Microwave surface resistance of superconductors with grain boundaries

Yasunori Mawatari
Energy Technology Research Institute, National Institute of Advanced Industrial Science and Technology, Tsukuba, Ibaraki 305-8568, Japan

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Microwave-field distribution, dissipation, and surface impedance are theoretically investigated for superconductors with laminar grain boundaries (GBs). In the present theory we adopt the two-fluid model for intragrain transport current in the grains, and the Josephson-junction model for intergrain tunneling current across GBs. Results show that the surface resistance R_s nonmonotonically depends on the critical current density J_{cj} at GB junctions, and R_s for superconductors with GBs can be smaller than the surface resistance R_{s0} for ideal homogeneous superconductors without GBs.

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I. INTRODUCTION

High-temperature superconductors contain many grain boundaries (GBs), where the order parameter is locally suppressed due to the short coherence length.[1] GBs have attracted much interest for their basic physics as well as for their applications in superconductors,[2–4] and play a crucial role in microwave response and surface resistance R_s of high-temperature superconducting films.[5–13]

Electrodynamics of GB junctions can be described using the Josephson-junction model, and one of the most important parameters that characterize GB junctions is the critical current density J_{cj} for Josephson tunneling current across GBs.[14–16] The J_{cj} strongly depends on the misorientation angle of GBs.[17,18] In $YBa_2Cu_3O_{7-\delta}$ films, J_{cj} can be enhanced[19] and R_s reduced[13] by Ca doping. The investigation of the relationship between R_s and J_{cj} is needed to understand the behavior of R_s and J_{cj} in Ca doped $YBa_2Cu_3O_{7-\delta}$ films. The J_{cj} dependence of R_s, however, has not yet been clarified, and it is not trivial whether GBs enhance the microwave dissipation that is proportional to R_s.

In this paper, we present theoretical investigation on the microwave field and dissipation in superconductors with laminar GBs. Theoretical expressions of the surface impedance $Z_s=R_s-iX_s$ of superconductors with GBs are derived as functions of J_{cj} at GB junctions.

II. BASIC EQUATIONS

A. Superconductors with grain boundaries

We consider penetration of a microwave field (i.e., magnetic induction $\boldsymbol{B}=\mu_0\boldsymbol{H}$, electric field $\boldsymbol{E}$, and current density $\boldsymbol{J}$) into superconductors that occupy a semi-infinite area of $x>0$. We investigate linear response for small microwave power limit, such that the time dependence of the microwave field is expressed by the harmonic factor $e^{-i\omega t}$, where $\omega/2\pi$ is the microwave frequency that is much smaller than the energy-gap frequency of the superconductors. Magnetic induction $\boldsymbol{B}$ is assumed to be less than the lower critical field, such that no vortices are present in the superconductors. (See Ref. 20 for microwave response of vortices.)

The GBs are modeled to have laminar structures as in Ref. 21; the laminar GBs that are parallel to the xz plane are situated at $y=ma$, where a is the spacing between grains (i.e., effective grain size) and $m=0,\pm 1,\pm 2,\dots,\pm\infty$. The thickness of the barrier of GB junctions d_j is much smaller than both a and the London penetration depth λ and, therefore, we investigate the thin-barrier limit of $d_j\to 0$, namely, GB barriers situated at $ma-0<y<ma+0$.

B. Two-fluid model for intragrain current

We adopt the standard two-fluid model[15,16] for current transport in the grain at $ma+0<y<(m+1)a-0$. The intragrain current $\boldsymbol{J}=\boldsymbol{J}_s+\boldsymbol{J}_n$ is given by the sum of the supercurrent $\boldsymbol{J}_s=i\sigma_s\boldsymbol{E}$ and the normal current $\boldsymbol{J}_n=\sigma_n\boldsymbol{E}$, where $\sigma_s=1/\omega\mu_0\lambda^2$ and σ_n is the normal-fluid conductivity in the grains. The displacement current $\boldsymbol{J}_d=-i\omega\epsilon\boldsymbol{E}$ with the dielectric constant ϵ can be neglected for a microwave range of $\omega/2\pi\sim$GHz. Ampère's law $\mu_0^{-1}\nabla\times\boldsymbol{B}=(\sigma_n+i\sigma_s)\boldsymbol{E}$ is thus reduced to

$$\boldsymbol{E}=-i\omega\Lambda_g^2\nabla\times\boldsymbol{B}, \tag{1}$$

where Λ_g is the intragrain ac field penetration depth defined by

$$\Lambda_g^{-2}=\omega\mu_0(\sigma_s-i\sigma_n)=\lambda^{-2}-i\omega\mu_0\sigma_n. \tag{2}$$

