ohmic absorption

First, consider the case of the simple Ohmic absorption with cooling only due to the contacts in the sequential tunneling regime with the rate given by Eq. (18). At small detunings ($U \ll T$) we have

$$W_{\mathrm{out}}(T) = \frac{\gamma}{\delta}\left[\frac{\pi^2T^2}{12} - \frac{U^2}{4} + O(U^4)\right], \tag{44}$$

so that the stationary temperature is given by

$$T(U) = \frac{2}{\pi}\sqrt{\frac{3W_0}{\gamma/\delta}}\left[1 + \frac{(\gamma/\delta)U^2}{8W_0} + O(U^4)\right]. \tag{45}$$

The temperature $T(U=0)$ determines the curvature of the Coulomb blockade peak at $U=0$: from Eq. (15) we have

$$\frac{G(U)}{G_0} = 1 - \frac{1}{4\ln 2}\frac{U^2}{T^2(U=0)}. \tag{46}$$

At large detunings $\gg T$ we can approximate the right-hand side of Eq. (18) by $|x|e^{-2|x|}$ and write

$$\frac{W_0}{(\gamma/\delta)T^2} \approx |x|e^{-2|x|}, \quad T \approx \frac{U}{\ln[(\gamma/\delta)U^2/(2W_0)]}, \qquad (47)$$

with the logarithmic precision. It is correct if the logarithm in the denominator is large, or $U \gg T(U=0)$. This result means that the tails of the Coulomb blockade peak have the form

$$\frac{G(U)}{G_0} \approx \frac{2W_0}{(\gamma/\delta)U^2}\ln\left[\frac{(\gamma/\delta)U^2}{2W_0}\right]. \qquad (48)$$

The weak power-law fall-off of the tails is drastically different from the exponential one occurring in equilibrium: $G(U)/G_0=(U/T)/\sinh(U/T)$ (Ref. 15). The reason for this difference is very simple: as the gate voltage is turned away from the degeneracy point, the exchange of electrons between the dot and the contacts becomes weaker, so the cooling rate decreases leading to an increase in the temperature and hence in the conductance.

At large enough detunings the cooling becomes dominated by the inelastic cotunneling, Eq. (25), rather then sequential tunneling, Eq. (18). In this regime the dot temperature and the conductance are given by

$$T=\left(\frac{15}{2\pi^3}\frac{W_0U^2}{(\gamma/\delta)^2}\right)^{1/4}, \quad \frac{G(U)}{G_0}=\sqrt{\frac{10}{3\pi^2}}\frac{\sqrt{W_0}}{U}. \qquad (49)$$

The switching to the inelastic cotunneling occurs at

$$U \sim \sqrt{W_0}\frac{\delta}{\gamma}\ln^2\frac{\delta}{\gamma}, \quad \frac{G(U)}{G_0} \sim \frac{\gamma/\delta}{\ln^2(\delta/\gamma)}. \qquad (50)$$

The logarithmic precision of these estimates, however, makes them applicable only for extremely small γ [such that $\ln(\delta/\gamma) \gg 1$]. In reality, if one takes directly the expressions (18) and (25) for the cooling rate, for $W_0=30\ \mu\text{eV}^2 \approx 46\ \mu\text{eV/s}$, $\gamma/\delta=0.2$ (see Ref. 30 for the numbers) the contribution of the inelastic cotunneling starts to affect the stationary electronic temperature noticeably [as compared to the precision of Eq. (47)] only as far as $V>1$ meV. At $V=1$ meV the conductance $G(1\ \text{meV})/G_0 \approx 0.01$, and about 18% of it is still due to the sequential tunneling.

If one takes now into account cooling by phonons with the rate (43), it sets the upper limit for the electronic temperature $T_{\max}=(W_0T_{\text{ph}}^4)^{1/6}$. If the pumping is strong enough (or the dot is closed enough), $T_{\max} \ll \sqrt{W_0\delta/\gamma}$, the phonon cooling mechanism dominates, the dot temperature is constant and equal to $T_{\max}$ for all U, so that the Coulomb blockade peak shape is given explicitly by Eq. (15), and its tails by Eq. (25). In the opposite limiting case, $T_{\max} \gg \sqrt{W_0\delta/\gamma}$ the electronic temperature in the peak region is determined by electron escape, and only in the peak tails, when the dot effectively becomes more and more closed, phonon emission starts to dominate. This will manifest itself as a crossover from the $1/U$ tail (49) to the $1/U^2$ one given by Eq. (25) at fixed $T=T_{\max}$. This crossover occurs at $U \sim (\gamma/\delta)T_{\max}^2/\sqrt{W_0}$, $G/G_0 \sim (\delta/\gamma)W_0/T_{\max}^2$ (Ref. 31). We plot the tails of the Coulomb blockade peak $G(U)/G_0$ in Fig. 1 for three cases when (a) only sequential tunneling is taken into account, (b) cotunneling is added, and (c) cooling by phonons is present as well.[30] In Fig. 2 we plot the electronic temperature $T(U)$ for the same three cases.

