Paraconductivity of a mesoscopic superconducting tube in the nonohmic regime

Carlos Carballeira* and Victor V. Moshchalkov
Nanoscale Superconductivity and Magnetism Group, Laboratory for Solid State Physics and Magnetism, K. U. Leuven, Celestijnenlaan 200 D, 3001 Leuven, Belgium
(Received 8 July 2004; published 18 February 2005)

By using a generalized version of the time-dependent Ginzburg-Landau theory, we have calculated the paraconductivity, $\Delta\sigma$, of a superconducting mesoscopic tube beyond the conventional ohmic approximation. Among other dimensionality effects, the so-obtained paraconductivity expressions show that, close to T_c and deep inside the nonohmic region, a dimensional crossover from the one-dimensional to the two-dimensional fluctuation regimes can be induced which, in turn, may be understood in terms of an effective shrinkage of the superconducting coherence length with the electrical field. These nonohmic effects on $\Delta\sigma$ may be observed in Al mesoscopic tubes and carbon nanotubes, thus opening new experimental possibilities to investigate the superconducting behavior of these nanostructured materials.

DOI: 10.1103/PhysRevB.71.052503 PACS number(s): 74.20.De, 74.25.Fy, 74.40.+k, 74.78.Na

During the last years, advances in different microfabrication techniques have attracted the interest of many researchers to the properties of superconductors with dimensions of the order of $\xi(0)$, the amplitude of the Ginzburg-Landau (GL) superconducting coherence length.[1] One of the most striking conclusions in this field is that the superconducting critical parameters of these so-called mesoscopic samples may be appreciably enhanced by nanoscale modulation of the boundary conditions.[1,2] This outstanding property broadens the perspectives for both the design of innovative devices based on these materials and the improvement of the practical applications of superconductivity.

The study of the response of these mesoscopic superconductors to external magnetic fields and, in particular, to applied electrical currents is a crucial issue for the development of the novel applications mentioned above. Up to date, the dimensionality effects on the resistivity, $\rho(T)$, have been mainly studied in mesoscopic loops and wires, where it has been found that the behavior of $\rho(T)$ around the critical temperature at zero applied magnetic field, T_{c0}, is dominated by order parameter fluctuations.[3–7] These fluctuations create below T_{c0} phase slips that considerably enhance the resistivity, whereas above the transition they lead to a depletion of the normal state resistivity (the so-called paraconductivity) due to the presence of short-lived thermally activated Cooper pairs. Such precursive superconducting effects above T_{c0}, also studied through the diamagnetic response to an applied magnetic field,[8–12] have been proven to be a useful tool to investigate the properties of the mesoscopic superconductors.

Our present paper aims to extend these studies of the dimensionality effects on the electrical transport properties of mesoscopic superconductors to the paraconductivity ($\Delta\sigma$) of a mesoscopic tube [i.e., a tube with radius, R, of the order of $\xi(0)$] at high applied electrical fields. This regime is characterized by an electrical field dependence of $\Delta\sigma$ which, therefore, departs from the conventional ohmic behavior. The physical origin of this nonlinear effect is the acceleration of the Cooper pairs along the fluctuation's size which, if the electrical field is high enough, may increase their kinetic energy up to suppress the fluctuation itself.[13] To the best of our knowledge, this high field regime of the paraconductivity has only been investigated in thin films of conventional low-T_c superconductors (LTSC).[14] However, the existing studies have also raised interesting questions concerning the nonohmic region of $\Delta\sigma$ in high-T_c superconductors (HTSC) as, for instance, those related to the possible influence of the electrical field on the fluctuations' dimensionality.[15]

