Anisotropic conductivity tensor on a half-filled high Landau level

I. S. Burmistrov
Landau Institute for Theoretical Physics, Kosygina str. 2, 117940 Moscow, Russia and
Institute for Theoretical Physics, University of Amsterdam, Valckenierstraat 65, 1018XE Amsterdam, The Netherlands
(Received 8 May 2004; revised manuscript received 4 November 2004; published 21 January 2005)

We study two-dimensional interacting electrons in a weak perpendicular magnetic field with the filling factor $\nu \gg 1$ and in the presence of a quenched disorder. As it is known, the unidirectional charge density wave state can exist near a half-filled high Landau level at low temperatures if disorder is weak enough. We show that the existence of the unidirectional charge density wave state at temperature $T < T_c$ where T_c is the transition temperature leads to the anisotropic conductivity tensor. We find that the anisotropic part of conductivity tensor is proportional to $(T_c - T)/T_c$ below the transition. The order parameter fluctuations wash out the mean-field cusp at $T = T_c$ and the conductivity tensor becomes anisotropic even above the mean-field transition temperature T_c.

DOI: 10.1103/PhysRevB.71.035331 PACS number(s): 72.10.−d

I. INTRODUCTION

A two-dimensional electron gas (2DEG) in a perpendicular magnetic field (H) has remained a subject of intensive studies, both theoretical and experimental, for several decades.[1] Recently the phenomenon of sharp anisotropy of magnetoresistance near half-filled Landau levels with Landau level index $N=2,3,4$ at low temperatures has been discovered.[2,3] Right away the magnetoresistance anisotropy has been related with the possible existence of unidirectional charge density wave (UCDW) state near half-filling of a high Landau level theoretically predicted[4–6] in the absence of disorder by the mean-field treatment in the limit $N \gg 1$. However it is well-known that the UCDW state (smectic) is destroyed by thermal fluctuations in two dimensions in the absence of crystal field anisotropy.[7] At zero temperature the UCDW state can exist only on high Landau levels, $N \gg 1$, whereas at moderate values of N a nematic phase should appear.[8] At nonzero temperatures (T) the nematic phase is stable against fluctuations and the isotropic-to-nematic transition may lead to anisotropic magnetoresistance.[9] This scenario is supported by experimental measurements[10] of magnetoresistance dependence on in-plane magnetic fields near half-filled Landau levels with Landau level index $N=2,3,4$. However, in spite of enormous efforts,[11] a microscopical theory for the temperature dependence of magnetoresistance on Landau levels with moderate N is absent so far.

In practice the UCDW state can exist on lengths L smaller than lengthscales $\xi_\perp$ and $\xi_\parallel$ at which translational order is destroyed in directions perpendicular and parallel to a stripe, respectively. The microscopic analysis of smectic state[12] yields that $\xi_{\perp,\perp}$ increases as a power law with growing of N and the temperature. In the presence of disorder the UCDW state is destroyed on a scale of the order of the Larkin length[13] that grows with N as a power law also.[14] One can therefore expect to find the UCDW state on high Landau levels (with $N \gg 1$) in samples of small size.

If a random potential created by impurities near 2DEG is *weak* then the Landau level broadening $1/\tau$ is much less than the spacing ω_H between Landau levels, $1/\tau \ll \omega_H$. Here the ω_H is cyclotron frequency $\omega_H = eH/m$ with e and m being the electron charge and band mass, respectively (we use the units with $\hbar = 1$, $c = 1$, and $k_B = 1$). In the framework of Hartree-Fock analysis one can show that there is mean-field isotropic-to-UCDW transition at some temperature $T_c = T_c(1/\tau)$ on a half-filled high Landau level.[16] The disorder decreases the transition temperature such that there is the critical value of the Landau level broadening $1/\tau_c = 8T_0/\pi$ where $T_0 = T_c(1/\tau = 0)$ at which the UCDW state disappears (see Fig. 1). Surprisingly, the disorder does not affect the period of the UCDW state that remains of the order of cyclotron radius $R_c = \sqrt{2N+1}\, l_H$ where $l_H = 1/\sqrt{m\omega_H}$ denotes the magnetic field length.[4–6] It has been shown[16] that the correction due to Gaussian fluctuations of the UCDW order parameter (weak crystallization) to the mean-field value of T_c is of the order of $\delta T_c \propto T_c N^{-2/3} \ll T_c$. In accordance with general expectations about the effect of disorder on ordering in two dimensions, these fluctuations destroy the UCDW state on the lengthscale $\xi = R_c g(1/4\pi T\tau) N^{1/3}$ where $g(x)$ monotonically increases from the value 0.55 at $x=0$ to infinity when x tends to infinity.[17] Thus, the effect of Gaussian fluctuations

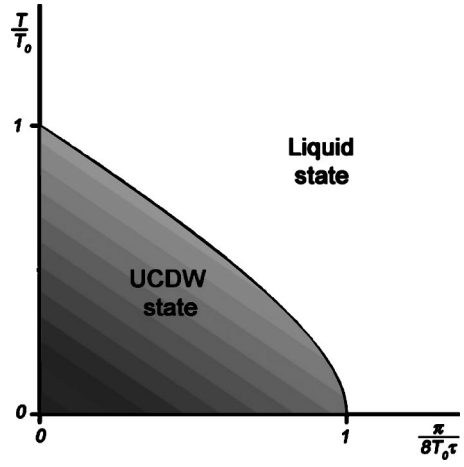

FIG. 1. Mean-field phase diagram for electrons on half-filled high Landau levels, $N \gg 1$.

of the UCDW order parameter is qualitatively the same as in more elaborate treatments.[12,14] The fluctuations about the mean-field UCDW state are negligible on high Landau levels, $N \gg 1$, for samples of size $L \ll \xi$.

The main objective of the paper is to present the analysis of temperature dependence of the conductivity tensor on a half-filled high Landau level near the isotropic-to-UCDW transition ($|T_c - T| \ll T_c$) where the expansion in the UCDW order parameter is legitimate. The effect of the order parameter fluctuations that are enhanced near the phase transition on the conductivity tensor both below and above the transition temperature T_c will be investigated also.