Combining Eq. (1) with Faraday's law, $\nabla\times\boldsymbol{E}=i\omega\boldsymbol{B}$, we obtain the London equation for magnetic induction $\boldsymbol{B}=B_z(x,y)\hat{\boldsymbol{z}}$ for $y\neq ma$ as

$$B_z-\Lambda_g^2\nabla^2 B_z=0. \tag{3}$$

For ideal homogeneous superconductors without GBs, Eq. (3) is valid for $-\infty<y<+\infty$ and the solution is simply given by $B_z(x)=\mu_0H_0e^{-x/\Lambda_g}$, and the electric field is obtained from Eq. (1) as $E_y(x)=-i\omega\mu_0\Lambda_gH_0e^{-x/\Lambda_g}$. The surface impedance $Z_{s0}=R_{s0}-iX_{s0}$ for homogeneous superconductors is given by $Z_{s0}=E_y(x=0)/H_0=-i\omega\mu_0\Lambda_g$. The surface resistance $R_{s0}=\mathrm{Re}(Z_{s0})$ and reactance $X_{s0}=-\mathrm{Im}(Z_{s0})$ of ideal homogeneous superconductors without GBs are given by[16]

$$R_{s0}=\mu_0^2\omega^2\lambda^3\sigma_n/2, \tag{4}$$

$$X_{s0} = \mu_0\omega\lambda \tag{5}$$

for $\sigma_n/\sigma_s \ll 1$ well below the superconducting transition temperature T_c.

C. Josephson-junction model for intergrain current

We adopt the Josephson-junction model[14–16] for tunneling current across GBs at $y=ma$. Behavior of the GB junctions is determined by the gauge-invariant phase difference across GBs $\varphi_j(x)$ and the voltage induced across GB $V_j(x)$ is given by the Josephson's relation

$$\int_{ma-0}^{ma+0} E_y dy = V_j = \frac{\phi_0}{2\pi}(-i\omega\varphi_j), \tag{6}$$

where ϕ_0 is the flux quantum. The tunneling current parallel to the y axis is given by the sum of the superconducting tunneling current (i.e., Josephson current) $J_{sj}=J_{cj}\sin\varphi_j$ and the normal tunneling current (i.e., quasiparticle tunneling current) $J_{nj}=\gamma_{nj}V_j$. The critical current density J_{cj} at GB junctions is one of the most important parameters in the present paper, and the resistance-area product of GB junctions corresponds to $1/\gamma_{nj}$. We neglect the displacement current across GBs, $J_{dj}=-i\omega C_j V_j$ where C_j is the capacitance of the GB junctions.

Here we define the Josephson length λ_J and the characteristic current density J_0 as

$$\lambda_J = (\phi_0/4\pi\mu_0 J_{cj}\lambda)^{1/2}, \tag{7}$$

$$J_0 = \phi_0/4\pi\mu_0\lambda^3. \tag{8}$$

The ratio $J_{cj}/J_0=(\lambda/\lambda_J)^2$ characterizes the coupling strength of GB junctions.[22] For weakly coupled GBs, namely, $J_{cj}/J_0=(\lambda/\lambda_J)^2\ll 1$ (e.g., high-angle GBs), electrodynamics of the GB junctions can be well described by the weak-link model.[14–16] For strongly coupled GBs, namely, $J_{cj}/J_0=(\lambda/\lambda_J)^2\gtrsim 1$ (e.g., low-angle GBs), the Josephson-junction model is still valid but requires appropriate boundary condition at GBs, as given in Eq. (4) in Ref. 22, as pointed out by Gurevich; see also Refs. 21 and 23.