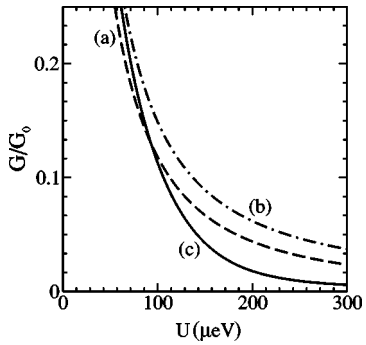

FIG. 1. Normalized conductance versus reduced gate voltage (Coulomb blockade tail): (a) only sequential tunneling is taken into account, (b) cotunneling is added, and (c) cooling by phonons is present as well (Ref. 30).

B. Dynamic localization

As we have discussed in Sec. II, in the strong dynamic localization regime the residual absorption is determined by dephasing. Using the results of the previous sections we can identify three sources of dephasing.

(i) *Escape to the contacts*. The quasiparticle relaxation rates for the sequential tunneling and inelastic cotunneling are given by Eqs. (19) and (25).

(ii) *Phonon emission*. According to the arguments given in the end of Sec. IV A, we can write

$$\gamma_{\text{ph}}(T) \sim \delta\left(\frac{T}{T_{\text{ph}}}\right)^4. \qquad (51)$$

(iii) *Electron-electron collisions*. The corresponding quasiparticle relaxation rate in a quantum dot was calculated by Sivan, Imry, and Aronov:[32]

$$\gamma_{e\text{-}e}(T) \sim \delta\left(\frac{T}{E_{\text{Th}}}\right)^2, \qquad (52)$$

where E_{Th} is the Thouless energy. The derivation of this expression implies the effective continuity of the many-particle

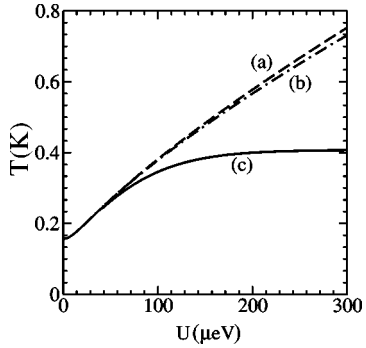

FIG. 2. Electronic temperature in K versus reduced gate voltage: (a) only sequential tunneling is taken into account, (b) cotunneling is added, and (c) cooling by phonons is present as well (Ref. 30).

spectrum, which imposes a condition $T_* \gg \sqrt{E_{\rm Th}\delta/\ln(E_{\rm Th}/\delta)}$ (Ref. 33). Obviously, for the dynamic localization to have any chance to develop, the condition $\gamma_\phi(T_*)t_* \ll 1$ should be satisfied.

Suppose for a moment that dephasing is dominated by electron-electron collisions, while cooling is dominated by the escape to the contacts (later we will analyze the conditions for this to be true). From Eqs. (15), (18), and (25) it is seen that for both sequential tunneling and cotunneling G/G_0 and $W_{\rm out}(U)/[(\gamma/\delta)T^2]$ are functions of $x \equiv U/(2T)$ only. This allows us to write a relation

$$W_{\rm out} = (\gamma/\delta)T^2\mathcal{W}(G/G_0), \tag{53}$$

where $\mathcal{W}(G/G_0)$ is a given function, whose overall behavior is quite similar to G/G_0 itself.[13] Equation (53) is actually analogous to the Wiedemann-Franz law (an extra power of T arises because here we consider hot electrons in the dot, while those in the contacts are at $T=0$, instead of a small temperature difference used in the definition of heat conductivity). The energy balance condition takes the form

$$W_{\rm in} \sim T_*^2\frac{T^2}{E_{\rm Th}^2} = W_{\rm out} = \frac{\gamma}{\delta}T^2\mathcal{W}(G/G_0) \tag{54}$$

or $(\gamma/\delta)\mathcal{W}(G/G_0) = (T_*/E_{\rm Th})^2$. Since U and T have dropped out, the solution of this equation for G is independent of U, leading to a flat plateau on the Coulomb blockade curve $G(U)$ (Ref. 13). If one approximates $\mathcal{W}(G/G_0) \sim G/G_0$, the level of the plateau is given by $G/G_0 \sim (\delta/\gamma)(T_*/E_{\rm Th})^2$.