We believe that, due to their relatively large values of $\xi(0)$ and intrinsic intermediate dimensionality, the mesoscopic low-T_c superconductors are appropriate systems to observe the interplay between dimensionality and electrical field effects addressed here. Therefore, by using a generalized version of the time-dependent Ginzburg-Landau theory (TDGL), we will first present detailed calculations of the paraconductivity for a mesoscopic tube which extend the existing results for the ohmic regime[16] to the finite electrical field limit. The so-obtained $\Delta\sigma$-expressions will show that the superconducting fluctuations (SFs) in the tube, which may vary between the one-dimensional (1D) and two-dimensional (2D) regimes, are not only controlled by the reduced temperature, $\epsilon \equiv \ln(T/T_{c0})$, but also by the applied electrical field. In fact, these results predict that, close to T_{c0}, a 1D to 2D crossover may be induced when going deep inside the nonlinear region. We will argue here that this behavior, which contrasts with the one expected for other superconducting systems showing a crossover between two different dimensionalities,[15] may be understood in terms of an effective shrinkage of the superconducting coherence length when the electrical field increases. Finally, we will discuss the possibility to reach the nonohmic regime in Al mesoscopic tubes, as well as the usefulness of our present results for studies of the superconducting effects in carbon nanotubes.

To calculate the paraconductivity of a mesoscopic tube beyond the conventional Ohmic approximation, we have adapted to this particular geometry the GL formalism proposed in Ref. 17 for bulk isotropic 3D superconductors and 2D thin films. This procedure consists in combining the standard GL expression for the thermally averaged current of the superconducting condensate,

$$\mathbf{J}=\frac{2e}{m^*}\frac{\hbar}{i}\langle\Psi^*\nabla\Psi-\Psi\nabla\Psi^*\rangle-\frac{4e^2}{m^*c}\mathbf{A}(t)\langle\Psi^*\Psi\rangle, \quad (1)$$

with the generalized Langevin equation for the order parameter,

$$\frac{\partial\Psi(\mathbf{r},t)}{\partial t}=\frac{8k_BT_{c0}}{\pi\hbar a_0}\Big(-\hat{H}\Psi(\mathbf{r},t)+G(\mathbf{r},t)\Big). \quad (2)$$

In these equations $\hat{H}=a_0\epsilon+(1/2m^*)[(\hbar/i)\nabla-(2e/c)\mathbf{A}(t)]^2$ is the GL Hamiltonian, m^* is the effective mass of the Cooper pairs, e is the electron charge, $\hbar$ is the reduced Planck constant, $a_0=\hbar^2/2m^*\xi^2(0)$ is the GL normalization constant, k_B is the Boltzmann constant, $\mathbf{A}(t)$ is the vector potential, and $G(\mathbf{r},t)$ is a Langevin random force uncorrelated in space and time. The latter implies that $G(\mathbf{r},t)$ will verify $\langle G^*(\mathbf{r},t)G(\mathbf{r}',t')\rangle=a\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')$, where $a=\pi\hbar a_0/4$ is a normalization constant.

By using cylindrical coordinates, the presence of a homogeneous electrical field (E) applied parallel to the z axis at the instant $t=0$ may be introduced into the formalism through the following gauge choice,

$$A_\rho=A_\phi=0,$$

$$A_z(t)=\begin{cases}-cEt & \text{if } t\geqslant 0,\\ 0 & \text{if } t<0,\end{cases} \quad (3)$$

where ρ, ϕ, and z are, respectively, the radial coordinate, the azimuthal angle, and the axial coordinate. In order to calculate the thermal averages involved in Eqs. (1) and (2), it is also useful to perform an expansion of the superconducting order parameter in terms of its Fourier components, $\Psi_{\mathbf{k}}(t)\equiv V^{-1}\int d^3\mathbf{r}\Psi(\mathbf{r},t)\exp(-i\mathbf{kr})$, where $V=2\pi RdL_z$ is the sample's volume[18] [here d and L_z are, respectively, the thickness and length of the tube, which verify $d\ll R$, $\xi(0)\ll L_z$] and $\mathbf{k}$ is the so-called wave vector of the fluctuations. As a direct consequence of the boundary conditions, the latter may be expressed as $\mathbf{k}\equiv(0,n/R,k_z)$, where n is an integer number and k_z is the component of the fluctuations' wave vector in the z direction. Equations (1) and (2) are then transformed into

$$J_z=-V^{-1}\sum_{n,k_z}\frac{2e}{m^*}(\hbar k_z-2eEt)\langle|\Psi_{n,k_z}|^2\rangle, \quad (4)$$

and, respectively,

$$\frac{\partial\Psi_{n,k_z}(t)}{\partial t}=\frac{8k_BT_{c0}}{\pi\hbar a_0}[-E_{n,k_z}\Psi_{n,k_z}(t)+G_{n,k_z}(t)]. \quad (5)$$