In the small-microwave-power limit such that $\sin\varphi_j\simeq\varphi_j=2\pi V_j/(-i\omega\phi_0)$ for $|\varphi_j|\ll 1$, the J_{cj} is reduced to

$$J_{sj} \simeq J_{cj}\varphi_j = i\gamma_{sj}V_j, \tag{9}$$

where $\gamma_{sj}=2\pi J_{cj}/\omega\phi_0=1/2\omega\mu_0\lambda\lambda_J^2$. The total tunneling current across GB is thus given by

$$-\frac{1}{\mu_0}\frac{\partial B_z}{\partial x}\bigg|_{y=ma} = J_{sj}+J_{nj} = (i\gamma_{sj}+\gamma_{nj})V_j. \tag{10}$$

Integration of Faraday's law $\partial E_y/\partial x-\partial E_x/\partial y=i\omega B_z$ yields

$$\begin{aligned} &E_x(x,y=ma+0)-E_x(x,y=ma-0)\\ &\quad=\int_{ma-0}^{ma+0} dy\left[\frac{\partial E_y(x,y)}{\partial x}-i\omega B_z(x,y)\right]=\frac{\partial V_j(x)}{\partial x}, \end{aligned} \tag{11}$$

where we used Eq. (6). The static version (i.e., $\omega\to 0$) of Eq. (11) corresponds to Eq. (4) in Ref. 22. Substitution of Eqs. (1) and (10) into Eq. (11) yields the boundary condition for B_z at $y=ma$

$$-\frac{\partial B_z}{\partial y}\bigg|_{y=ma+0} + \frac{\partial B_z}{\partial y}\bigg|_{y=ma-0} = \frac{a\Lambda_j^2}{\Lambda_g^2}\frac{\partial^2 B_z}{\partial x^2}\bigg|_{y=ma}, \tag{12}$$

where Λ_j is the characteristic length for ac field penetration into GBs defined by

$$\Lambda_j^{-2} = \omega\mu_0 a(\gamma_{sj}-i\gamma_{nj}) = \mu_0 a(2\pi J_{cj}/\phi_0 - i\omega\gamma_{nj}). \tag{13}$$

III. SURFACE IMPEDANCE

A. Microwave field and surface impedance

Equations (3) and (12) are combined into a single equation for $x>0$ and $-\infty<y<+\infty$ as

$$B_z-\Lambda_g^2\nabla^2 B_z = a\Lambda_j^2\sum_{m=-\infty}^{+\infty}\frac{\partial^2 B_z}{\partial x^2}\delta(y-ma), \tag{14}$$

whose solution is calculated as

$$\begin{aligned}\frac{B_z(x,y)}{\mu_0 H_0} &= e^{-x/\Lambda_g}+\frac{2}{\pi}\int_0^\infty dk\frac{\cosh[K(y-a/2)]}{\Lambda_g^2K^2\sinh(Ka/2)}\\ &\quad\times\frac{k\sin kx}{(2K\Lambda_g^2/a\Lambda_j^2)+k^2\coth(Ka/2)}\end{aligned} \tag{15}$$

for $0<y<a$, where $K=(k^2+\Lambda_g^{-2})^{1/2}$. The right-hand side of Eq. (14) and the second term of the right-hand side of Eq. (15) reflect the GB effects. See the Appendix for the derivation of Eq. (15) from Eq. (14).

Electric field in the grains is obtained from Eq. (1) as $E_y=i\omega\Lambda_g^2\partial B_z/\partial x$, and voltage induced across GB is obtained from Eq. (10) as $V_j=i\omega a\Lambda_j^2\partial B_z/\partial x|_{y=0}$. The mean electric field $\overline{E_s}$ at the surface of the superconductor is thus calculated as

$$\begin{aligned}\overline{E_s} &\equiv \frac{1}{a}\int_{-0}^{a-0} dy E_y(x=0,y)\\ &=\frac{1}{a}\left[V_j(x=0)+\int_{+0}^{a-0}dyE_y(x=0,y)\right]\\ &= i\omega\left[\Lambda_j^2\frac{\partial B_z}{\partial x}\bigg|_{x=y=0}+\frac{\Lambda_g^2}{a}\int_{+0}^{a-0}dy\frac{\partial B_z}{\partial x}\bigg|_{x=0}\right].\end{aligned} \tag{16}$$

Substitution of Eq. (15) into Eq. (16) yields the surface impedance $Z_s=R_s-iX_s\equiv\overline{E_s}/H_0$ as

$$\begin{aligned}\frac{Z_s}{-i\omega\mu_0\Lambda_g} &= 1+\frac{2}{\pi}\int_0^\infty dk\frac{1}{\Lambda_g^3K^3}\\ &\quad\times\frac{1}{(K\Lambda_g^2/\Lambda_j^2)+(k^2a/2)\coth(Ka/2)}.\end{aligned} \tag{17}$$

The surface resistance and reactance are given by $R_s=\mathrm{Re}(Z_s)$ and $X_s=-\mathrm{Im}(Z_s)$, respectively.