One of the main results of the paper is the fact that the conductivity tensor σ_{ab} (we measure conductivity in units of e^2/h) of two-dimensional electrons on a high Landau level in the UCDW state acquires an anisotropic part proportional to temperature deviation from T_c. Such temperature dependence can be understood in the following way. Since the UCDW state appears at nonzero wave vector the first nonvanishing contribution to the conductivity tensor due to the UCDW induced potential is of the second order in the UCDW order parameter. It is proportional to $\sqrt{(T_c - T)/T_c}$ in the Landau theory. Hence the conductivity tensor in the UCDW state should acquire corrections, isotropic as well as anisotropic, proportional to $(T_c - T)/T_c$. Other principal results of the present paper is that in the vicinity of the isotropic-to-UCDW transition in the presence of an anisotropy with typical energy E_A there are additional anisotropic contributions to the conductivity tensor proportional to $(E_A/T_c)(|T_c - T|/T_c)^{-3/2}$ that we shall refer to as fluctuational. In general, in the presence of crystal field anisotropy E_A the conductivity tensor has an anisotropic part at all temperatures. Compared to this background anisotropy of the conductivity tensor the anisotropic correction due to fluctuations becomes anomalously enhanced in the vicinity of T_c. We mention that it is analogous to fluctuational contribution to the conductivity of a normal metal due to superconducting paring above the critical temperature.[18]

We start out with an introduction to the formalism that mainly follows one introduced in the previous paper.[16] In order to consider the transport properties of electrons on the Nth high Landau level we integrate electron degrees of freedom on all Landau levels except the $(N-1)$th, Nth, and $(N+1)$th levels (Sec. II). Next we prove that scattering electrons from the Nth Landau level to the $(N \pm 1)$th Landau levels by the UCDW induced potential results in corrections of the order of $\mathcal{O}(N^{-1})$ and, consequently, can be neglected (Sec. II F). The conductivity tensor in the UCDW state is evaluated in Sec. III. Effect of the UCDW order parameter fluctuations on the conductivity tensor is investigated in Sec. IV. In Sec. V the results obtained are discussed in relation with recent experiments.[2,3] We end the paper with Sec. VI.

Some of the results of the present paper have been published in a brief form in Ref. 19.

II. FORMALISM

A. Introduction

To start out consider the system of two-dimensional interacting electrons in the presence of a random potential $V_{\rm dis}(\mathbf{r})$ and a perpendicular magnetic field H. The conventional parameter that characterizes the strength of the Coulomb interaction is $r_s = \sqrt{2}e^2/\varepsilon R_c \omega_H$ with the ε being the dielectric constant of a media. We assume that the Coulomb interaction between the electrons is weak

$$r_s \ll 1, \tag{1}$$

and the magnetic field obeys the condition

$$N \gg r_s^{-1}, \tag{2}$$

where the Landau level index $N = [\nu/2]$ is the integer part of half of filling factor ν. In addition, we assume that the Landau level broadening $1/\tau$ is not too small,[17] $1/\tau \gg \omega_H N^{-1} \ln \sqrt{2} r_s N$. In this case it is possible to construct an effective field theory for electrons on the highest partially filled Landau level by integrating out all other degrees of freedom.[20,21]

Also we consider the case when the Landau levels are spin resolved that occurs according to Ref. 20 if $1/\tau \ll \Delta_{\rm ex} = (r_s \omega_H/\pi\sqrt{2}) \ln 2\sqrt{2}/r_s$. Therefore, the Landau level broadening should be restricted from below and from above as

$$\frac{\omega_H}{N} \ln \sqrt{2} r_s N \ll \frac{1}{\tau} \ll \frac{r_s \omega_H}{\pi\sqrt{2}} \ln \frac{2\sqrt{2}}{r_s}. \tag{3}$$

Throughout the paper we characterize the UCDW state by the order parameter $\Delta(\mathbf{Q})$. All calculations are performed under the assumption

$$N \gg r_s^{-2}. \tag{4}$$

In this case the Hartree-Fock approximation is well-justified.[6] As it was shown the corrections to the Hartree-Fock approximation are controlled by the small parameter $a_B/l_H = 1/N r_s^2 \ll 1$, where $a_B = \varepsilon/me^2$ stands for the Bohr radius.

Thermodynamic potential of the system in hand can be written as

$$\Omega = -\frac{T}{N_r} \int \mathcal{D}[\overline{\psi}, \psi] \int \mathcal{D}[V_{\rm dis}] \mathcal{P}[V_{\rm dis}] \exp \mathcal{S}[\overline{\psi}, \psi, V_{\rm dis}], \tag{5}$$

where action $\mathcal{S}[\overline{\psi}, \psi, V_{\rm dis}]$ is written in Matsubara representation

$$\begin{aligned} \mathcal{S} = \int d\mathbf{r} \sum_{\alpha, \omega_n, \omega_m} \overline{\psi^{\alpha,\sigma}_{\omega_n}}(\mathbf{r}) \{[i\omega_n + \mu - V_{\rm dis}(\mathbf{r})]\delta_{nm} - \hat{\mathcal{H}}\} \\ \times \psi^{\alpha,\sigma}_{\omega_m}(\mathbf{r}) - \frac{T}{2} \sum_{\omega_n, \omega_m, \nu_l} \int d\mathbf{r} d\mathbf{r}' \overline{\psi^{\alpha,\sigma}_{\omega_n}}(\mathbf{r}) \psi^{\alpha,\sigma}_{\omega_n - \nu_l}(\mathbf{r}) \\ \times U_0(\mathbf{r} - \mathbf{r}') \overline{\psi^{\alpha,\sigma'}_{\omega_m}}(\mathbf{r}) \psi^{\alpha,\sigma'}_{\omega_m + \nu_l}(\mathbf{r}'). \end{aligned} \tag{6}$$