Note that the largest possible value of $\mathcal{W}(G/G_0)$ is $\pi^2/12$ reached at $G/G_0 = 1$ (corresponding to $U=0$). Therefore, the solution exists only if

$$\frac{\gamma}{\delta} \gg \left(\frac{T_*}{E_{\rm Th}}\right)^2. \tag{55}$$

Physically, this means that the dot should be sufficiently open, so that the cooling is intense enough and the stationary temperature is not too high to destroy the localization. Note that for the observation of the plateau the condition $\gamma \ll 1/t_*$ *is not* necessary: even if at $U=0$ the dynamic localization is absent, as U is increased, the dot becomes effectively more closed, so the dephasing by escape becomes less efficient. Of course, for the Coulomb blockade itself to be present, the condition $\gamma/\delta \ll 1$ should be satisfied.[34]

In the very top of the peak ($U=0$) we can include the dephasing due to both escape and electron-electron collisions, and write the condition $W_{\rm in} = W_{\rm out}$ as

$$\frac{\gamma}{\delta}T_*^2 + \frac{T^2}{E_{\rm Th}^2}T_*^2 \sim \frac{\gamma}{\delta}T^2. \tag{56}$$

Here the first term on the left-hand side comes from the dephasing due to escape, and the second term represents the contribution from collisions. The condition (55) ensures the smallness of the second term as compared to the right-hand side, so Eq. (56) gives $T(U=0) \sim T_*$. Thus, for the dynamic localization to be possible the dephasing in the very peak of the Coulomb blockade *must* be dominated by escape.

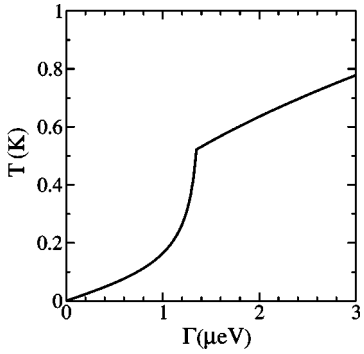

FIG. 3. Dependence of the electronic temperature in the peak $T(U=0)$ on the perturbation intensity Γ for $\gamma/\delta = 0.02$, $\delta = 0.3$ μeV, $\omega = 3$ μeV, $E_{\rm Th} = 100$ μeV; in reality the sharp angle is replaced by a smooth crossover.

Equation (46) remains valid in the dynamic localization regime as well, as it does not depend on the details of heating and cooling mechanisms. Thus, one can extract the temperature of the stationary state at $U=0$ measuring the curvature of the peak, and study its dependence on control parameters: intensity Γ and coupling to the contacts γ. From Eq. (56) it is seen that this dependence is the strongest when γ/δ is close to $T_*^2/E_{\rm Th}^2$ (up to a numerical coefficient), i.e., when the dynamic localization in the peak is about to be destroyed. If we plot $T(U=0)$ versus Γ (Fig. 3), we see that destruction of the dynamic localization manifests itself as a crossover from the linear dependence $T \propto \Gamma$ deep in the DL regime (small Γ) to $T \propto \sqrt{\Gamma}$ in the Ohmic regime. According to the abovesaid, this crossover can be quite pronounced (as is shown in the figure) when

$$\gamma \sim \delta\frac{T_*^2}{E_{\rm Th}^2} \ll \frac{1}{t_*}. \tag{57}$$

As U is detuned away from the peak, the dot becomes effectively more closed, and simultaneously the electronic temperature grows and electron-electron collisions become more frequent. Thus, the crossover from the peak to the plateau occurs where the two mechanisms equally contribute to the dephasing. With the logarithmic precision this happens at

$$T \sim T_*, \quad U \sim U_{\rm min} \sim T_* \max\left\{1, \frac{\gamma}{\delta}\frac{E_{\rm Th}}{T_*}\right\}, \tag{58}$$

depending on whether the plateau is in the region of sequential tunneling, $\gamma/\delta \ll T_*/E_{\rm Th}$, or of inelastic cotunneling, $\gamma/\delta \gg T_*/E_{\rm Th}$. The plateau ends when the temperature of the dot becomes so large that the dynamic localization is destroyed by dephasing. Obviously, this happens when the horizontal line $G/G_0 = (\delta/\gamma)(T_*/E_{\rm Th})^2$ hits the curve (48) or (49), which happens at

$$U \sim U_{\rm max} \sim E_{\rm Th}\sqrt{\frac{\delta}{\Gamma}}\max\left\{1, \frac{\gamma}{\delta}\frac{E_{\rm Th}}{T_*}\right\}, \tag{59}$$

again, with logarithmic precision. The two boundaries (58) and (59) give a nonzero range of U (i.e., $U_{\rm min} < U_{\rm max}$), if $T_* \ll E_{\rm Th}\sqrt{\delta/\Gamma}$, which can be equivalently rewritten as