B. Microwave dissipation and surface resistance

The time-averaged electromagnetic energy passing through the surface of a superconductor at $x=0$ and $-0<y<a-0$ is given by the real part of

$$\mathcal{E}=\frac{1}{2\mu_0}\int_{-0}^{a-0}dy(E_yB_z^*)_{x=0}=\frac{a}{2}\overline{E_s}H_0^*, \tag{18}$$

where $\overline{E_s}=Z_sH_0$ is defined by Eq. (16), and $(B_z)_{x=0}=\mu_0H_0$. Poynting's theorem[24] states that $\mathcal{E}$ is identical to the energy stored and dissipated in the superconductor

$$\mathcal{E}=\frac{1}{2}\int_0^\infty dx\left[\int_{+0}^{a-0}dy(\sigma_n-i\sigma_s)|\boldsymbol{E}|^2+(\gamma_{nj}-i\gamma_{sj})|V_j|^2-\int_{-0}^{a-0}dy\frac{i\omega}{\mu_0}|B_z|^2\right]. \tag{19}$$

The real parts of Eqs. (18) and (19) show that the surface resistance $R_s=\mathrm{Re}(\overline{E_s}/H_0)=\mathrm{Re}(Z_s)$ is composed of two terms:

$$R_s=R_{sg}+R_{sj}. \tag{20}$$

The intragrain contribution R_{sg} is from the energy dissipation in the grains, and the intergrain contribution R_{sj} is from the dissipation at GBs:

$$R_{sg}=\frac{1}{a|H_0|^2}\int_0^\infty dx\int_{+0}^{a-0}dy\sigma_n|\boldsymbol{E}|^2, \tag{21}$$

$$R_{sj}=\frac{1}{a|H_0|^2}\int_0^\infty dx\gamma_{nj}|V_j|^2. \tag{22}$$

Both the intragrain current $|\boldsymbol{J}_g|$ around GBs and the intergrain tunneling current $|J_j|$ across GBs are suppressed by the GBs, and are increasing functions of J_{cj}. With increasing J_{cj}, the intragrain electric field $|\boldsymbol{E}|=|\boldsymbol{J}_g/(\sigma_n+i\sigma_s)|$ also increases, whereas the intergrain voltage $|V_j|=|J_j/(\gamma_{nj}+i\gamma_{sj})|$ decreases because $\gamma_{sj}\propto J_{cj}$. The dissipation in the grains $\sigma_n|\boldsymbol{E}|^2/2$ and the intragrain contribution to the surface resistance R_{sg}, therefore, tend to *increase* with increasing J_{cj}. The dissipation at GBs $\gamma_{nj}|V_j|^2/2$ and the intergrain contribution to the surface resistance R_{sj}, on the other hand, *decrease* with increasing J_{cj}.

The surface reactance $X_s=-\mathrm{Im}(Z_s)$ is also divided into two contributions

$$X_s=X_{sg}+X_{sj}, \tag{23}$$

where the intragrain contribution X_{sg} and the intergrain contribution X_{sj} are given by

$$X_{sg}=\frac{1}{a|H_0|^2}\int_0^\infty dx\int_{+0}^{a-0}dy\left(\sigma_s|\boldsymbol{E}|^2+\frac{\omega}{\mu_0}|B_z|^2\right), \tag{24}$$

$$X_{sj}=\frac{1}{a|H_0|^2}\int_0^\infty dx\gamma_{sj}|V_j|^2. \tag{25}$$

Both X_{sg} and X_{sj} decrease with increasing J_{cj}.

C. Simplified expressions for surface impedance

The following Eqs. (26)–(34) show simplified expressions of the surface impedance Z_s, the surface resistance $R_s=\mathrm{Re}(Z_s)$, and the surface reactance $X_s=-\mathrm{Im}(Z_s)$ for certain restricted cases, assuming $\sigma_n/\sigma_s\ll 1$ and $\gamma_{nj}/\gamma_{sj}\ll 1$ well below the transition temperature.