Here, $\psi^{\alpha,\sigma}_{\omega_n}(\mathbf{r})$ and $\overline{\psi^{\alpha,\sigma}_{\omega_n}}(\mathbf{r})$ are annihilation and creation electron operators. T stands for temperature, μ chemical potential, σ and σ' spin indices, and $\omega_n = \pi T(2n+1)$ fermionic frequency whereas $\nu_n = 2\pi T n$ bosonic one. Matrix $\hat{\mathcal{H}}$ is defined as

$$\hat{\mathcal{H}} = \sum_{\alpha \nu_n} \mathcal{H}(\nu_n) I_n^{\alpha}, \tag{7}$$

with matrices

$$(I_n^{\alpha})_{kl}^{\beta\gamma} = \delta^{\alpha\beta}\delta^{\alpha\gamma}\delta_{k-l,n} \tag{8}$$

being $U(1)$ generators. One-particle hamiltonian $\mathcal{H}$ describes a two-dimensional electron in constant perpendicular magnetic field $H = \epsilon_{ab}\partial_a A_b$ and in time-dependent magnetic field with vector-potential $\mathbf{a}$,

$$\mathcal{H} = \frac{1}{2m}(-i\nabla - e\mathbf{A} - e\mathbf{a})^2. \tag{9}$$

As usual, we assume the white-noise distribution for the random potential

$$\mathcal{P}[V_{\mathrm{dis}}(\mathbf{r})] = \frac{1}{\sqrt{2\pi g}} \exp\left(-\frac{1}{2g}\int d\mathbf{r} V_{\mathrm{dis}}^2(\mathbf{r})\right). \tag{10}$$

In order to average over disorder we introduce N_r replicated copies of the system[22] labeled by the replica indices $\alpha = 1, \ldots, N_r$.

It is convenient to rewrite one-particle Hamiltonian (9) with the help of covariant derivative

$$\mathbf{D} = \nabla - ie\mathbf{A}, \tag{11}$$

in order to extract the time-dependent vector potential $\mathbf{a}(\nu_n)$

$$\mathcal{H} = -\frac{1}{2m}\mathbf{D}^2 + K(\nu_n), \tag{12}$$

$$K(\nu_n) = -\frac{e}{m}\mathbf{a}(\nu_n)\mathbf{D} + \frac{e^2}{2m}\sum_{\nu_m} \mathbf{a}(\nu_{n-m})\mathbf{a}(\nu_m).$$

B. Effective action of "three-level" model

To investigate the thermodynamic properties of electrons on the Nth Landau level one can integrate out electrons on all other Landau levels.[20] However, to find conductivity tensor projection on the single Nth Landau level is not appropriate because of covariant derivative $\mathbf{D}$ has nonzero matrix elements only for transitions between adjacent Landau levels. It is necessary therefore to consider not only the Nth Landau level alone but two adjacent ones, the $(N-1)$th and $(N+1)$th Landau levels.

Extending the projection to the Nth Landau level only of Refs. 20 and 21, we obtain effective action for electrons on the $(N-1)$th, Nth, and $(N+1)$th Landau level as follows:

$$\begin{aligned}\mathcal{S} = \int d\mathbf{r} \sum_{\alpha,\omega_n,\omega_m} \overline{\psi_{\omega_n}^{\alpha,\sigma}}(\mathbf{r})\{[i\omega_n + \mu - V_{\mathrm{dis}}(\mathbf{r})]\delta_{nm} - \hat{\mathcal{H}}\} \\ \times \psi_{\omega_m}^{\alpha,\sigma}(\mathbf{r}) - \frac{T}{2}\sum_{\omega_n,\omega_m,\nu_l} \int d\mathbf{r}d\mathbf{r}' \overline{\psi_{\omega_n}^{\alpha,\sigma}}(\mathbf{r})\psi_{\omega_n-\nu_l}^{\alpha,\sigma}(\mathbf{r}) \\ \times U_{\mathrm{scr}}(\mathbf{r}-\mathbf{r}')\overline{\psi_{\omega_m}^{\alpha,\sigma'}}(\mathbf{r})\psi_{\omega_m+\nu_l}^{\alpha,\sigma'}(\mathbf{r}').\end{aligned} \tag{13}$$

Here $\psi_{\omega_n}^{\alpha,\sigma}(\mathbf{r})$ and $\overline{\psi_{\omega_n}^{\alpha,\sigma}}(\mathbf{r})$ are annihilation and creation operators of an electron on the $(N-1)$th, Nth, and $(N+1)$th Landau levels

$$\psi_{\omega_n}^{\alpha,\sigma}(\mathbf{r}) = \sum_{p=N-1}^{N+1} \psi_{p\omega_n}^{\alpha,\sigma}(\mathbf{r}), \quad \overline{\psi_{\omega_n}^{\alpha,\sigma}}(\mathbf{r}) = \sum_{p=N-1}^{N+1} \overline{\psi_{p\omega_n}^{\alpha,\sigma}}(\mathbf{r}). \tag{14}$$

The screened electron-electron interaction $U_{\mathrm{scr}}(\mathbf{r})$ has the following Fourier transform:

$$\begin{aligned}U_{\mathrm{scr}}(q) = \frac{2\pi e^2}{\varepsilon q}\Bigg[1 + \frac{2}{qa_B}\left(1 - \frac{\pi}{6\omega_H\tau}\right) \\ \times [1 - \mathcal{J}_0^2(qR_c) - 2\mathcal{J}_1^2(qR_c)]\Bigg]^{-1}.\end{aligned} \tag{15}$$

It is different from one obtained in Refs. 21 and 23. The reason for that is exclusion of contributions from the $(N-1)$th and $(N+1)$th Landau level from the polarization operator.

Effective action (13) was obtained under assumptions (1)–(3) discussed above. Hereafter, for reasons to be explained shortly we neglect small correction $\pi/(6\omega_H\tau) \ll 1$ in the screened electron-electron interaction (15).

C. Hartree-Fock decoupling

Effective action (13) involves electron states with spin-up and spin-down projections. Electron-electron interaction can flip electron spin. Therefore, a charge density wave (CDW) state is characterized by an order parameter $\Delta_{p_1p_2}^{\sigma_1\sigma_2}(\mathbf{Q})$ that is matrix in the space of Landau level and spin indices. However, as it will be clear from discussion below, if the Landau levels are spin resolved, i.e., $\Delta_{\mathrm{ex}} \gg \max\{T, \tau^{-1}\}$, the charge density wave state creates only on the Nth Landau level with certain spin projection. Then Landau levels with different spin projection become completely separated and can be ignored. Thus, we can consider the charge density wave order parameter to be matrix only in the space of Landau level indices. It is related with distortion of electron density on the $(N-1)$th, Nth, and $(N+1)$th Landau levels as

$$\langle\rho(\mathbf{q})\rangle = Sn_L \sum_{p_1,p_2=N-1}^{N+1} \Delta_{p_1p_2}(\mathbf{q})F_{p_1p_2}(\mathbf{q}), \tag{16}$$

where S stands for the area of two-dimensional electron gas and form-factor $F_{p_1p_2}(\mathbf{q})$ is defined as

$$F_{p_1p_2}(\mathbf{q}) = n_L^{-1}\sum_k \phi_{p_1k}^*(0)\phi_{p_2k}(\mathbf{q}l_H^2)\exp\left(\frac{i}{2}q_xq_yl_H^2\right). \tag{17}$$

Here $n_L = 1/2\pi l_H^2$ and $\phi_{pk}(\mathbf{r})$ are the eigenfunctions of the free electron Hamiltonian with k being pseudomomentum.

