# Loss of Andreev backscattering in superconducting quantum point contacts

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We study effects of magnetic field on the quasiparticle energy spectrum in a superconducting quantum point contact. The supercurrent induced by the magnetic field leads to intermode transitions between the electron waves that pass and do not pass through the constriction. The latter experience normal reflections which couple the states with opposite momenta inside the quantum channel and create a minigap in the low-energy spectrum that depends on the magnetic field.

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### I. INTRODUCTION

Transport in superconducting/normal-metal hybrid structures is governed by normal and Andreev reflections. Competition between these processes is determined by insulating barriers at the interfaces, mismatch in the material parameters, impurities, etc. The devices where the degree of normal and Andreev reflections can be tuned to control the system conductance are in the focus of current nanoscale physics research. One of the possible ways to manipulate the conductance is to use an external magnetic field either to influence the cyclotron trajectories of particles and holes in a normal part of a hybrid system<sup>1,2</sup> or to induce interference between partial reflected waves as in Ref. 3 (Andreev interferometer). Another mechanism to change the trajectories and to affect the interplay between the Andreev and normal reflections is to interfere with the fundamental property of the Andreev reflection, i.e., with its almost exact backscattering. During the Andreev reflection, the angle of divergence between the trajectories of an incoming particle and the reflected hole does not exceed  $(k_F \xi)^{-1}$ , where  $\xi$  is the superconducting coherence length and  $k_F$  is the Fermi wave vector. Generally, small deviations from exact backscattering come from interaction of electron and hole waves with an inhomogeneity in spatial distribution of the order-parameter phase. For example, such deviations can be caused by the transverse force on particles and holes from the supercurrent<sup>4,5</sup> induced by a magnetic field.

Although the deviations from the exact backscattering are small, they become noticeable if the divergence between the particle and hole trajectories is comparable with the size of the system involved. The exemplary device where such a condition is achieved is a ballistic quantum point contact having the form of a narrow channel (constriction) between the two superconductor electrodes. In the present Brief Report we show that breaking down the exact Andreev back-scattering produces a dramatic change in the low energy spectrum of quantum contact. The loss of backscattering mixes the modes passing through the channel with the modes that do not penetrate inside but are normally reflected from the channel end. The normal reflections couple the waves propagating through the channel in the opposite directions,

which leads to formation of a minigap in the energy spectrum at a superconducting phase difference  $\pi$  in a way similar to that for contacts with normal scatterers. <sup>6-9</sup> The deviation from backscattering is produced by an exchange, during the Andreev process, of a Cooper pair momentum induced in the electrodes by an applied magnetic field. Varying the magnetic field one can tune the degree of normal reflection together with the minigap thus controlling the transport properties of the contact.

#### II. MODEL

Shown in Fig. 1(a) is the model device that illustrates the loss of exact backscattering during the Andreev reflection: A single-mode channel with the radius  $a \sim k_F^{-1}$  opens into a normal semispherical region with the radius b much larger than the superconducting coherence length  $\xi$ . The normal region is surrounded by a superconductor which carries the supercurrent with the momentum  $\hbar \mathbf{k}_s$ . For  $b \ge \xi$  the quasiparticle propagation is well described via a trajectory representation. Due to the transfer of  $\hbar \mathbf{k}_s$  the Andreev reflected trajectory deviates from its initial direction<sup>4</sup> and can miss the constriction to experience a normal reflection from the insulating barrier. The trajectory returns to the constriction after several reflections; this results in coupling of states propagating through the channel in the opposite directions. The momentum transfer of  $\hbar \mathbf{k}_s$  deflects a trajectory by an angle  $k_s/k_F$  and produces its divergence by  $k_sb/k_F$  over a distance b. The probability of normal reflection thus depends on the ratio of the trajectory divergence to the transverse channel dimension a. For a single-mode quantum channel,  $a \sim k_F^{-1}$ , this ratio becomes  $k_s b/k_F a \sim k_s b$ .

