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Bubble domains in disc-shaped ferromagnetic particles

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We study the fundamental magnetic states of disc-shaped ferromagnetic particles with a uniaxial anisotropy along the symmetry axis. Besides the monodomain, a bidomain state is also identified and studied both numerically and theoretically. This bidomain state in the symmetry broken system is the analog of the vortex state, consists of two coaxial oppositely magnetized cylindrically symmetric domains, and is stable at zero-bias field, unlike magnetic bubbles in ferromagnetic films. For a given disc thickness we find the critical radius above which the magnetization configuration falls into the bidomain bubble state. The critical radius depends strongly on the film thickness, especially for ultrathin films. In an external field the bidomain state remains stable over a range of field strengths. The switching from the bidomain to the monodomain takes place through an abrupt domain annihilation mechanism.

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Vortices have been studied extensively in many physical systems, and specifically in magnetism, where they are associated with configurations of magnetic-flux closure and thus correspond to states of low magnetostatic energy. In particular, magnetic particles with small magnetocrystalline anisotropies tend to fall into the magnetic vortex state above the critical size for single-domain formation. This corresponds, therefore, to a fundamental state which is expected to be present whenever the magnetostatic energy dominates over the other energy terms. Extensive work has been done in circular particles with in-plane anisotropy.[1–3] Recently, ferromagnetic rings were studied[4,5] where the vortex state is the state of lowest energy for a wide range of geometrical parameters. We found that highly symmetric domain configurations mediate the switching between a high moment (onion) state and low moment (vortex) state.[6]

In view of the recent development of materials with large perpendicular anisotropies[7,8] and experiments in Permalloy discs,[9] we study in this Communication the magnetic structure of particles in the presence of a strong symmetry-breaking anisotropy field and identify the analog of the vortex. Motivated by the work on rings and the importance of the vortex state we search in the present case for high-symmetry stable states in disc particles with perpendicular anisotropy. We identify bidomain states which exist even in the absence of a bias field but are otherwise the analogs of the magnetic bubbles observed in ferromagnetic films. The bidomain states have a circulation similar to the vortices and are thus expected to be generic and of a corresponding importance to the vortices. Our study also predicts the size of the magnetic domains supported in the particle, corresponding to the periodicity of stripes observed in films.

Furthermore, the bidomain state is robust and remains stable for a range of applied fields. Its domain-wall position depends on the applied field and it is abruptly expelled out of the particle when it comes close to the outer radius. Likewise, it shrinks abruptly to a point when the bubble becomes very small. This procedure therefore gives rise to an unusual type of switching mechanism. The importance of the present findings is that, again, very simple, high-symmetry domain structures are found to be stable and to mediate the magnetic switching process, in contrast to the complex behavior which usually occurs in small elements.

Static, as well as dynamical properties of the magnetization $\boldsymbol{m}$ are governed by the Landau-Lifshitz equation. The constant length of the magnetization is normalized to unity: $\boldsymbol{m}^2=1$. An important length scale of the system is the exchange length

$$\ell_{\mathrm{ex}} = \sqrt{\frac{A}{2\pi M_0^2}}, \tag{1}$$

where A is the exchange constant and M_0 is the saturation magnetization. In the following, we shall use ℓ_{ex} as the unit of length. Another important quantity is the dimensionless quality factor

$$\kappa = \frac{K}{2\pi M_0^2}, \tag{2}$$

where K is the anisotropy constant. The significance of the quality factor can be seen in two important quantities. First, the domain-wall width is $\ell_{\mathrm{ex}}/\sqrt{\kappa}$. Second, κ controls the relative strength of the magnetostatic field which has a demagnetizing effect, with respect to the anisotropy field which favors alignment along a direction perpendicular to the film. We shall suppose here that $\kappa > 1$, which means that the anisotropy is, in general, stronger than the demagnetizing field.

The Landau-Lifshitz equation is the basis for all our calculations and we are interested only in its static solutions. We find such magnetic configurations by a relaxation algorithm, the details of which were explained in Refs. 10 and 11. We only note here that we use finite differences with a typical lattice spacing of 0.2. We introduce a Gilbert damping term in the equation and feed the algorithm with an initial guess state. This eventually converges to a static solution at a local minimum of the energy functional.

The most demanding part of the method is by far the calculation of the magnetostatic field which requires the solution of a boundary value Poisson problem. A huge reduction of the numerical calculations is obtained if we confine our interest to axially symmetric configurations, and in the rest of this paper we shall be concerned only with such magnetic states. We expect that this is not a serious constraint, at least for the most basic magnetic states of small particles which will be our main focus. Indeed, it is reasonable to assume that the lowest-lying states of the system will have the symmetry of the geometry of the particle. We call z the axis of symmetry of the disc which is also the direction of the easy axis. We use cylindrical coordinates and suppose that the radial (m_ρ), azimuthal (m_ϕ), and longitudinal (m_z) components of the magnetization vector are functions of ρ and z only: $m_\rho = m_\rho(\rho,z)$, $m_\phi = m_\phi(\rho,z)$, $m_z = m_z(\rho,z)$.