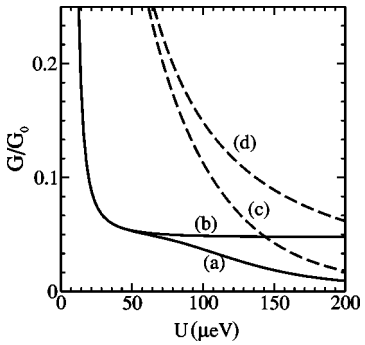

FIG. 4. Normalized conductance versus reduced gate voltage (Coulomb blockade tail): dynamic localization regime with [curve (a)] and without [curve (b)] phonon cooling taken into account, and the same for the purely Ohmic absorption [curves (c) and (d)] (Ref. 30).

$\gamma_{e\text{-}e}(T_*) \ll 1/t_*$, i.e., a necessary condition for the dynamic localization itself.

So far, when analyzing the dynamic localization, we did not take phonons into account. Now consider another extreme case: both cooling and dephasing are entirely due to phonons. Then the energy balance condition in the localization regime reads

$$W_{\rm in} \sim T_*^2 \frac{T^4}{T_{\rm ph}^4} = \frac{T^6}{T_{\rm ph}^4} = W_{\rm out}, \tag{60}$$

giving $T \sim T_*$. Note that this conclusion is independent of the power of temperature in the phonon cooling rate [or of α appearing in Eq. (36)]. Obviously, phonons will dominate if $T_{\rm ph} \ll T_*(\delta/\gamma)^{1/4}$, $T_{\rm ph} \ll \sqrt{T_* E_{\rm Th}}$. In this case the shape of the peak is given explicitly by Eq. (15), its tails by Eq. (25), and the width corresponds to the electronic temperature of the dot. The signature of the dynamic localization effect would be the linear dependence of the temperature on the microwave power, in contrast to the $1/6$ power for the Ohmic absorption case (see the previous subsection). The localization regime exists as long as $\gamma_{\rm ph}(T_*) \ll 1/t_*$, or $T_*^5 \ll \omega T_{\rm ph}^4$. The solution for the Ohmic regime is $T_{\max} = (\omega T_* T_{\rm ph}^4)^{1/6}$, and it is stable as long as $\gamma_{\rm ph}(T_{\max}) \gg 1/t_*$, which gives $T_*^5 \gg \omega T_{\rm ph}^4$. Thus, at a certain intensity such that $T_* \sim (\omega T_{\rm ph}^4)^{1/5}$ there is a crossover between the localization and Ohmic regimes.

Including all mechanisms, we can note that if the electron-phonon interaction is weak enough, $T_{\rm ph}^2 \gg E_{\rm Th}^3 \omega / T_*^2$, the phonon cooling plays any role only in the Ohmic part of the Coulomb blockade tail. Otherwise, phonons start to "eat up" the plateau from the large U side.[35] The plateau will disappear at $T_{\rm ph} \sim \sqrt{T_* E_{\rm Th}}$. As an illustration, for the intermediate case, we plot the Coulomb blockade tail in Fig. 4 in the dynamic localization regime with and without phonon cooling (lower and upper solid curves, respectively) together with the corresponding Ohmic curves shown by dashed lines.

VI. INTENSITY DEPENDENCE OF THE ABSORPTION

In this section we wish to remark that conductance measurements are not necessarily the only possible way to detect the dynamic localization. An isolated mesoscopic sample can be put into a microwave cavity, and the energy absorption rate $W_{\rm in}$ can be measured as it affects the Q factor of the cavity.[36] In this case the only cooling mechanism is phonon emission, while the dephasing can be due to electron-electron interactions as well. Equating $W_{\rm in} = W_{\rm out}$ and finding the stationary temperature, as before, we find that the dynamic localization regime the absorption rate depends nonlinearly on the ac field intensity

$$W_{\rm in} \sim \left(\frac{\Gamma\omega}{E_{\rm Th}\delta}\right)^3 T_{\rm ph}^2 \propto \Gamma^3, \tag{61}$$

if the dephasing is dominated by electron-electron collisions ($T_{\rm ph} \gg \sqrt{E_{\rm Th}\Gamma\omega/\delta}$), or

$$W_{\rm in} \sim \left(\frac{\Gamma\omega}{\delta}\right)^6 \frac{1}{T_{\rm ph}^4} \propto \Gamma^6, \tag{62}$$

if the dephasing is dominated by electron-phonon interaction ($T_{\rm ph} \ll \sqrt{E_{\rm Th}\Gamma\omega/\delta}$).