In a superconducting point contact, the wave functions for subgap states decay at distances  $b \sim \xi$ , thus  $k_s b \sim k_s \xi \lesssim 1$ . However, the trajectory divergence  $k_s \xi / k_F$  is less than the wavelength, and the trajectory description is not adequate. A single-mode channel with a radius  $a \sim k_F^{-1}$  irradiates an electronic wave determined by diffraction. Let us consider the right superconductor occupying a half-space z > 0 and introduce the spherical coordinates  $x = r \sin \theta \cos \phi$ ,  $y = r \sin \theta \sin \phi$ ,  $z = r \cos \theta$  with the origin at the right channel outlet. Far from it,  $r \gg a$ , particlelike and holelike wave functions are

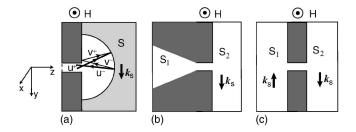


FIG. 1. (a) A single-mode channel is open to a normal region (white semicircle) in a contact with a superconductor (gray region). Andreev reflected trajectories deviate from initial direction due to the transverse pair momentum  $\hbar \mathbf{k}_s$  and experience normal reflections from the insulator surface (black), which couple right-moving  $u^+$  and left-moving  $u^-$  states. (b) Asymmetric and (c) symmetric point contacts.

$$\begin{pmatrix} u^{\pm} \\ v^{\pm} \end{pmatrix} = r^{-1} e^{\pm ik_F r} \begin{pmatrix} U^{\pm} \\ V^{\pm} \end{pmatrix}.$$
 (1)

microscopic wave functions vanish superconductor/insulator boundary  $\theta = \pi/2$  which is assumed specular for simplicity; thus the amplitudes  $U^{\pm}$  and  $V^{\pm}$  can be expanded in spherical functions  $P_{l,m}(\theta,\phi)$  with odd angular momenta l. Inside the single-mode channel there are two particle and two hole waves  $\propto e^{\pm ik_xz}$  with amplitudes  $u_0^{\pm}$  and  $v_0^{\pm}$ , respectively, corresponding to the momentum projections  $\pm \hbar k_z$  on the z axis. To match the channel modes with the quasiparticle waves in the superconducting half-space we note that, for a waveguide  $a \leq k_F^{-1}$ , the radiated/incident diffraction field is  $\exp(\pm ik_F r)\cos\theta/r$ . We now assume that it is only one mode in the diffraction field, Eq. (1), proportional to  $P_{1,0} = \cos \theta$  that ideally transforms into the channel mode  $u_0^{\pm}$ ,  $v_0^{\pm}$  without reflections, while all other modes  $l \neq 1$  are normally reflected from the waveguide end without transmission into the channel. Thus the wave-function amplitudes have the form

$$\begin{pmatrix} U^{\pm} \\ V^{\pm} \end{pmatrix} = P_{1,0} \begin{pmatrix} u_0^{\pm} \\ v_0^{\pm} \end{pmatrix} + \begin{pmatrix} \Psi_u^{\pm} \\ \Psi_v^{\pm} \end{pmatrix}.$$
 (2)

The amplitudes  $\Psi_u^\pm$  and  $\Psi_v^\pm$  stand for the modes with  $l\neq 1$  which experience normal reflections at the channel end. They are coupled by a normal reflection matrix  $\check{R}_\epsilon\colon \Psi_u^+ = \check{R}_\epsilon \Psi_u^-$ ,  $\Psi_v^+ = \check{R}_- \epsilon \Psi_v^-$ . The functions  $\Psi_u^\pm$ ,  $\Psi_v^\pm$  are orthogonal to  $P_{1,0}$ :  $\langle P_{1,0} | \Psi_{u,v}^\pm \rangle = 0$ . The angular brackets denote the angular average within  $0 < \theta < \pi/2$ .

We use the Bogoliubov-de Gennes equations

$$\left[\frac{1}{2m}\left(-i\hbar\,\nabla - \frac{e}{c}\mathbf{A}\right)^2 - E_F\right]u + \Delta v = Eu,\tag{3}$$

$$\left[\frac{1}{2m}\left(-i\hbar \nabla + \frac{e}{c}\mathbf{A}\right)^2 - E_F\right]v - \Delta^* u = -Ev, \qquad (4)$$

where **A** is the vector potential of the magnetic field **B** =  $B(z)\hat{\mathbf{x}}$ . The gap function has the form  $\Delta = \Delta_0 e^{i(\chi_R + \mathbf{k}_{SR}\mathbf{r})}$  or  $\Delta = \Delta_0 e^{i(\chi_L + \mathbf{k}_{SL}\mathbf{r})}$  in the right (left) superconducting electrode;  $\chi_{R,L} = \pm \chi$  is the zero-field constant phase in the right (left)