As a first step we calculate the simplest possible state. This is expected to be a single-domain state in which all spins are driven by anisotropy and lie roughly along the symmetry z axis. We use the uniform $\boldsymbol{m}=(0,0,1)$ state as an initial guess in the relaxation algorithm. This then quickly converges to a quasiuniform static state, at least for strong anisotropy $\kappa > 1$. The magnetization vector deviates from the z axis only around the edges of the disc. The quasiuniform state is a monodomain state and is thus expected to be the ground state for sufficiently small particles. The transformation $\boldsymbol{m} \to -\boldsymbol{m}$ gives a second monodomain state.

As the size of the particle becomes larger it is anticipated that the magnetic configuration will break up into domains. We conjecture a bidomain state that is axially symmetric. This consists of an inner cylindrical domain of "down" magnetization surrounded by the outer domain of "up" magnetization. A domain wall has to separate the two domains. We shall call these axially symmetric bidomain states "bubbles" because they bear some essential similarities to the so-called magnetic bubbles observed in abundance in ferromagnetic films.

Ferromagnetic films with a strong perpendicular anisotropy were studied experimentally and theoretically around the 1970s. These early studies were largely driven by technological interest in magnetic bubbles whose statics and dynamics were analyzed in detail.[12] A magnetic bubble is a circular domain of opposite magnetization in an otherwise uniformly magnetized film with magnetization perpendicular to the film. The presence of an external bias field is essential for the stabilization of these structures.[13,14] If the bias field is lifted, then the magnetostatic field destroys the bubble which expands and eventually transforms into stripe domains. In contrast, the bubble states calculated here for sufficiently small ferromagnetic particles remain stable even in the absence of a bias field.

We thus return to our conjecture and proceed to test it numerically. As a standard example we choose the following values for the quality factor, the film thickness, and the radius:

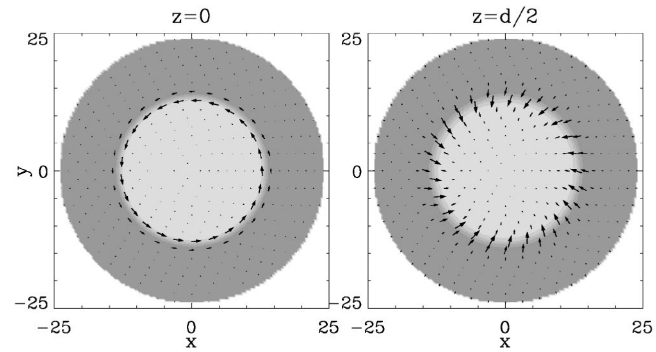

FIG. 1. The bubble state illustrated at the middle ($z=0$) plane of the disc and at the top ($z=d/2$). The magnetization is such that $m_z=-1$ at the center and $m_z \approx 1$ in the outer domain. The arrows show the projection of the magnetization on the (x,y) plane, which has a significant value at the domain wall. This is a Bloch-like wall in the middle ($z=0$) plane and it turns almost Néel-like at the top and bottom surfaces ($z=\pm d/2$). In all cases that the algorithm converges, the magnetization satisfies the parity relations $m_\rho(\rho,z) = -m_\rho(\rho,-z)$, $m_\phi(\rho,z) = m_\phi(\rho,-z)$, $m_z(\rho,z) = m_z(\rho,-z)$.

$$\kappa = 2, \quad d = 8\ell_{\mathrm{ex}}, \quad R = 24\ell_{\mathrm{ex}}, \tag{3}$$

and this choice will be explained later in the text. In the case of the FePt films of Ref. 7, where $M_0 = 1150\ \mathrm{emu/cm^3}$, $A = 10^{-6}\ \mathrm{erg/cm}$, the exchange length is $\ell_{ex} = 3.5\ \mathrm{nm}$ and thus the values of Eq. (3) are translated to $d = 28\ \mathrm{nm}$, $R = 84\ \mathrm{nm}$. The value $\kappa = 2$ for the quality factor corresponds to $K = 1.6 \times 10^7\ \mathrm{erg/cm^3}$ for the anisotropy constant, as is typical in FePt films. We feed our numerical algorithm with an initial condition which has the gross features of the bubble state described above with a domain wall of the Bloch type smoothly connecting the domains. The algorithm converges to a static bubble state which has a complicated domain-wall structure shown in Fig. 1. Also, the magnetization deviates to some extent from the z direction at the side surface of the particle. The profile of this structure is sufficiently interesting and deserves attention. The magnetostatic energy is the driving force here and it clearly favors a bidomain state with opposite magnetization where the total magnetization would roughly vanish. On the other hand, the anisotropy and exchange energies are significant at the domain wall and they put a tension on it to shrink. In the final result the bubble has an inner domain with volume smaller than the outer domain. Thus the total magnetization points along the symmetry axis and it is nonzero. As mentioned already, contrary to the situation in films, we suppose here that no external bias field is present.