After Hartree-Fock decoupling of interaction term in effective action (13) (see Ref. 24), we obtain

$$\mathcal{S} = -\frac{N_r \Omega_\Delta}{T} + \int d\mathbf{r} \sum_{p_1,p_2} \sum_{\alpha,\omega_n,\omega_m} \overline{\psi^\alpha_{p_1\omega_n}}(\mathbf{r}) \times [(i\omega_n + \mu - V_{\rm dis}(\mathbf{r}))\delta_{nm} - \hat{\mathcal{H}} + \lambda_{p_1p_2}(\mathbf{r})]\psi^\alpha_{p_2\omega_m}(\mathbf{r}), \tag{18}$$

where

$$\Omega_\Delta = \frac{n_L S^2}{2} \sum_{p_i} \int \frac{d\mathbf{q}}{(2\pi)^2} U_{p_1p_2p_3p_4}(\mathbf{q}) \Delta_{p_1p_4}(\mathbf{q}) \Delta_{p_3p_2}(-\mathbf{q}). \tag{19}$$

Potential $\lambda_{p_1p_2}(\mathbf{r})$ in Eq. (18) appears as a consequence of distortion of uniform electron density by the charge density wave and is related with the order parameter as

$$\lambda_{p_1p_2}(\mathbf{q}) = S \sum_{p_3p_4} \frac{U_{p_3p_4p_1p_2}(\mathbf{q})}{F_{p_1p_2}(-\mathbf{q})} \Delta_{p_3p_4}(\mathbf{q}), \tag{20}$$

where $U_{p_1p_2p_3p_4}(\mathbf{q})$ denotes the generalized Hartree-Fock potential

$$U_{p_1p_2p_3p_4}(\mathbf{q}) = -n_L \Bigg[U_{\rm scr}(\mathbf{q}) F_{p_1p_2}(\mathbf{q}) F_{p_3p_4}(-\mathbf{q}) - \int \frac{d\mathbf{p}}{(2\pi)^2 n_L} \times e^{-i\mathbf{q}p l_H^2} U_{\rm scr}(\mathbf{p}) F_{p_1p_4}(\mathbf{p}) F_{p_3p_2}(-\mathbf{p}) \Bigg]. \tag{21}$$

The Hartree-Fock approximation that we use is justified under condition (4) as we have mentioned above.

D. Average over disorder

After standard average over the random potential $V_{\rm dis}(\mathbf{r})$ (see Ref. 25), effective action (18) becomes

$$\mathcal{S} = -\frac{N_r \Omega_\Delta}{T} + \int d\mathbf{r} \psi^\dagger(\mathbf{r})(i\omega + \mu - \hat{\mathcal{H}} + \check{\lambda} + iQ)\psi(\mathbf{r}) - \frac{1}{2g} \int d\mathbf{r} \ {\rm tr}\, Q^2(\mathbf{r}), \tag{22}$$

where we introduce new field $Q(\mathbf{r})$, that is unitary matrix in Matsubara and replica spaces. For convenience we use the following notation:

$$\psi^\dagger \check{\lambda} \psi = \sum_{p_1p_2} \sum_{\alpha,\omega_n} \overline{\psi^\alpha_{p_1\omega_n}}(\mathbf{r}) \lambda_{p_1p_2}(\mathbf{r}) \psi^\alpha_{p_2\omega_n}(\mathbf{r}). \tag{23}$$

Let us recall that action (22) at zero temperature, i.e., for $\omega_n \to 0$, and in the absence of the induced potential $\check{\lambda}(\mathbf{r})$ and the time-dependent vector-potential $\mathbf{a}$ has the following saddle-point solution:

$$Q_{\rm sp} = V^{-1} P_{\rm sp} V, \quad (P_{\rm sp})^{\alpha\beta}_{nm} = P^n_{\rm sp} \delta_{nm} \delta^{\alpha\beta}, \tag{24}$$

where V is arbitrary global unitary rotation and $P^n_{\rm sp}$ obeys the equation

$$P^n_{\rm sp} = igG^{\omega_n}(\mathbf{r},\mathbf{r}), \quad G^{\omega_n}(\mathbf{r},\mathbf{r}') = \sum_{p=N-1}^{N+1} G^{\omega_n}_p(\mathbf{r},\mathbf{r}'). \tag{25}$$

Here Green's function $G^{\omega_n}_p(\mathbf{r},\mathbf{r}')$ is as follows:

$$G^n_p(\mathbf{r},\mathbf{r}') = \sum_k \phi^*_{pk}(\mathbf{r}) G_p(\omega_n) \phi_{pk}(\mathbf{r}'), \tag{26}$$

$$G_p(\omega_n) = [i\omega_n + \mu_N + \epsilon_N - \epsilon_p + iP^n_{\rm sp}]^{-1},$$

where chemical potential μ_N is measured from the Nth Landau level. The $\epsilon_p = \omega_H(p+1/2)$ are the eigenvalues of the free electron Hamiltonian. In the case of weak disorder $\omega_H \tau \gg 1$, the solution of Eq. (25) yields

$$P^n_{\rm sp} = \frac{{\rm sign}\ \omega_n}{2\tau}, \quad \frac{1}{2\tau} = \sqrt{g n_L}. \tag{27}$$

The fluctuations of the V field are responsible for the localization corrections to the conductivity (in the weak localization regime they correspond to the maximally crossed diagrams). However, in the considered case, these corrections are of the order of $\ln N/N \ll 1$ and, therefore, can be neglected. For this reason we simply put $V=1$.