In this last equation, $G_{n,k_z}(t)$ is the Langevin random force in momentum space, which now obeys

$$\langle G^*_{n,k_z}(t)G_{n',k_z'}(t')\rangle=a\delta(n-n')\delta(k_z-k_z')\delta(t-t'), \quad (6)$$

whereas E_{n,k_z} holds for the eigenvalues of the GL Hamiltonian, which are given by

$$E_{n,k_z}(t)=a_0\epsilon+\frac{1}{2m^*}(\hbar k_z+2eEt)^2+\frac{\hbar^2n^2}{2m^*R^2}. \quad (7)$$

The thermally averaged current density at arbitrarily high electrical fields may be now obtained by introducing in Eq. (4) the $\langle|\Psi_{n,k_z}|^2\rangle$ expression that results from combining the solution of Eq. (5) with Eq. (6). This gives

$$J_z=-\frac{e}{m^*}\frac{2k_B^2T_{c0}^2}{\pi^5\hbar a_0Rd}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}dk_z(\hbar k_z-2eEt)\times\int_{-\infty}^{t}dt'\exp\left(-\frac{16k_BT_{c0}}{\pi\hbar a_0}\int_{t'}^{t}dt''E_{n,k_z}(t'')\right), \quad (8)$$

where we have applied that the sums in momentum space may be written as $\Sigma_{n,k_z}\equiv\Sigma_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}dk_z/(2\pi/L_z)$.

As discussed in Ref. 17, to calculate the integrals involved in Eq. (8) it is convenient to introduce the variables p_z, u, and u' defined as, respectively, $\hbar p_z=\hbar k_z-eE(t+t')$, $u=t'-t$, and $u'=t+t'-2t''$. Then, the paraconductivity may be calculated by considering the limit $t\to\infty$ and taking also into account that $J_z=\Delta\sigma E$. This leads to

$$\Delta\sigma(\epsilon,E)=\frac{A_{AL}^{2D}}{r\sqrt{\pi}\epsilon^{3/2}}\int_0^\infty dx\sqrt{x}\exp\left\{-x\left[1+\left(\frac{E}{E^*(\epsilon)}x\right)^2\right]\right\}\times\vartheta_3\left[0,\exp\left(-\frac{x}{r^2\epsilon}\right)\right], \quad (9)$$

where $r=R/\xi(0)$, $x=16k_BT_{c0}\epsilon u/\pi\hbar$ is a dimensionless time variable, $A_{AL}^{2D}=e^2/16\hbar d$ is the Aslamazov-Larkin paraconductivity amplitude in two dimensions,[19] ϑ_3 is the elliptic theta function of index three defined by $\vartheta_3(u,q)=1+2\Sigma_{n=1}^{\infty}q^{n^2}\cos 2nu$, and $E^*(\epsilon)$ is an electrical field characteristic of each material given by,

$$E^*(\epsilon)=\frac{16\sqrt{3}k_BT_c}{\pi e\xi(0)}\cdot\epsilon^{3/2}=E_0^*\cdot\epsilon^{3/2}. \quad (10)$$