For small grains of $a\ll\lambda$ such that $\coth(Ka/2)\simeq 2/Ka$, Eq. (17) is reduced to

$$Z_s\simeq -i\omega\mu_0(\Lambda_g^2+\Lambda_j^2)^{1/2}. \tag{26}$$

The right-hand side of Eq. (14) is reduced to $\Lambda_j^2\partial^2B_z/\partial x^2$ for $a\ll\lambda$, and the effective ac penetration depth is given by $\Lambda_{\mathrm{eff}}=(\Lambda_g^2+\Lambda_j^2)^{1/2}$ as in Ref. 21, resulting in the surface impedance given by Eq. (26). The R_s and X_s for small grains is obtained as

$$\frac{R_s}{R_{s0}}\simeq\left(1+\frac{2\lambda}{a}\frac{J_0}{J_{cj}}\right)^{-1/2}\left[1+\frac{4\lambda^2\gamma_{nj}}{a\sigma_n}\left(\frac{J_0}{J_{cj}}\right)^2\right], \tag{27}$$

$$\frac{X_s}{X_{s0}}\simeq\left(1+\frac{2\lambda}{a}\frac{J_0}{J_{cj}}\right)^{+1/2}, \tag{28}$$

where R_{s0}, X_{s0}, and J_0 are defined by Eqs. (4), (5), and (8), respectively. Equation (27) is decomposed into the intragrain R_{sg} and intergrain R_{sj} contributions, as $R_{sg}/R_{s0}\simeq(1+2\lambda J_0/aJ_{cj})^{-1/2}$ and $R_{sj}/R_{sg}\simeq(4\lambda^2\gamma_{nj}/a\sigma_n)(J_0/J_{cj})^2$, respectively.

Equation (26) is further simplified when $a\ll 2\lambda_J^2/\lambda$ for small grain and weakly coupled GBs as

$$Z_s\simeq -i\omega\mu_0\Lambda_j \tag{29}$$

and we have

$$\frac{R_s}{R_{s0}}\simeq\frac{2\gamma_{nj}\lambda}{\sigma_n}\left(\frac{2\lambda}{a}\right)^{1/2}\left(\frac{J_0}{J_{cj}}\right)^{3/2}, \tag{30}$$

$$\frac{X_s}{X_{s0}}\simeq\left(\frac{2\lambda}{a}\right)^{1/2}\left(\frac{J_0}{J_{cj}}\right)^{1/2}. \tag{31}$$

Thus, we obtain the dependence of R_s and X_s on the material parameters as $R_s\propto\gamma_{nj}a^{-1/2}J_{cj}^{-3/2}$ and $X_s\propto a^{-1/2}J_{cj}^{-1/2}$, which are independent of λ. The R_s given by Eq. (30) for the small grain and weakly coupled GBs is mostly caused by intergrain dissipation $R_s\simeq R_{sj}\gg R_{sg}$. For X_s given by Eq. (31), on the other hand, both intragrain X_{sg} and intergrain X_{sj} contribute to the total $X_s=X_{sg}+X_{sj}$.

For large J_{cj} (i.e., strong-coupling limit) such that $K\Lambda_g^2/\Lambda_j^2\gg(k^2a/2)\coth(Ka/2)$, Eq. (17) for the surface impedance Z_s is simplified as

$$Z_s\simeq -i\omega\mu_0(\Lambda_g+\Lambda_j^2/2\Lambda_g), \tag{32}$$

and we have

$$\frac{R_s}{R_{s0}}\simeq 1-\frac{\lambda}{a}\frac{J_0}{J_{cj}}+\frac{4\lambda^2\gamma_{nj}}{a\sigma_n}\left(\frac{J_0}{J_{cj}}\right)^2, \tag{33}$$