The presence of the induced potential $\check{\lambda}(\mathbf{r})$ and the time-dependent vector potential $\mathbf{a}$ results in a shift of the saddle-point value (27) due to the coupling to the fluctuations of the P field. The corresponding effective action for the P field follows from Eq. (19) after integrating out fermions

$$\mathcal{S} = \int d\mathbf{r} \ {\rm tr} \ln G^{-1} - \frac{N_r \Omega_\Delta}{T} - \frac{1}{2g} \int d\mathbf{r} \ {\rm tr}(P_{\rm sp} + P)^2 + \int d\mathbf{r} \ {\rm tr} \ln[1 + (iP + \hat{K} + \check{\lambda})G]. \tag{28}$$

Finally, the thermodynamic potential can be written as

$$\Omega = -\frac{T}{N_r} \ln \int \mathcal{D}[P] I[P] \exp \mathcal{S}, \tag{29}$$

where following Ref. 26 the integration measure $I[\delta P]$ is given by

$$\ln I[P] = -\frac{1}{(\pi\rho)^2} \int \sum_{nm}^{\alpha\beta} [1 - \Theta(nm)] P^{\alpha\alpha}_{nn} P^{\beta\beta}_{mm}, \tag{30}$$

with ρ being the thermodynamic density of states and $\Theta(x)$ the Heaviside step function.

The quadratic in P part of the action (28) together with

the contribution (30) from the integration measure determines the propagator of the P fields (see Ref. 21 for details)

$$\langle P^{\alpha\beta}_{m_1m_2}(\mathbf{q})P^{\gamma\delta}_{m_3m_4}(-\mathbf{q})\rangle = \frac{g\delta_{m_1m_4}\delta_{m_2m_3}\delta^{\alpha\delta}\delta^{\beta\gamma}\Theta(m_1m_3)}{1+g\pi^{\omega_{m_1}}(\nu_{m_3-m_1},\mathbf{q})} - \frac{2[1-\Theta(m_1m_3)]}{(\pi\rho)^2}\frac{g\delta_{m_1m_2}\delta^{\alpha\beta}}{1+g\pi^{\omega_{m_1}}(0,\mathbf{q})} \times \frac{g\delta_{m_3m_4}\delta^{\delta\gamma}}{1+g\pi^{\omega_{m_3}}(0,\mathbf{q})}, \quad (31)$$

where the bare polarization operator $\pi^{\omega_m}(\nu_n,\mathbf{q})$ involves Green's functions for the $(N-1)$th, Nth, and $(N+1)$th Landau levels only

$$\pi^{\omega_m}(\nu_n,\mathbf{q}) = \sum_{p_1p_2}\pi^{\omega_m}_{p_1p_2}(\nu_n,\mathbf{q}) = -n_L\sum_{p_1p_2}G_{p_2}(\omega_m) \times G_{p_1}(\omega_m+\nu_n)F_{p_1p_2}(\mathbf{q})F_{p_2p_1}(-\mathbf{q}). \quad (32)$$

E. Thermodynamic potential. Second order contribution

In the absence of the time-dependent vector potential $\mathbf{a}$ effective action (28) should contain only $\Delta_{NN}(\mathbf{q})\equiv\Delta(\mathbf{q})$ order parameter in the limit $\max\{T,\tau^{-1}\}\ll\Delta_{\mathrm{ex}}\ll\omega_H$. To demonstrate it, we find the second order contribution to the thermodynamic potential for $\mathbf{a}=0$.

Performing evaluation similar to one presented in Ref. 16, we obtain

$$\Omega = \Omega^{(0)} + \Omega^{(2)} + \cdots, \quad (33)$$

where

$$\Omega^{(0)}(\mu) = \int d\mathbf{r}\ \mathrm{tr}\ln G^{-1} - \frac{1}{2g}\int d\mathbf{r}\ \mathrm{tr}\ P^2_{\mathrm{sp}} \quad (34)$$

is the thermodynamic potential of the isotropic state and

$$\Omega^{(2)} = \frac{n_LS^2}{2T}\sum_{p_1\cdots p_4}\int\frac{d\mathbf{q}}{(2\pi)^2}\Bigg[U_{p_1p_2p_3p_4}(\mathbf{q}) - T\sum_{\omega_n}\sum_{p_5\cdots p_8}\frac{U_{p_1p_2p_5p_6}(\mathbf{q})}{F_{p_5p_6}(\mathbf{q})}\frac{U_{p_3p_4p_7p_8}(-\mathbf{q})}{F_{p_7p_8}(-\mathbf{q})} \times\left(\delta_{p_5p_8}\delta_{p_6p_7}-\frac{g\pi^{\omega_n}_{p_8p_7}(0,\mathbf{q})}{1+g\pi^{\omega_n}(0,\mathbf{q})}\right)\pi^{\omega_n}_{p_5p_6}(0,\mathbf{q})\Bigg] \times\Delta_{p_1p_2}(\mathbf{q})\Delta_{p_3p_4}(-\mathbf{q}) \quad (35)$$

is the contribution to the thermodynamic potential quadratic in the order parameter $\Delta_{p_1p_2}(\mathbf{q})$.

It is worthwhile to mention that polarization operators $\pi^{\omega_n}_{p_1p_2}(\nu_n,\mathbf{q})$ obey the following hierarchy with respect to small parameter, $\max\{T,\tau^{-1}\}/\omega_H\ll1$:

$$\pi^{\omega_n}_{p_1p_2}\sim\begin{cases}\mathcal{O}(1), & p_1=p_2=N,\\ \mathcal{O}\left(\dfrac{\max\{T,\tau^{-1}\}}{\omega_H}\right), & p_1=N \text{ or } p_2=N,\\ \mathcal{O}\left(\left[\dfrac{\max\{T,\tau^{-1}\}}{\omega_H}\right]^2\right), & p_1\neq N \text{ and } p_2\neq N.\end{cases} \quad (36)$$

According to the hierarchy (36) we can write

$$\pi^{\omega_n}_{p_1p_2}(0,\mathbf{q}) \approx \pi^{\omega_n}_0(0,q)\delta_{p_1N}\delta_{p_2N}, \quad (37)$$

where we introduce $\pi^{\omega_n}_0(0,q)\equiv\pi^{\omega_n}_{NN}(0,q)$. Thus in the leading order in small parameter $\max\{T,\tau^{-1}\}/\omega_H\ll1$ we obtain from Eq. (35)

$$\Omega^{(2)} = \frac{n_LS^2}{2T}\sum_{p_1\cdots p_4}\int\frac{d\mathbf{q}}{(2\pi)^2}\Bigg[U_{p_1p_2p_3p_4}(\mathbf{q}) + T\sum_{\omega_n}\frac{n_LG^2_N(\omega_n)}{1+g\pi^{\omega_n}_0(0,\mathbf{q})}U_{p_1p_2NN}(\mathbf{q})U_{p_3p_4NN}(-\mathbf{q})\Bigg] \times\Delta_{p_1p_2}(\mathbf{q})\Delta_{p_3p_4}(-\mathbf{q}). \quad (38)$$