As it can be seen from Eq. (9), the behavior of the paraconductivity in a mesoscopic tube is only controlled by $r^2\epsilon$ and the ratio $E/E^*(\epsilon)$. The latter defines, in particular, the different electrical field regimes of the paraconductivity which we are going to discuss separately. Note first that for $E\ll E^*(\epsilon)$ Eq. (9) reduces to

$$\Delta\sigma(\epsilon)=\frac{A_{AL}^{2D}}{r\sqrt{\pi}\epsilon^{3/2}}\int_0^\infty dx\sqrt{x}e^{-x}\times\vartheta_3[0,e^{-x/r^2\epsilon}], \quad (11)$$

a field-independent expression that corresponds to the well-known ohmic regime of the paraconductivity, where the $\Delta\sigma$ behavior is entirely governed by $r^2\epsilon$. For instance, if $r^2\epsilon\ll 1$, the SF will be confined in the direction parallel to the axis of the tube and, subsequently, $\Delta\sigma$ will show a 1D behavior. In fact, Eq. (11) at $r^2\epsilon\ll 1$ simplifies to

$$\Delta\sigma_{AL}^{1D}(\epsilon)=\frac{A_{AL}^{2D}}{2r}\cdot\frac{1}{\epsilon^{3/2}}, \quad (12)$$

an expression that shows the critical exponent of $-3/2$ characteristic of the Aslamazov-Larkin contribution to the paraconductivity in 1D superconductors.[19,20] Contrary to that, for $r^2\epsilon\gg 1$ the 2D fluctuations may be created over the whole surface of the tube as it becomes topologically equivalent to a thin film. This is confirmed by the behavior of Eq. (11) in this limiting case, which is given by

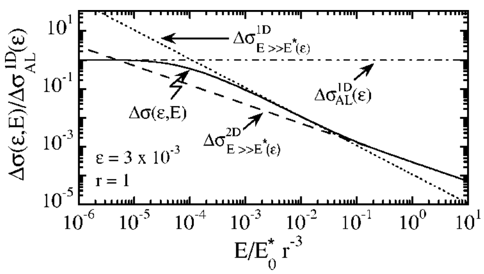

FIG. 1. An overview of electrical field dependence of $\Delta\sigma(\epsilon,E)$ [scaled by $\Delta\sigma_{AL}^{1D}(\epsilon)$] at $\epsilon=3\times10^{-3}$ in a mesoscopic tube with $r=1$, together with its different limit-behaviors. Deep inside the non-Ohmic region, for $E/E_0^* r^{-3}\gtrsim 5\times10^{-2}$, the paraconductivity shows an unexpected dimensionality increase from the 1D to the 2D fluctuational regimes. See main text for details.

$$\Delta\sigma_{AL}^{2D}(\epsilon)=A_{AL}^{2D}\cdot\frac{1}{\epsilon}, \tag{13}$$

in perfect agreement with the ohmic paraconductivity for 2D superconductors.[19,20]

Another important limiting case of Eq. (9) is the so-called high electrical field regime, which corresponds to $E\gg E^*(\epsilon)$. By applying this condition to Eq. (9) we obtain

$$\Delta\sigma_{E\gg E^*(\epsilon)}(E)=\frac{A_{AL}^{2D}}{r\sqrt{\pi}}\cdot\frac{E_0^*}{E}\int_0^\infty dy\sqrt{y}e^{-y^3}\times\vartheta_3\left[0,\exp\left[-\frac{y}{r^2}\cdot\left(\frac{E_0^*}{E}\right)^{2/3}\right]\right], \tag{14}$$

where $y=[E/E^*(\epsilon)]^{2/3}x$. As it can be seen from Eq. (14), the behavior of the paraconductivity at high applied electrical fields is temperature independent and controlled only by r^3E/E_0^*. In particular, note that for $E\ll E_0^* r^{-3}$ Eq. (14) reduces to

$$\Delta\sigma_{E\gg E^*(\epsilon)}^{1D}(E)=A_{AL}^{2D}\cdot\frac{1}{3r}\cdot\frac{E_0^*}{E}, \tag{15}$$

an expression that is proportional to $(E/E_0)^{-1}$ as expected at high-E in 1D superconductors.[13,17] On the other hand, the asymptotic limit of Eq. (14) at $E\gg E_0^* r^{-3}$ is given by

$$\Delta\sigma_{E\gg E^*(\epsilon)}^{2D}(E)=A_{AL}^{2D}\cdot\Gamma\left(\frac{4}{3}\right)\cdot\left(\frac{E_0^*}{E}\right)^{2/3}, \tag{16}$$

where Γ is the gamma function. Equation (16) coincides with the high electrical field limit of the paraconductivity in 2D superconductors.[13,17]