electrode with  $2\chi$  phase difference between them;  $\mathbf{k}_{sR,L}$  is a constant wave vector in each electrode. It enters the superconducting velocity  $m\mathbf{v}_s = \hbar \mathbf{k}_s - (2e/c)\mathbf{A}$  that determines the difference in the eikonals of particle and hole wave functions,  $(m/\hbar) \int \mathbf{v}_s \cdot d\mathbf{r}$ . The magnetic field is screened in the superconductor at the London length λ. In the right superconductor, for example,  $\mathbf{v}_s(z) = -(2e/mc)B_0\lambda\hat{\mathbf{y}}e^{-z/\lambda}$ . Assuming for simplicity  $\lambda \gg \xi$  we can neglect **A** in the region r  $\sim \xi$  where the low-energy wave functions are localized:  $\mathbf{k}_{sR} = (2\pi/\phi_0)\lambda B_0\hat{\mathbf{y}}$ , where  $\phi_0 = \pi\hbar c/|e|$  is the flux quantum. The parameter  $k_s \xi$  that determines the relative weight of normal reflections at the channel end is  $k_s \xi \sim B_0/H_{cm}$  where  $H_{cm} \sim \phi_0/(\lambda \xi)$  is the thermodynamic critical field. The gap  $\Delta_0$  is suppressed by the magnetic field. However, this does not change the backscattering properties of Andreev reflection; we ignore it in what follows.

# III. SCATTERING MATRIX

In the normal channel, particles  $u_0^+$  and holes  $v_0^-$  propagate in the +z direction while particles  $u_0^-$  and holes  $v_0^+$  propagate in -z direction. Using the scheme employed in Ref. 8, we introduce the scattering matrices  $\hat{S}_R(\epsilon, \chi, \mathbf{k}_{sR})$  and  $\hat{S}_L(\epsilon, -\chi, \mathbf{k}_{sL})$  that relate the incident and outgoing wave amplitudes, respectively, at the right, z=0, and left, z=-d, ends of the channel:

$$\begin{pmatrix} u_0^- \\ v_0^+ \end{pmatrix}_R = \hat{S}_R \begin{pmatrix} u_0^+ \\ v_0^- \end{pmatrix}_R, \quad \begin{pmatrix} u_0^+ \\ v_0^- \end{pmatrix}_L = \hat{S}_L \begin{pmatrix} u_0^- \\ v_0^+ \end{pmatrix}_L.$$
 (5)

Here  $d \le \xi$  is the channel length. The wave functions at different ends of the channel have different phase factors:

$$\begin{pmatrix} u_0^{\pm} \\ v_0^{\pm} \end{pmatrix}_{P} = e^{\pm ik_z d} \begin{pmatrix} u_0^{\pm} \\ v_0^{\pm} \end{pmatrix}_{I}. \tag{6}$$

The solvability condition of Eqs. (5) and (6) yields

$$\det(1 - e^{i\hat{\sigma}_z k_z d} \hat{S}_R e^{i\hat{\sigma}_z k_z d} \hat{S}_L) = 0. \tag{7}$$

For  $|E| < \Delta_0$  the matrix  $\hat{S}$  is unitary:  $\hat{S}\hat{S}^{\dagger} = 1$ . Indeed, Eqs. (3) and (4) conserve the quasiparticle flow

$$\operatorname{div} \left[ u^* \left( -i\hbar \, \boldsymbol{\nabla} - \frac{e}{c} \mathbf{A} \right) u + u \left( i\hbar \, \boldsymbol{\nabla} - \frac{e}{c} \mathbf{A} \right) u^* \right.$$
$$\left. - v^* \left( -i\hbar \, \boldsymbol{\nabla} + \frac{e}{c} \mathbf{A} \right) v - v \left( i\hbar \, \boldsymbol{\nabla} + \frac{e}{c} \mathbf{A} \right) v^* \right] = 0.$$

Since this flow vanishes deep in the superconductor where  $\mathbf{v}_s = 0$ , it should be zero also in the channel, whence  $|u_0^+|^2 + |v_0^-|^2 = |u_0^-|^2 + |v_0^+|^2$  which results in  $\hat{S}\hat{S}^\dagger = 1$ . The unitarity implies that those quasiparticles which are scattered normally at the superconductor surface and the channel end will eventually return into the channel either as particles or as holes after certain number of Andreev reflections at the superconducting side.