To find the possible nonzero order parameters $\Delta_{p_1p_2}(\mathbf{q})$, we should diagonalize the 9×9 matrix in Eq. (38). Fortunately, the nontrivial part of Eq. (38) can be written as

$$\delta\Omega^{(2)} = \frac{n_LS^2}{2T}\int\frac{d\mathbf{q}}{(2\pi)^2}T_0(q)\left(\Delta(\mathbf{q}),\frac{T_1(q)}{T_0(q)}\varphi(\mathbf{q})\right) \times\begin{pmatrix}a(q) & a(q)\\ a(q) & a(q)+2b(q)\end{pmatrix}\begin{pmatrix}\Delta(-\mathbf{q})\\ \dfrac{T_1(q)}{T_0(q)}\varphi(-\mathbf{q})\end{pmatrix}. \quad (39)$$

Here $\varphi(\mathbf{q})$ involves a linear combination of all order parameters $\Delta_{p_1p_2}(\mathbf{q})$ except $\Delta(\mathbf{q})$. Characteristic energies $T_0(q)$ and $T_1(q)$ is related with Hartree-Fock potential (21) as follows

$$T_0(q) = \frac{U_{NNNN}(q)}{4}, \quad T_1(q) = e^{i\phi}\frac{U_{N,N\pm1,NN}(\mathbf{q})}{4}, \quad (40)$$

where ϕ denotes angle of vector $\mathbf{q}$ with respect to the x axis. We emphasize that quantity $T_1(q)$ depends only on the absolute value q of vector $\mathbf{q}$. Matrix element $a(q)$ is given by

$$a(T,\tau^{-1},q) = 1+4T\sum_{\omega_n}\frac{n_LT_0(q)G^2_N(\omega_n)}{1+g\pi^{\omega_n}(0,q)}, \quad (41)$$

whereas function $b(q)$ is defined as

$$b(q) = \frac{T_0(q)}{2T_1(q)} - \frac{1}{2}. \quad (42)$$

The eigenvalues of the 2×2 matrix in Eq. (39) can be easily found

$$\lambda_\pm(q) = a(q)+b(q)\pm\sqrt{[a(q)]^2+[b(q)]^2} \quad (43)$$

As one can check, the eigenvalue $\lambda_+(q)$ has the same sign as $b(q)$ for all values of $a(q)$ whereas the eigenvalue $\lambda_-(q)$ changes its sign at point $a(q)=0$. Therefore, the instability

appears at the same condition as if we consider only one CDW order parameter $\Delta(\mathbf{q})$ as it has usually done.[4–6,16] According to the result derived in Appendix I, characteristic energy $T_1(q)$ is of the order of $T_0(q)/N \ll T_0(q)$. By using the condition $b(q) \gg a(q)$, we find therefore

$$\Omega^{(2)} = \frac{n_L S^2}{2T} \int \frac{d\mathbf{q}}{(2\pi)^2} T_0(q) \left[a(q) \left(1 - \frac{T_1(q)}{T_0(q)} a(q) \right) \times \Delta_-(\mathbf{q}) \Delta_-(-\mathbf{q}) + \frac{T_1(q)}{T_0(q)} \Delta_+(\mathbf{q}) \Delta_+(-\mathbf{q}) \right], \quad (44)$$

where

$$\Delta_-(\mathbf{q}) = \Delta(\mathbf{q}) - \left(\frac{T_1(q)}{T_0(q)} \right)^2 a(q) \varphi(\mathbf{q}), \quad (45)$$

$$\Delta_+(\mathbf{q}) = \varphi(\mathbf{q}) + a(q) \Delta(\mathbf{q}).$$

Minimum of the free energy[27] is reached at $\Delta_+(q)=0$. Neglecting the difference of the order of $\mathcal{O}(N^{-2})$ between $\Delta_-(q)$ and $\Delta(q)$, we obtain finally

$$\Omega^{(2)} = \frac{n_L S^2}{2T} \int \frac{d\mathbf{q}}{(2\pi)^2} T_0(q) a(q) \left(1 - \frac{T_1(q)}{T_0(q)} a(q) \right) \times \Delta(\mathbf{q}) \Delta(-\mathbf{q}). \quad (46)$$

Thus, the fact that the order parameters $\Delta_{p_1 p_2}(\mathbf{q})$ with p_1 and p_2 different from N can exist leads to correction of the order of $\mathcal{O}(N^{-1})$. Hereafter we write therefore that

$$\Delta_{p_1 p_2}(\mathbf{q}) = \Delta(\mathbf{q}) \delta_{p_1 N} \delta_{p_2 N} \quad (47)$$

and neglect the second term of the order of $\mathcal{O}(N^{-1})$ in brackets in Eq. (46).

F. "Three-level" model

The results of the previous section allows us to establish finally an effective action for the "three-level" model.

According to definition (20), CDW on the Nth Landau level with the order parameter $\Delta(\mathbf{q})$ results in the induced potential $\lambda_{N,N\pm1}(\mathbf{q})$, scattering electrons from the Nth Landau level to the $(N\pm1)$th Landau level. However, the induced potential is of the order of $T_1(q)$ and, consequently, leads to small corrections of the order of $\mathcal{O}(N^{-1})$. For the reasons to be explained shortly, we write

$$\lambda_{p_1 p_2}(\mathbf{q}) = S U(q) F_N^{-1}(q) \Delta(\mathbf{q}) \delta_{p_1 N} \delta_{p_2 N}. \quad (48)$$