An interesting aspect of the paraconductivity results summarized above is the fact that close to T_{c0} a dimensional crossover may be induced deep inside the nonlinear region. Surprisingly, this crossover happens between the 1D and 2D cases, in spite of the fact that a depairing perturbation like an external field normally leads to a decrease of the fluctuations dimensionality.[19,20] To illustrate this nonintuitive feature of the $\Delta\sigma$ behavior in a mesoscopic tube, we first present in Fig. 1 an overview of the electrical field dependence of the paraconductivity at $\epsilon=3\times10^{-3}$ that can be calculated by using Eq. (9) (solid line). To visualize better the different $\Delta\sigma$ regimes, we have imposed $r=1$ and the vertical axis has been normalized by the 1D ohmic paraconductivity. At low applied electrical fields [for $E/E_0^* r^{-3}\lesssim 2\times10^{-5}$, which corresponds to $E/E^*(\epsilon)\lesssim 0.1$] the paraconductivity coincides with $\Delta\sigma_{AL}^{1D}(\epsilon)$ (dotted-dashed line). This behavior is just the one expected in this low ϵ-region, where the GL coherence length, $\xi(\epsilon)=\xi(0)\epsilon^{-1/2}$, is much larger than R and, due to that, the SFs in the tube are one-dimensional in nature. Note also that this dimensionality is preserved at moderate electrical fields, for $2\times10^{-5}\lesssim E/E_0^* r^{-3}\lesssim 5\times10^{-2}$, where $\Delta\sigma(\epsilon,E)$ deviates from the ohmic regime but tends to be proportional to E^{-1} as predicted for the high electrical field limit of the 1D paraconductivity (dotted line). However, when going deeper inside the nonohmic region (for $E/E_0^* r^{-3}\gtrsim 5\times10^{-2}$), the power law in E that describes the $\Delta\sigma$ behavior progressively changes to $E^{-2/3}$ in perfect agreement with $\Delta\sigma_{E\gg E^*(\epsilon)}^{2D}$ (dashed line). In fact, $\Delta\sigma(\epsilon,E)$ already shows a pure 2D behavior for $E/E_0^* r^{-3}\gtrsim 0.1$. We see, therefore, that if E is comparable to $E_0^* r^{-3}$, the paraconductivity will be two-dimensional even close to the transition, where the SFs are one-dimensional in the absence of external perturbations.

The results presented in Fig. 1 are in striking contrast with the E dependence of $\Delta\sigma$ expected for other superconducting systems showing a crossover between two different dimensionalities. This is the case, for instance, of the moderately anisotropic HTSC, where close to T_{c0} the electrical field decreases the fluctuation's dimensionality from the 3D to the 2D case.[15] The physical origin of these differences may be understood in terms of the qualitative description of the nonlinear effects considered in Ref. 13: The electrical field accelerates the Cooper pairs along the fluctuating areas, so that if E is high enough it can increase their kinetic energy up to the depairing energy. In this case, the Cooper pairs will be destroyed after covering distances much shorter than $\xi(\epsilon)$, thus leading to an effective decrease of the coherence length. In a moderately anisotropic HTSC this shrinkage of the order parameter tends to confine the motion of the Cooper pairs in the superconducting CuO_2 layers and, subsequently, $\Delta\sigma$ behaves as two-dimensional even very close to the transition.[15] However, in the case of a mesoscopic tube, the field-induced decrease in $\xi(\epsilon)$ may allow the creation of SFs over the whole sample's surface so that, even close to T_{c0}, $\Delta\sigma$ will show a 2D behavior.