We now calculate the matrix  $\hat{S}_R$  explicitly. Consider first low energies such that  $|E+E_s(0)| < \Delta_0$  where  $E_s(0)$  is the Doppler shift associated with the supercurrent near the plane

z=0. We assume, of course,  $|E_s(0)| < \Delta_0$ . Wave functions decaying for  $z \rightarrow \infty$  at distances of the order of  $\xi$  obey the relations

$$v_R^+ = e^{-i/2(\chi + \mathbf{k}_{sR}\mathbf{r})} \check{a}_{\epsilon}^+(\mathbf{k}_{sR}) e^{-i/2(\chi + \mathbf{k}_{sR}\mathbf{r})} u_R^+, \tag{8}$$

$$u_R^- = e^{i/2(\chi + \mathbf{k}_{SR}\mathbf{r})} \check{a}_{\epsilon}^+(\mathbf{k}_{SR}) e^{i/2(\chi + \mathbf{k}_{SR}\mathbf{r})} v_R^-, \tag{9}$$

which couple the electron and hole amplitudes near the channel end  $|\mathbf{r}| \ll \xi$ . Here

$$\check{a}_{\epsilon}^{\pm}(\mathbf{k}_{s}) = \epsilon + \check{\epsilon}_{s}(0) \mp i\sqrt{1 - [\epsilon + \check{\epsilon}_{s}(0)]^{2}},$$

 $\xi = \hbar v_F / \Delta_0$ ,  $\epsilon = E / \Delta_0$ , and  $\check{\epsilon}_s = i\hbar v_s(0) \nabla / 2\Delta_0$ . For the left superconductor, similar expressions hold with the replacement  $\chi \rightarrow -\chi$  and  $\check{a}^+ \rightarrow \check{a}^-$ .

The Andreev relations (8) and (9) should be applied to the entire functions with the amplitudes U and V from Eq. (2) at a hemisphere with a radius  $a \ll r \ll \xi$  where  $\mathbf{k}_s \mathbf{r} \sim k_s r \sin \theta \sin \phi \ll 1$ . Taking the derivatives only of the rapidly varying radial exponents in Eq. (1) we obtain

$$v_0^+ P_{1,0} + \check{R}_{-\epsilon} \Psi_v^- = e^{-i\varphi_+} (u_0^+ P_{1,0} + \check{R}_{\epsilon} \Psi_u^-), \tag{10}$$

$$u_0^- P_{1,0} + \Psi_u^- = e^{i\varphi_-} (v_0^- P_{1,0} + \Psi_v^-), \tag{11}$$

where

$$e^{i\varphi_{\pm}} = \epsilon \pm \epsilon_s \pm i\sqrt{1 - (\epsilon \pm \epsilon_s)^2},$$
 (12)

 $\epsilon_s = -\frac{1}{2} \xi k_s \sin \theta \sin \phi$ , and the index R is omitted. Equations (10) and (11) are written for  $\chi = 0$ , the phase can be recovered by  $\hat{S}(\chi) = e^{i\chi\sigma_z/2} \hat{S}(\chi = 0) e^{-i\chi\sigma_z/2}$ .

For higher energies, it may happen that the Doppler-shifted energy exceeds the gap,  $|\epsilon \pm \epsilon_s| > 1$ , where the sign depends on the momentum direction and on the sign of  $\epsilon$ . The full Andreev reflection then occurs at a point  $r_0 \sim \lambda/\cos\theta$  where the corresponding Doppler-shifted energy is equal to the gap energy  $|\epsilon \pm \epsilon_s(r_0)| = 1$  due to the screening of  $v_s$ . In this case, one of the corresponding factors,  $e^{i\varphi_\pm}$ , in Eqs. (10) and (11) should be modified. The new factors can be found using the WKB approximation for  $\lambda \gg \xi$ . We do not present these expressions here.