Finally, the effective action for the "three-level" model becomes

$$\mathcal{S}_{TL}[P] = \int d\mathbf{r} \, \mathrm{tr} \ln G^{-1} - \frac{N_r \Omega_\Delta}{T} - \frac{1}{2g} \int d\mathbf{r} \, \mathrm{tr}(P_{\mathrm{sp}} + P)^2 + \int d\mathbf{r} \, \mathrm{tr} \ln[1 + (iP + \hat{K} + P_N \lambda P_N) G], \quad (49)$$

where

$$P_N(\mathbf{r}_1, \mathbf{r}_2) = \sum_k \phi^*_{Nk}(\mathbf{r}_2) \phi_{Nk}(\mathbf{r}_1) = n_L \exp\left(i \frac{(y_1 - y_2)(x_1 + x_2)}{2 l_H^2} \right) \times \exp\left(- \frac{|\mathbf{r}_1 - \mathbf{r}_2|^2}{4 l_H^2} \right) L_N\left(\frac{|\mathbf{r}_1 - \mathbf{r}_2|^2}{2 l_H^2} \right) \quad (50)$$

is the projection operator on the Nth Landau level [$L_N(x)$ denotes the Laguerre polynomial] and

$$\Omega_\Delta = \frac{n_L S^2}{2} \int \frac{d\mathbf{q}}{(2\pi)^2} U(q) \Delta(\mathbf{q}) \Delta(-\mathbf{q}), \quad (51)$$

Here, for a brevity we introduce $U(q) \equiv U_{NNNN}(q)$.

III. CONDUCTIVITY OF THE UCDW STATE AT $T_c - T \ll T_c$

A. UCDW state

Effective action (49) allows us to evaluate conductivity of the system in the CDW state. As the most interesting case we consider the half-filled Nth Landau level where the UCDW can exist. Let us recall that the UCDW order parameter is written as[24]

$$\Delta(\mathbf{q}) = \frac{(2\pi)^2}{S} \Delta[\delta(\mathbf{q} + \mathbf{Q}_0) + \delta(\mathbf{q} - \mathbf{Q}_0)], \quad (52)$$

where vector $\mathbf{Q}_0$ that determines period and direction of the UCDW state can be oriented along spontaneously chosen direction. Usually, its direction is fixed either by intrinsic anisotropy of the system or by small magnetic field applied parallel to 2DEG.[28–32] We assume that the vector $\mathbf{Q}_0$ is directed at an angle ϕ with respect to the x axis. The temperature of the isotropic-to-UCDW transition $T_c = \max T(q)$ where $T(q)$ is solution of equation $a[T(q), \tau^{-1}, q] = 0$. The absolute value of vector $\mathbf{Q}_0$ equals the value $q = Q_0$ at which the $T(q)$ reaches its maximum. It was shown in Ref. 16 that $Q_0 = r_0/R_c$ with $r_0 \approx 2.4$ being the first zero of the Bessel function $\mathcal{J}_0(x)$ and the T_c is determined as the solution of the following equation (see Fig. 1)

$$\frac{T_c}{T_0} = \frac{2}{\pi^2} \zeta\left(2, \frac{1}{2} + \frac{1}{4\pi T_c \tau} \right). \quad (53)$$

Here $\zeta(k,z)$ denotes the generalized Riemann zeta function and we have introduced $T_0 = T_0(Q_0)$ for the brevity. It has meaning of the transition temperature in clean case $T_0 = T_c(\tau^{-1} = 0)$. According to Refs. 4 and 6,

$$T_0 = \frac{r_s \omega_H}{4\pi\sqrt{2}} \left[\ln\left(1 + \frac{c}{r_s} \right) - \frac{c}{\sqrt{2} + r_s} \right], \quad \frac{1}{N} \ll r_s \ll 1, \quad (54)$$

where $c = 1/(\sqrt{2} r_0) \approx 0.3$. It is worthwhile to mention that $T_0 \ll \Delta_{\mathrm{ex}} \ll \omega_H$ and it is determined by the characteristic energy of the screened electron-electron interaction (15). Near the transition temperature, $T_c - T \ll T_c$, the UCDW order parameter equals[16]

$$\Delta = \zeta\left(2, \frac{1}{2} + \frac{1}{4\pi T_c \tau} \right) \sqrt{\mathcal{G}\left(\frac{1}{4\pi T_c \tau} \right)} \sqrt{\frac{T_c - T}{T_c}}. \quad (55)$$

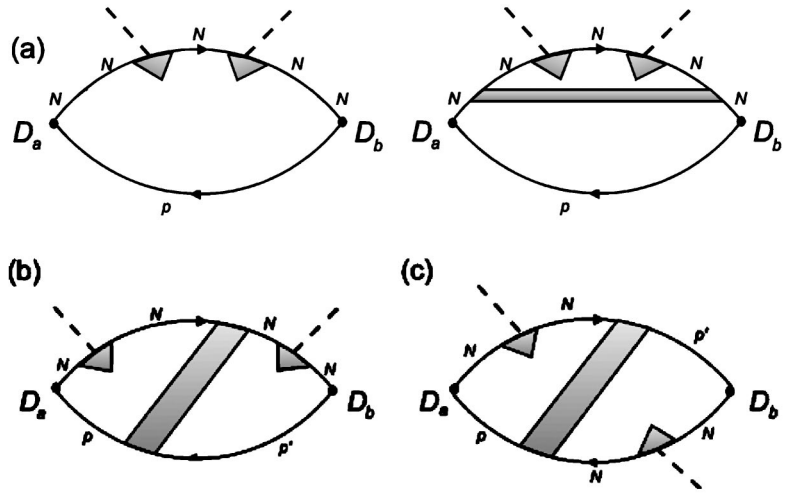

FIG. 2. Diagrams for corrections to the conductivity tensor σ_{ab}. Solid line denotes Green's function, $N/p/p'$ Landau level indices, dashed line the induced potential $\lambda(\mathbf{r})$, and shaded block the impurity ladder.

Here the function $\mathcal{G}(z)$ is determined as

$$\mathcal{G}(z)=\frac{\zeta\left(2,\frac{1}{2}+z\right)-z\zeta\left(3,\frac{1}{2}+z\right)}{-3\zeta\left(4,\frac{1}{2}+z\right)+4\Phi_0(z)+2\Phi_2(z)}, \tag{56}$$

where

$$\Phi_n(z)=\frac{\zeta\left(2,\frac{1}{2}+z\right)}{z^2\mathcal{J}_0^2(nr_0)}-\frac{\mathrm{Im}\,\psi\left[\frac{1}{2}+z+iz\mathcal{J}_0(nr_0)\right]}{z^3\mathcal{J}_0^3(nr_0)}. \tag{57}$$

The $\psi(z)$ stands for the Euler di-gamma function.