From the experimental point of view, the most severe limitation to observe the nonlinear effects in $\Delta\sigma$ is the Joule heating caused, both in the sample and in the contacts, by the high electrical currents required to access the nonohmic region. In spite of these complications, which strongly depend on sample's geometry, the high-E regime of the paraconductivity was observed 30 years ago by using dc-currents in Al thin films.[14] In these experiments it was already possible to reach electrical fields as high as $E/E_0^*\sim 10^{-2}$, where E_0^* is of the order of 10^5 mV cm^{-1} in Al. Indeed, these E values also allow us to reach the nonlinear region in Al tubes which, as mentioned before, are topologically equivalent to thin films. However, to observe the field-induced dimensional crossover close to T_{c0}, it will be necessary to use electrical fields amplitudes approximately one order of magnitude larger than those used in Ref. 14 (i.e., $E/E_0^*\simeq 10^{-1}$, which in Al corresponds to $E \simeq 10^4$ mV cm^{-1}). The latter seems to be well within the present experimental capabilities, which include high pulsed current sources that may be particularly suitable for this type of measurements. The high electrical field experiments could be also carried out on carbon nanotubes where, recently, evidences of an intrinsic superconducting behavior were found which, deeply affected by order parameter fluctuations, may vary between the 1D and 2D cases.[21] The reported T_{c0} and $\xi(0)$ values in these materials are within $0.5\ \mathrm{K} \lesssim T_{c0} \lesssim 15\ \mathrm{K}$ and, respectively, $40\ \text{Å} \lesssim \xi(0) \lesssim 3000\ \text{Å}$, which leads to 10^5 mV/cm $\lesssim E_0^* 10^7$ mV/cm. This means that, if the GL theory is applicable to describe the superconducting state of carbon nanotubes, the nonohmic regime could be achieved in these materials. Therefore, the experimental observation of the nonlinear effects on $\Delta\sigma$ and their subsequent analysis in terms of the results presented here would help to clarify the origin of the superconducting state in these systems. In such a case, the difference $\Delta\sigma(\epsilon) - \Delta\sigma(\epsilon, E)$ could be also used to detect and investigate the superconducting effects above T_{c0} in carbon nanotubes while, at the same time, avoiding several experimental difficulties (as, for instance, those related to the estimate of the normal state contribution to the resistivity).

To summarize, by using a generalized version of the time-dependent Ginzburg-Landau theory, we have calculated the paraconductivity of a mesoscopic superconducting tube beyond the conventional ohmic approximation. Among other dimensionality effects for the SF, our results predict at high-E an unexpected 1D to 2D crossover close to T_{c0} which may be attributed to an effective shrinkage of the superconducting coherence length with the electrical field. These nonlinear effects in the paraconductivity can be observed in Al tubes and, also, they might be particularly useful for a better understanding of the superconducting behavior of carbon nanotubes. Other aspects of the nonohmic behavior of the paraconductivity in a mesoscopic tube deserve further analysis. For instance, it will be interesting to study whether the decrease of ξ with E may induce the appearance of the so-called short-wavelength effects, which dominate the behavior of the SF when the coherence length approaches $\xi(0)$.[19,20] If so, the extension of the GL formalisms to the high-E region would also require the introduction of a total energy cutoff in the fluctuations spectrum which, in particular, takes into account the uncertainty principle limitations to the shrinkage of the superconducting wave function.[22]

This work has been supported by the Belgian Interuniversity Attraction Poles, the Research Fund K.U. Leuven GOA/2004/02, the Fund for Scientific Research Flanders (FWO) and the ESF "VORTEX" programmes. C.C. acknowledges financial support from Fudación Ramón Areces (Spain) through Post-Doctoral Studentship Grants and from CICYT MAT2004-04364 (Spain).