Since the normal reflection at the channel end is associated with the momentum transfer  $\sim \hbar k_F$ , one can neglect the energy dependence of  $\check{R}$  on the scale  $\Delta_0$  and take  $\check{R} = -e^{i\varphi_r}$  with a constant phase shift  $\varphi_r$  which is a reasonable approximation at least for specular reflection. Solving Eqs. (10) and (11) for  $\Psi_u$  and  $\Psi_v$  and then applying the orthogonality  $\langle P_{1,0}|\Psi_{u,v}^-\rangle=0$ , yields two equations coupling  $u_0^+$ ,  $v_0^-$  and  $u_0^-$ ,  $v_0^+$  through the matrix

$$\hat{S} = \frac{1}{1 - c_{+}c_{-}^{*}} \begin{pmatrix} e^{-i\varphi_{r}}(|c_{+}|^{2} - 1) & e^{i\chi}(c_{-} - c_{+}) \\ e^{-i\chi}(c_{+}^{*} - c_{-}^{*}) & e^{i\varphi_{r}}(|c_{-}|^{2} - 1) \end{pmatrix}, \quad (13)$$

where

$$c_{\pm} = \frac{\langle P_{1,0} (1 - e^{i(\varphi_{\mp} - \varphi_{\pm})})^{-1} P_{1,0} \rangle}{\langle P_{1,0} (e^{i\varphi_{\pm}} - e^{i\varphi_{\mp}})^{-1} P_{1,0} \rangle}.$$
 (14)

Using  $e^{i\varphi_+(\epsilon_s)} = e^{-i\varphi_-(-\epsilon_s)}$  and applying the transformation  $\phi \to \pi + \phi$  in the integral over the angles in Eq. (14) we find

 $c_{-}^{*}=c_{+}$ . For small  $\epsilon_{s}$ , it is sufficient to take Eq. (12) whence

$$c_{+} \simeq e^{i\eta} - \frac{i(k_{s}\xi)^{2} \langle P_{1,0}^{2} \sin^{2}\theta \sin^{2}\phi \rangle e^{2i\eta}}{8 \langle P_{1,0}^{2} \rangle \sin^{3}\eta},$$
 (15)

where  $e^{i\eta} = \epsilon + i\sqrt{1 - \epsilon^2}$ . Without  $k_s$  one has  $|c_+| = |c_-| = 1$ . As a result, the diagonal components of  $\hat{S}$  vanish, thus the  $+p_z$  and  $-p_z$  states are decoupled.

In the diffraction picture, the transitions that couple the penetrating and nonpenetrating modes are caused by the angle-dependent Doppler shift  $\epsilon_s$  in Eq. (12) which distorts the wave fronts of reflected holes as compared to those of incident particles. The interference of these waves results in the suppression of the amplitude of the Andreev reflected wave entering the channel.

### IV. RESULTS

Consider first the zero-bias conductance of a normal-metal/quantum-channel/superconductor junction  $G_s = (e^2/\pi\hbar)(1-|S_{11}|^2+|S_{12}|^2)$  where  $|S_{11}|^2$  and  $|S_{12}|^2$  are probabilities of normal and Andreev reflection, respectively. We get for small  $k_s \xi$ 

$$G_s = \frac{e^2}{\pi \hbar} \left[ 2 - \frac{2(|c_+|^2 - 1)^2}{|c_+^2 - 1|^2} \right]_{\epsilon = 0} \simeq \frac{e^2}{\pi \hbar} \left[ 2 - \frac{1}{2} \left( \frac{B_0}{H_c} \right)^4 \right].$$

Here we introduce a field  $H_c \sim H_{cm}$  through

$$\frac{B_0^2}{H_c^2} = \frac{(k_s \xi)^2 \langle P_{1,0}^2 \sin^2 \theta \sin^2 \phi \rangle}{4 \langle P_{1,0}^2 \rangle} = \frac{(k_s \xi)^2}{20}.$$
 (16)

Consider now an asymmetric structure that consists of a superconducting tip with a curvature radius smaller than  $\lambda_L$  in a contact with a bulk superconductor, see Fig. 1(b). In this case  $\mathbf{k}_{sL} = 0$  while  $\mathbf{k}_{sR} = \mathbf{k}_s \neq 0$ . On the right end of the channel the matrix  $\hat{S}_R = \hat{S}(\boldsymbol{\epsilon}, \boldsymbol{\chi}, \mathbf{k}_s)$  is determined by Eq. (13). On the left end the matrix  $\hat{S}_L = \hat{S}(\boldsymbol{\epsilon}, -\boldsymbol{\chi}, 0)$  assumes an Andreev form  $\hat{S}_L = e^{-i\eta}e^{-i\chi\sigma_z}\hat{\sigma}_x$ . The phase shift  $k_zd - \varphi_r$  drops out and Eq. (7) yields

$$(1 - c_+^2)e^{i\eta} - (1 - c_+^{*2})e^{-i\eta} = 2(c_+^* - c_+)\cos(2\chi).$$
 (17)