The domain wall resembles those discussed in the literature in related calculations.[10,15] It is Bloch in the central plane (set at $z=0$ here) and it progressively becomes Néel towards the surfaces. The Néel wall is significantly wider than the Bloch wall. The radius of the bubble is larger at the center than near the surfaces, but this is a small effect. It is easy to understand that for this type of wall the magnetostatic energy and the total-energy density are larger near the surfaces than at the disc center. In short, the surfaces disfavor the bubble domain wall. The final and important result is

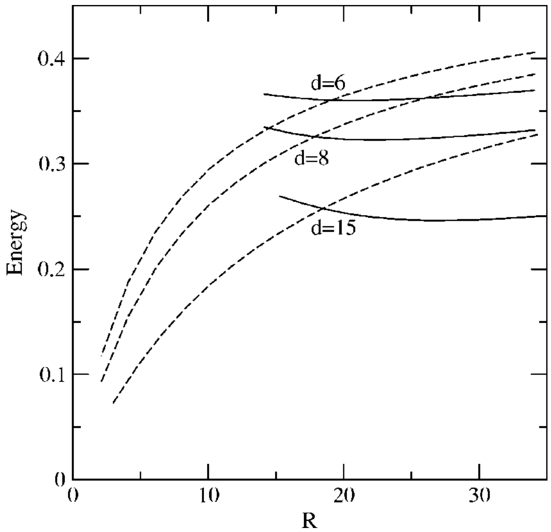

FIG. 2. Energy per unit volume (in units of $2\pi M_0^2$) of the monodomain (dashed lines) and of the bubble state (solid lines) as a function of the disc radius R for three values of the disc thickness d (R and d in exchange length units). The bubble exists only for R greater than a critical radius R_1 and it has a lower energy for $R>R_c$ where R_c is yet another critical radius which corresponds to the intersection of the two lines for each value of d. Both R_1 and R_c depend on d.

that, for the parameters (3), the bubble state has a lower energy than the monodomain state.

We now proceed to a systematic numerical study of the bubble state. We first fix the disc thickness at $d=8$ and vary the radius R. We find a bubble state when the radius is larger than some critical radius $R_1 \approx 14$. For smaller radii $R<R_1$ our algorithm always converges to the quasiuniform state irrespective of the initial condition. Our results thus indicate that the inner bubble domain cannot be sustained if it is too small. For a disc radius slightly larger than R_1 the bubble radius is small and the total magnetization of the structure is large. As the radius of the disc increases the inner bubble domain expands and the absolute value of the total magnetization decreases.

The energy per unit volume of the bubble state as a function of the disc radius is given in Fig. 2 for three values of the thickness d, along with the corresponding energy for the monodomain state. The latter exists as a local minimum of the energy for any radius up to the largest that we checked. The energy per unit volume of the bubble is greater than that of the monodomain state at the lowest radius R_1 where the bubble first appears. It decreases for larger radii and becomes lower than that of the monodomain state above a critical radius R_c. It eventually becomes an increasing function of the radius but apparently remains lower than the energy of the monodomain state for all $R>R_c$. We consider R_c as marking the size for the breakup of the magnetization configuration into domains. We could find the bubble as a local minimum of the energy for all radii $R>R_1$ that we have checked. However, it is expected that a multidomain state will eventually set in for a sufficiently large radius, with energy lower than the energy of both the monodomain and the bubble.

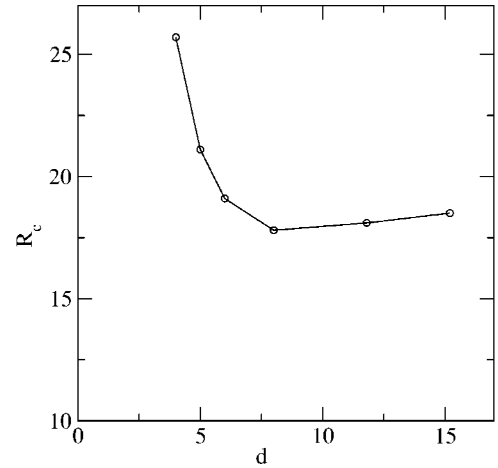

FIG. 3. The critical radius R_c as a function of the thickness of the disc d. For $R>R_c$ the bubble has a lower energy than the monodomain state.