---

*On leave from Laboratorio de Baixas Temperaturas e Superconductividade, Departamento de Física da Materia Condensada, Universidade de Santiago de Compostela, E-15782 Santiago de Compostela, Spain. Electronic address: Carlos.CarballeiraRomero@fys.kuleuven.ac.be

[1] For a review, see, e.g., V. V. Moshchalkov *et al.*, in *Handbook of Nanostructured Materials and Nanotechnology*, edited by H. S. Naiwa (Academic, San Diego, 1999), Vol. 3, Chap. 9, p. 451. See also V. V. Moshchalkov *et al.*, Nature (London) **373**, 319 (1995).

[2] L. F. Chibotaru, A. Ceulemans, V. Bruyndoncx, and V. V. Moshchalkov, Nature (London) **408**, 833 (2000).

[3] V. V. Moshchalkov, L. Gielen, G. Neuttiens, C. Van Haesendonck, and Y. Bruynseraede, Phys. Rev. B **49**, 15 412 (1994).

[4] J. Jorritsma and J. A. Mydosh, Phys. Rev. B **62**, 9703 (2000).

[5] A. Bezryadin, C. N. Lau, and M. Tinkham, Nature (London) **404**, 971 (2000); C. N. Lau, N. Markovic, M. Bockrath, A. Bezryadin, and M. Tinkham, Phys. Rev. Lett. **87**, 217003 (2001).

[6] K. A. Matveev, A. I. Larkin, and L. I. Glazman, Phys. Rev. Lett. **89**, 096802 (2002).

[7] D. Y. Vodolazov, B. J. Baelus, and F. M. Peeters, Phys. Rev. B **66**, 054531 (2002).

[8] L. P. Lévy, G. Dolan, J. Dunsmuir, and H. Bouchiat, Phys. Rev. Lett. **64**, 2074 (1990).

[9] L. I. Glazman, F. W. J. Hekking, and A. Zyuzin, Phys. Rev. B **46**, 9074 (1992).

[10] N. E. Israeloff, F. Yu, and A. M. Goldman, Phys. Rev. Lett. **71**, 2130 (1993).

[11] V. Ambegaokar and U. Eckern, Phys. Rev. Lett. **65**, 381 (1990); Europhys. Lett. **13**, 733 (1990); Phys. Rev. B **44**, 10 358 (1991).

[12] A. I. Buzdin and A. A. Varlamov, Phys. Rev. Lett. **89**, 076601 (2002).

[13] J. P. Hurault, Phys. Rev. **179**, 494 (1969).

[14] K. Kajimura and N. Mikoshiba, Phys. Rev. Lett. **26**, 1233 (1971); K. Kajimura, N. Mikoshiba, and K. Yamaji, Phys. Rev. B **4**, 209 (1971).

[15] A. A. Varlamov and L. Reggiani, Phys. Rev. B **45**, 1060 (1992).

[16] D. V. Livanov and A. A. Varlamov, Phys. Rev. B **66**, 104522 (2002).

[17] A. Schmid, Phys. Rev. **180**, 527 (1969).

[18] Instead, in Ref. 16 the ohmic paraconductivity is calculated by using $V = \pi R^2 L_z$. This leads to $\Delta\sigma$-expressions that, in particular, do not depend on the tube thickness. As a consequence, these calculations do not reproduce the expected topological equivalence between a tube with $R \gg \xi(\epsilon)$ and a thin film.

[19] See, e.g., M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1996), Chap. 8.

[20] See, e.g., W. J. Skocpol and M. Tinkham, Rep. Prog. Phys. **38**, 1094 (1975).

[21] Z. K. Tang *et al.*, Science **292**, 2462 (2001); M. Kociak *et al.*, Phys. Rev. Lett. **86**, 2416 (2001).

[22] F. Vidal, C. Carballeira, S. R. Currás, J. Mosqueira, M. V. Ramallo, J. A. Veira, and J. Viña, Europhys. Lett. **59**, 754 (2002). See also, C. Carballeira, S. R. Currás, J. Viña, J. A. Veira, M. V. Ramallo, and F. Vidal, Phys. Rev. B **63**, 144515 (2001); J. Mosqueira, C. Carballeira, and F. Vidal, Phys. Rev. Lett. **87**, 167009 (2001).