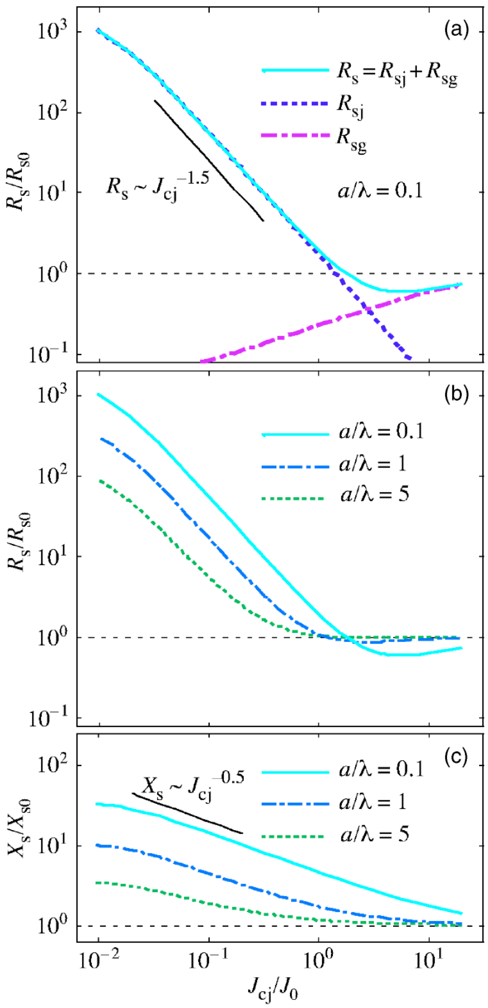

FIG. 1. (Color online) Dependence of surface resistance $R_s = \mathrm{Re}(Z_s)$ and surface reactance $X_s = -\mathrm{Im}(Z_s)$ [i.e., Eq. (17) with Eqs. (2) and (13)] on critical current density J_{cj} at GB junctions. R_s is normalized to the surface resistance without GB, i.e., R_{s0} given by Eq. (4), X_s is normalized to X_{s0} given by Eq. (5), and J_{cj} is normalized to J_0 defined as Eq. (8). Parameters are $\omega/2\pi = 10$ GHz, $\lambda = 0.2\ \mu$m, $\sigma_n = 10^7\ \Omega^{-1}\,\mathrm{m}^{-1}$, and $\gamma_{nj} = 10^{13}\ \Omega^{-1}\,\mathrm{m}^{-2}$, which yield $R_{s0} = 0.25$ mΩ, $X_{s0} = 16$ mΩ, and $J_0 = 1.6 \times 10^{10}$ A/m^2. (a) Total surface resistance $R_s = R_{sj} + R_{sg}$, intergrain contribution R_{sj} given by Eq. (22), and intragrain contribution R_{sg} given by Eq. (21) for $a/\lambda = 0.1$. (b) R_s and (c) X_s for $a/\lambda = 0.1$, 1, and 5.

$$\frac{X_s}{X_{s0}} \simeq 1 + \frac{\lambda}{a}\frac{J_0}{J_{cj}}. \tag{34}$$

The first and second terms of the right-hand side of Eq. (33) correspond to the intragrain contribution R_{sg}, whereas the third term corresponds to the intergrain contribution R_{sj}.

IV. DISCUSSION

Figures 1(a) and 1(b) show J_{cj} dependence of R_s. As shown in Fig. 1(a), the intergrain contribution R_{sj} is dominant for weakly coupled GBs (i.e., small J_{cj}/J_0 regime), whereas the intragrain contribution R_{sg} is dominant for strongly coupled GBs (i.e., large J_{cj}/J_0). The R_{sj} *decreases* with increasing J_{cj} as $R_{sj} \propto J_{cj}^{-1.5}$ [see Eq. (30)], whereas R_{sg} *increases* with J_{cj}. The resulting surface resistance $R_s = R_{sj} + R_{sg}$ *nonmonotonically* depends on J_{cj} and has a minimum, because R_s is determined by the competition between R_{sj} and R_{sg}. As shown in Fig. 1(c), on the other hand, X_s monotonically decreases with increasing J_{cj} [i.e., $X_s \propto J_{cj}^{-0.5}$ for weakly coupled GBs as in Eq. (31)].

The nonmonotonic dependence of R_s on the grain size a is also seen in Fig. 1(b). For small J_{cj}/J_0 the R_s decreases with increasing a as $R_s \propto a^{-0.5}$ [see Eq. (30)], whereas R_s increases with a for large J_{cj}/J_0.

The R_s for strongly coupled GBs can be *smaller* than R_{s0} for ideal homogeneous superconductors without GBs, namely, $R_s/R_{s0} < 1$ for $J_{cj}/J_0 \gtrsim 1$. The minimum surface resistance for $\lambda\gamma_{nj}/\sigma_n = 0.2$ is $R_s/R_{s0} \approx 0.97$ for $a/\lambda = 5$, $R_s/R_{s0} \approx 0.86$ for $a/\lambda = 1$, and $R_s/R_{s0} \approx 0.59$ for $a/\lambda = 0.1$. The minimum R_s/R_{s0} is further reduced when $\lambda\gamma_{nj}/\sigma_n$ is further reduced.