This nonlinear dependence represents a crossover between two qualitatively different linear regimes with $W_{\rm in} \propto \Gamma$. The high-intensity Ohmic regime with $W_{\rm in} = W_0$ can be viewed as "classical" linear response regime, as W_0 is independent of the inelastic scattering rate. At lower intensites one has the nonlinear regime described above, while at very low intesities $\Gamma \ll \delta$ one has to recover the linear response regime (which can be called "quantum") with the absorption entirely due to inelastic processes which allow the electron relaxation between essentially discrete energy levels (somewhat similarly to Ref. 37). This regime will take place when the intensity-dependent electronic temperature $T \ll \omega$, and the inelastic rate (with the corresponding value of the transferred energy $\sim \omega \gg T$) $\gamma_\phi(\omega) \ll \delta$. Then the absorption rate $W_{\rm in} \sim W_0 \gamma_\phi / \delta$. The condition of existence of such two distinct linear response regimes is the large difference of the corresponding proportionality coefficients, i.e., smallness of the level broadening due to inelastic processes compared to mean level spacing: $\gamma_\phi(\omega) \ll \delta$. It is precisely this inequality that opens the intensity range for the dynamic localization $\gamma_\phi(\omega) \ll \gamma_\phi(T_*) \ll 1/t_* \sim \delta^2/\Gamma \ll \delta$, assumed throughout the present paper.

VII. CONCLUSIONS

We have studied electronic conduction through a quantum dot in the Coulomb blockade regime under an external periodic perturbation. In contrast to the well-studied equilibrium case, the electronic temperature of the dot under pumping is different from that of the contacts and the substrate. It is determined by the balance between heating by the perturbation and cooling due to electron exchange with contacts and phonon emission. When the cooling is dominated by the former mechanism, its rate depends on the gate voltage, and so does the dot temperature. As the gate voltage is detuned away from the peak, the cooling rate decreases, and the temperature increases. As a result, the tails of the Coulomb blockade peak fall off less rapidly than in the equilibrium case: instead of the usual exponential fall-off for the sequen-

tial tunneling, under pumping one has a power-law dependence (48), while for the inelastic cotunneling the equilibrium power law is replaced by a weaker one, Eq. (49). At sufficiently high temperatures cooling by phonons becomes important, which sets an upper limit for the dot temperature (depending on the pumping intensity), which, however, can be significantly higher than the cryostat temperature.

In the strong dynamic localization regime the heating rate is determined by dephasing, as the usual linear absorption is blocked by quantum interference. The dephasing can be due to electron-electron collisions, electron escape to the contacts, as well as phonon emission. The most peculiar situation is realized when the cooling is due to the contacts, while the dephasing is due to electron-electron collisions: in this case the Coulomb blockade peak has a flat shoulder, where the conductance does not depend on the gate voltage. Such a shape could be an experimental signature of the dynamic localization effect.

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[30] The parameters used for the plots in Figs. 1, 2, and 4, are the following: $\delta = 0.3\ \mu$eV, $\Gamma = 1\ \mu$eV, $\omega = 3\ \mu$eV ($\omega/2\pi \approx 0.7$ GHz), $E_{\mathrm{Th}} = 100\ \mu$eV, $T_{\mathrm{ph}} = 1\ \mathrm{K} \approx 86\ \mu$eV.

[31] It is worth noting that due to the condition $T_{\max} \gg \sqrt{W_0 \delta/\gamma}$ the crossover to phonon cooling occurs after the crossover from sequential tunneling to cotunneling.

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[34] If one assumes that the dot is open, $\gamma/\delta \gg 1$, the condition for the dynamic localization to be present would be $\gamma \ll 1/t_*$, equivalent to $\gamma/\delta \ll \delta/\Gamma$. Since the theory of Refs. 8, 9, and 14 is applicable only at $\Gamma/\delta \gg 1$, the dot *must* be in the Coulomb blockade regime.

[35] Note that both for electron escape and phonon emission the ratio of the cooling rate to the dephasing rate is $\sim T^2/\delta$, i.e., of the order of the total electronic energy of the dot. This is due to the fact that each act of escape or phonon emission leads to a loss of energy of the order of T. As a result, the relative importance of these mechanisms is the same in cooling and dephasing.

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