In order to study the dependence of R_c on the thickness d we have repeated our calculation for a few values of d. From Fig. 2 one can extract the critical radii for three values of $d=6,8,15$. In Fig. 3 we give the R_c as a function of d inferred from six values of d. For small d the critical radius significantly exceeds the particle thickness because surface effects become important and disfavor the formation of a domain wall. On the other hand, the critical radius R_c levels off for higher values of d. The thickness for which R_c attains a minimum appears to be close to $d=8$ for which $R_c \approx 18$. This is actually the reason for choosing (3) as our standard parameters, along with the fact that for $d=8$ the energy of the bubble has a minimum at $R \approx 24$ as is seen in Fig. 2.

We have also repeated our calculation for the particle sizes employed in the experiment of Ref. 9 and have confirmed the existence of a bubble state. However, a detailed quantitative comparison cannot be made before one determines the strength of the deposition-induced anisotropy in the Permalloy used in the experiment.

Once we have established the existence of a bubble we would like to know how it behaves under an externally applied field. Apart from the apparent practical implications, this is an interesting question also because the field will affect the intricate balance of energies that is responsible for the stabilization of the bubble. The field is applied along the symmetry axis of the disc, i.e., it is of the form $\boldsymbol{h}_{\text{ext}}=(0,0,h_{\text{ext}})$.

We apply the field on a particle which is already in a bubble state. As a specific example, we choose our standard parameter values (3). Our results are given in Fig. 4. For $h_{\text{ext}}=0$ the total magnetization per unit volume, $\mu=1/V\int m_z dV$, is nonzero. If we choose the inner domain to point down then μ is positive (point C in Fig. 4). Applying a positive external field favors the outer domain, which expands at the expense of the inner domain. The system does reach a new equilibrium state which is again a bubble state, but with a smaller radius. This corresponds to an increased value for μ. For a high enough magnetic field the bubble becomes too small and it cannot be sustained by the system. In our example μ jumps to unity for $h_{\text{ext}}>h_B=0.13$, which

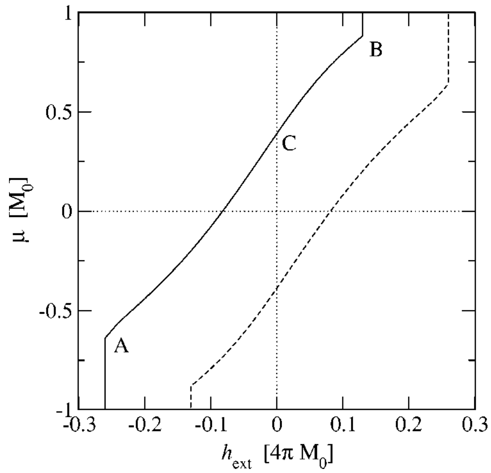

FIG. 4. The total magnetization μ per unit volume as a function of the applied field, for a bubble state in a disc of thickness $d=8$ and radius $R=24$.

signals that the bubble shrinks to zero radius, resulting in a monodomain state with magnetization pointing up (point B). On the other hand, if we start from point C but now reduce h_{ext} to negative values the inner domain is favored and pushes the domain wall to a larger radius, which is reflected in a smaller value for μ. The total magnetization μ crosses zero for $h_{\text{ext}}=-0.08$; it then becomes negative and eventually μ jumps to minus unity for $h_{\text{ext}}<h_A=-0.26$, which means that the system is in the monodomain state pointing down (point A). Below h_A the domain wall is attracted by the side surface and is expelled from the disc. The dashed line in Fig. 4 corresponds to the equivalent physical situation obtained by the symmetry transformation $\boldsymbol{m}\to-\boldsymbol{m}$, $\boldsymbol{h}_{\text{ext}}\to-\boldsymbol{h}_{\text{ext}}$.

The bubble is stable in the range $h_A<h_{\text{ext}}<h_B$ in which reversible behavior occurs. For $h<h_A$ and $h>h_B$ irreversible jumps in the magnetization occur, which correspond to the domain wall being attracted to the edge (point A) or shrinking to the center of the disc (point B). The size of the jumps in the magnetization reflect the size of the domain wall being annihilated (larger when the inner domain expands) and constitutes yet another example of how the geometry of the element constrains the shape of the domain wall and the details of the switching process.[16,17] On the other hand, if a particle with $R>R_c$ is saturated by a strong *in-plane field*, it will eventually relax into a bubble state after the field is removed.[9]

In conclusion, we have studied the fundamental states of disc-shaped magnetic particles with uniaxial anisotropy along the axis of the disc. A magnetic bubble state has been identified within our numerical calculation and has been studied in detail. The bubble is a particularly simple axially symmetric state and it is expected to play for perpendicular anisotropy materials the role that the vortex plays for in-plane anisotropy.

We recently became aware of Ref. 18, where experiments on Ni particles support our present theoretical results.

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