Theoretical results shown above may possibly be observed by measuring R_s, X_s, and J_{cj} in Ca doped $YBa_2Cu_3O_{7-\delta}$ films. The enhancement of J_{cj} (Ref. 19) and reduction of R_s (Ref. 13) by Ca doping are individually observed in $YBa_2Cu_3O_{7-\delta}$, but simultaneous measurements of J_{cj} and R_s are needed to investigate the relationship between R_s and J_{cj}. The nonmonotonic J_{cj} dependence of R_s for strongly coupled GBs may be observed in high quality films with small grains $a < \lambda$ and with large J_{cj} on the order of $J_0 \sim 10^{10}$ A/m^2 at low temperatures.

V. CONCLUSION

We have theoretically investigated the microwave-field distribution in superconductors with laminar GBs. The field calculation is based on the two-fluid model for current transport in the grains and on the Josephson-junction model for tunneling current across GBs. Results show that the microwave dissipation at GBs is dominant for weakly coupled GBs of $J_{cj} \ll J_0$, whereas dissipation in the grains is dominant for strongly coupled GBs of $J_{cj} \gg J_0$. The surface resistance R_s nonmonotonically depends on J_{cj}; the R_s decreases with increasing J_{cj} as $R_s \propto J_{cj}^{-1.5}$ for $J_{cj} \ll J_0$, whereas R_s increases with J_{cj} for $J_{cj} \gg J_0$. The intragrain dissipation can be suppressed by GBs, and the surface resistance of superconductors with GBs can be smaller than that of ideal homogeneous superconductors without GBs.

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APPENDIX

Equation (15) is derived by solving Eq. (14) with the boundary condition of $B_z = \mu_0 H_0$ at $x = 0$, as follows. We introduce the Fourier transform of $B_z(x,y)$ and $B_z(x,ma) = B_z(x,0)$ as

$$\tilde{b}(k,q) = \int_0^\infty dx \int_{-\infty}^{+\infty} dy B_z(x,y) e^{-iqy} \sin kx, \qquad (A1)$$

$$\tilde{b}_0(k) = \int_0^\infty dx B_z(x,0) \sin kx = \int_{-\infty}^{+\infty} \frac{dq}{2\pi} \tilde{b}(k,q), \qquad (A2)$$

respectively. The Fourier transform of Eq. (14) leads to

$$\frac{\tilde{b}(k,q)}{\mu_0 H_0} = 2\pi\delta(q)\frac{k}{K^2} + \frac{\alpha k}{K^2+q^2}\sum_m e^{-imqa}\left[1 - \frac{k\tilde{b}_0(k)}{\mu_0 H_0}\right], \qquad (A3)$$

where $K=(k^2+\Lambda_g^{-2})^{1/2}$ and $\alpha = a\Lambda_j^2/\Lambda_g^2$. Substituting Eq. (A3) into Eq. (A2), we have

$$\frac{\tilde{b}_0(k)}{\mu_0 H_0} = \frac{k}{K^2} + \alpha k\left[1 - \frac{k\tilde{b}_0(k)}{\mu_0 H_0}\right]\sum_m \int_{-\infty}^{+\infty} \frac{dq}{2\pi}\frac{e^{-imqa}}{K^2+q^2}, \qquad (A4)$$

which is reduced to

$$\frac{\tilde{b}_0(k)}{\mu_0 H_0} = \frac{1}{k} - \frac{2}{kK\Lambda_g^2}\frac{1}{2K + \alpha k^2 \coth(Ka/2)}. \qquad (A5)$$

$B_z(x,y)$ is calculated from $\tilde{b}(k,q)$ given by Eq. (A3) as

$$\begin{aligned}\frac{B_z(x,y)}{\mu_0 H_0} &= \frac{2}{\pi}\int_0^\infty dk \int_{-\infty}^{+\infty} \frac{dq}{2\pi}\frac{\tilde{b}(k,q)}{\mu_0 H_0} e^{iqy} \sin kx \\ &= e^{-x/\Lambda_g} + \frac{2\alpha}{\pi}\int_0^\infty dk k \sin kx \left[1 - \frac{k\tilde{b}_0(k)}{\mu_0 H_0}\right] \\ &\quad \times \sum_m \int_{-\infty}^{+\infty} \frac{dq}{2\pi}\frac{e^{iq(y-ma)}}{K^2+q^2}. \end{aligned} \qquad (A6)$$

Substitution of Eq. (A5) into Eq. (A6) yields Eq. (15).

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