Reply to "Comment on 'Lifetime of metastable states in resonant tunneling structures' "

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We find the true low bound of the size of a critical nucleus of the low-current state in double-barrier resonant tunneling structures. By changing the structural parameters of the device, the critical nucleus size can be made as low as the typical distance between electrons in the well. We thus reaffirm that the decay of the metastable upper current state via the nucleation mechanism can be achieved in resonant tunneling structures.

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The authors of the Comment[1] on our earlier paper[2] estimate the critical size of the initial nucleus of the low-current state in a double-barrier resonant tunneling structure (DBRTS). Their estimate is based on the theory of deterministic switching front propagation. They claim that for realistic parameters of DBRTS, the size of the critical nucleus r_{cr} is comparable with the sample size. Even when r_{cr} is smaller than the size of the sample, it is still macroscopic and, therefore, is too large for the switching to occur, as the switching time should increase exponentially with the area of the nucleus. They subsequently conclude that the decay of the metastable state via the nucleation mechanism is doubtful.

The conclusion that r_{cr} is macroscopically large is based on the numerical estimate rather than on physical argument. Therefore, we first discuss this numerical estimate and then give our physical estimate for r_{cr}. The authors of the Comment define the macroscopic size as "e.g., >10 μm." They claim that r_{cr} is given by the front width W, which is found using the theory of deterministic switching front propagation. This theory describes the deterministic growth of the nucleus of the new phase well after the critical size has been reached. On the other hand, the formation of the critical nucleus is not deterministic, but a stochastic process that we considered in our paper.[2] At the formation stage the theory of switching front propagation can be expected to provide an order of magnitude estimate for r_{cr} such as Eq. (2) in the Comment. Under certain assumptions this estimate can be reproduced from the theory of stochastic switching[2] (see below).

Substituting the values of the parameters quoted by the authors, namely $\mu=10^5$ cm^2/V s, $E_F=10$ meV, and $\Gamma_L=1$ meV, into Eq. (2) of the Comment, we find the numerical value of r_{cr} to be 0.26 μm. This estimate disagrees with the result of 10 μm reported by the authors of the Comment. The increase by a factor of 40 in the minimal r_{cr} is claimed on the basis of numerical simulations (Ref. 12 of the Comment). In our opinion, the discussion in the Comment and references therein does not convincingly explain the origin of the additional large numerical coefficient. It is conceivable that the result of 10 μm is a correct estimate of the front width W for the nuclei of the size well above the critical size. However, for these nucleus sizes, the front width W is obviously not r_{cr}. Thus in our opinion Eq. (2) of the Comment gives a more reliable estimate $r_{cr}=0.26$ μm than the subsequent numerical estimate $r_{cr}=10$ μm.

To further support our point of view, we estimate the true low bound of the critical nucleus size based on the theory of stochastic switching.[2] In Ref. 2 the radius of a critical nucleus is found to be $r_0=\sqrt{\eta}(\alpha\gamma)^{-1/4}$, where the parameters α, γ are given by Eqs. (4c) and (4d), and η is found in the paragraph preceding Eq. (15) of Ref. 2. The parameter r_{cr} introduced in the Comment coincides with r_0 at $e(V_{th}-V)\sim E_F$. Under these conditions we find

$$r_{cr}\sim\sqrt{\frac{\hbar\sigma T_L}{e^2 n T_R^2}}. \tag{1}$$

Here σ is the conductivity in the well, T_L and T_R are the transmission coefficients of the left and right barriers, and n is the electron density in the well. To obtain Eq. (1) we also assumed that the energy of the level in the well E_0 is of the order of E_F and estimated the capacitance of the device per unit area as $C\sim e^2\nu$, where ν is the two-dimensional density of states in the well. Note that the estimate (1) is consistent with Eq. (2) of the Comment. This can be shown by using the standard expressions for the mobility $\mu=\sigma/en$, the level widths $\Gamma_{L,R}\sim E_F T_{L,R}$, and by taking $\Gamma_L\sim\Gamma_R$ as implicitly assumed by the authors of the Comment.

To obtain the low bound of r_{cr}, the transmission coefficients of the barriers should be close to their maximum value $T_{L,R}\sim 1$, and the metallic conductivity should take the lowest possible value $\sigma\sim e^2/h$. Under these conditions we find $r_{cr}\sim d\sim 20$ nm, where $d\sim 1/\sqrt{n}$ is the typical distance between electrons in the well.

To explore the limits of applicability of our approach, let us estimate the exponent F of the mean switching time $\tau\propto e^F$. It is given by Eq. (16) of Ref. 2, and can be expressed as

$$F\sim\frac{1}{T_R}\frac{h\sigma}{e^2}\frac{e(V_{th}-V)}{E_F}. \tag{2}$$

Substituting $\sigma\sim e^2/h$, $T_R\sim 1$, and $e(V_{th}-V)\sim E_F$ into this result, we find $F\sim 1$. For more realistic structural parameters $\sigma\sim 10^2 e^2/h$ and $T_R\sim 0.2$, which correspond to $\mu\sim 10^5$ cm^2/V s, $E_F\sim 10$ meV, and $\Gamma_L\sim 1$ meV used by the authors of the Comment, we obtain $F\sim 500$. This value of F is too large for τ to be measured experimentally. To reduce it one has to tune the voltage closer to V_{th}, as we discussed in Ref. 2. If we take $e(V_{th}-V)\sim 0.02E_F$, we find $F\sim 10$. When

the voltage V approaches the threshold V_{th}, the critical size r_0 increases, albeit very slowly, $r_0 \propto [E_F/e(V_{th}-V)]^{1/4}$. In the above example $r_0 \sim 60d \sim 1$ μm.

The numerical coefficient in the exponent F in principle can be large. In this case, the increase of F should be compensated by tuning the voltage closer to the threshold value. Note that even for $e(V_{th}-V) \sim 10^{-4}E_F$ the critical nucleus size is of order $200d \sim 4$ μm for $\sigma \sim 10^2 e^2/h$ and $T_{L,R} \sim 0.2$. This r_0 is still small compared to the size of the sample.[3,4] Note that although the size of the critical nucleus approaches the "macroscopic" value 10 μm (as defined by the authors of the Comment), the measurement of the switchng time $\tau \propto e^F$ in this regime is possible because $F \to 0$ at $V \to V_{th}$. However, we agree with the authors of the Comment that for small $(V_{th}-V)$ the inaccuracy of the applied voltage may become an important issue.

To summarize, we have found that the true low bound of the size of the critical nucleus is the typical distance between electrons. We conclude that the switching via the nucleation mechanism driven by shot noise can be achieved with the proper choice of the parameters of DBRTS and applied bias.

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