For  $k_s$ =0 with  $c_+$ = $e^{i\eta}$  we obtain a standard gapless expression<sup>11,12</sup>  $\epsilon$ = $\pm$ cos  $\chi$ . For a nonzero  $k_s$ , a minigap opens in the spectrum. To see this consider Eq. (17) in the limit of small  $\epsilon$  and  $k_s$ . We have

$$\epsilon^2 = \cos^2 \chi + \frac{1}{8} [(ic_+ + 1)^2 + (ic_+^* - 1)^2]_{\epsilon=0}.$$

Within the leading terms in  $B/H_c$  we find

$$\epsilon^2 = \cos^2 \chi + \epsilon_g^2, \tag{18}$$

where the minigap in the spectrum is  $\epsilon_g = \frac{1}{4}(B_0/H_c)^2$ . The spectrum for energies close to  $\pm \Delta_0$  is not expected to change qualitatively.

In the case of a symmetric contact shown in Fig. 1(c) the solution of the screening problem yields  $\mathbf{k}_{sL} = -\mathbf{k}_{sR} = -\mathbf{k}_{s}$ . The spectral equation (7) with  $\hat{S}_R = \hat{S}(\epsilon, \chi, \mathbf{k}_s)$  and  $\hat{S}_L = \hat{S}(\epsilon, \chi, \mathbf{k}_s)$  reduces to

$$\frac{(c_{+} + c_{+}^{*})^{2}}{4} = \cos^{2} \chi + \frac{(|c_{+}|^{2} - 1)^{2}}{(ic_{+} - ic_{+}^{*})^{2}} \sin^{2}(k_{z}d - \varphi_{r}). \quad (19)$$

The spectrum has a gap for nonzero  $k_s$ . In the limit of low  $\epsilon$  and  $k_s$ , the right-hand side of Eq. (19) can be treated as a perturbation. We put  $(|c_+|^2-1)^2 \simeq (B_0/H_c)^4$  where  $H_c$  is defined in Eq. (16) while  $(c_+-c_+^*)^2 \approx -4$  and  $(c_++c_+^*)^2/4 \simeq \epsilon^2$ . Finally we get Eq. (18) where

$$\epsilon_g = \frac{1}{2} (B_0 / H_c)^2 |\sin(k_z d - \varphi_r)|.$$
 (20)

#### V. DISCUSSION

Since the wave vector  $k_s \leq \xi^{-1} \leq k_F$ , it induces transitions only between the modes with close transverse quantum numbers. Thus, the predicted effect is more easily seen in a contact transparent only for a few modes. On the contrary, in a multimode channel, coupling to the reflected modes that mixes  $\mathbf{p}$  and  $-\mathbf{p}$  states has a small weight while transitions occur mostly between the penetrating modes. In large area superconductor/normal-metal/superconductor junctions, these transitions result in an instability of the spectrum accompanied by formation of energy bands. 1

The minigap calculated here has the same origin as in the presence of normal scatterers. Note that in a symmetric contact, the gap, Eq. (20), vanishes for the phase difference  $k_z d - \varphi_r = \pi n$ . It is the result of resonant tunneling through a system of two barriers with equal reflection coefficients. The transmission probability  $|T|^2$  through such a system is unity at the resonance, and the gap  $\epsilon_e = 1 - |T|^2$  disappears. An

asymmetry in the scattering removes the resonant tunneling effect, thus a finite gap exists for any phase shift  $k_z d - \varphi_r$  as illustrated by Eq. (18). Similar effects of resonant tunneling and minigap oscillations as functions of  $k_z d$  can also take place for other mechanisms of normal reflection such as mismatch in the material parameters, interface barriers, etc. <sup>13</sup>

The predicted minigap is not small but can reach values of the order of  $\Delta_0$  for  $B_0 \sim H_{cm}$ . It can be monitored by varying the magnetic field and measuring the Josephson current that decreases in magnitude and acquires more sinusoidal shape with the increase in the minigap. The minigap affects dynamic properties of the contact; in particular, at voltage bias  ${\rm eV} < \epsilon_g$  the dc current is suppressed. Varying the magnetic field one can thus observe a transition from the ballistic to high-resistance behavior of the contact. For simplicity we assumed specular reflections from both the insulating surface and the channel end. However, the general arguments on the loss of backscattering and formation of a minigap in the energy spectrum of a single-mode contact should hold for an arbitrary rough surface as well.

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