B. Conductivity tensor $\sigma_{ab}(\omega)$

To determine the conductivity tensor $\sigma_{ab}(\omega)\equiv\sigma_{ab}(\omega,q=0)$ we use the Matsubara technique. We compute conductivity $\sigma_{ab}(i\nu_n)$ on imaginary discrete frequencies $\nu_n=2\pi Tn$ and then perform analytic continuation from upper half plane ($\nu_n>0$) to real axis, $i\nu_n\to\omega$. The conductivity tensor $\sigma_{ab}(i\nu_n)$ can be found after integration over $P(\mathbf{r})$ fields as the second derivative of logarithm of the effective action with respect to spatially constant time-dependent vector potential $\mathbf{a}(\nu_n)$,

$$\sigma_{ab}(i\nu_n)=\frac{\pi T}{SN_r\nu_n}\frac{\delta^2}{\delta a_a(\nu_n)\delta a_b(-\nu_n)}\times\ln\int\mathcal{D}P I[P]\exp\mathcal{S}_{TL}[P]\Big|_{\mathbf{a}=0}. \tag{58}$$

It is worthwhile to mention that Eq. (58) corresponds to the term $\mathbf{j}(\nu_n)\mathbf{a}(-\nu_n)$ with $\mathbf{j}(\nu_n)$ being current density in the effective action for the vector potential $\mathbf{a}(\nu_n)$. As one can check by inspection, contribution to the conductivity tensor of the first order in the order parameter $\Delta(\mathbf{q})$ vanishes. It occurs because the UCDW state appears at nonzero vector $\mathbf{Q}_0$. Thus, the first nonvanishing contribution to the conductivity tensor of the UCDW state is of the second order in the order parameter Δ.

In order to find it, we expand the effective action $\mathcal{S}_{TL}[P]$ to the second order in both the induced potential $\lambda(\mathbf{r})$ and the $\hat{K}$. Then, we integrate over $P(\mathbf{r})$ fields. We do not present the explicit calculations here since they are similar to ones presented in Ref. 16. We mention that there are three contributions of different structure to the conductivity tensor of the UCDW state. Diagrams for them are shown in Fig. 2.

The first and the second diagrams [Fig. 2(a)] correspond to the following contribution:

$$\sigma_{ab}^{(a)}(i\nu_n)=-\frac{8\pi\omega_H}{m\nu_n}T_0^2\Delta^2T\sum_{\omega_n}\frac{G_N^3(\omega_n)}{1+g\pi_0^{\omega_n}(0,0)}\times\sum_p\frac{D_{Np}^aD_{pN}^bG_p(\omega_n+\nu_n)}{[1+g\pi_0^{\omega_n}(0,Q_0)]^2}. \tag{59}$$

Here D_{Np}^a denotes matrix element of the covariant derivative

$$D_{Np}^a=\int d\mathbf{r}\,\phi_{Nk}^*(\mathbf{r})D_a\phi_{pk}(\mathbf{r})=\sqrt{n_L}[\delta_{p,N-1}\beta^a\sqrt{N}+\delta_{p,N+1}\gamma^a\sqrt{N+1}], \tag{60}$$

where

$$\gamma^x=i,\quad \gamma^y=1,\quad \beta^x=-i,\quad \beta^y=1. \tag{61}$$

The third diagram [Fig. 2(b)] is given by

$$\sigma_{ab}^{(b)}(i\nu_n)=\frac{8\pi\omega_H}{\nu_nm}T_0^2\Delta^2T\sum_{\omega_n}\sum_{pp'}\frac{G_N^4(\omega_n+\nu_n)}{[1+g\pi_0^{\omega_n}(0,Q_0)]^2}\times D_{pN}^aD_{Np'}^bG_p(\omega_n)G_{p'}(\omega_n)\Bigg[I_{Npp'N}(\mathbf{Q}_0)+\frac{n_LG_N(\omega_n)G_N(\omega_n+\nu_n)}{1+g\pi_0^{\omega_n}(\nu_n,Q_0)}\times I_{NpNN}(\mathbf{Q}_0)I_{NNp'N}(\mathbf{Q}_0)\Bigg]. \tag{62}$$

Symbol $I_{p_1p_2p_3p_4}(\mathbf{Q})$ denotes the impurity ladder in the Landau level index representation (see Fig. 3)

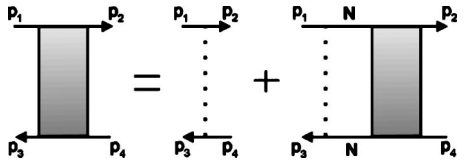

FIG. 3. Impurity ladder. Frequency $\omega_n+\nu_n$ runs from right to left, whereas ω_n runs from left to right.

$$I_{p_1p_2p_3p_4}(\mathbf{Q}) = g\int_q F_{p_1p_2}(\mathbf{q})F_{p_3p_4}(-\mathbf{q})\exp\left(-i\mathbf{q}\mathbf{Q}l_H^2\right). \tag{63}$$

Evaluation of $I_{p_1p_2p_3p_4}(\mathbf{Q}_0)$ is given in Appendix II. For convenience we present the results for quantities $I_{p_1p_2p_3p_4}(\mathbf{Q}_0)$ in Table I. As one can see, the $I_{NNNpN}(\mathbf{Q}_0)$, $I_{NpNNN}(\mathbf{Q}_0)$, and $I_{Npp'N}(\mathbf{Q}_0)$ are proportional to $\mathcal{J}_0(r_0)=0$. Thus, the contribution (62) vanishes

$$\sigma_{ab}^{(b)}(i\nu_n) = 0. \tag{64}$$

The last diagram [Fig. 2(c)] can be written as

$$\sigma_{ab}^{(c)}(i\nu_n) = \frac{8\pi\omega_H}{\nu_n m}T_0^2\Delta^2 T\sum_{\omega_n}\sum_{pp'}\frac{G_N^2(\omega_n)G_N^2(\omega_n+\nu_n)}{[1+g\pi_0^{\omega_n}(0,Q_0)]} \times \frac{G_p(\omega_n)G_{p'}(\omega_n+\nu_n)}{(1+g\pi_0^{\omega_n+\nu_n}(0,Q_0))} \times \frac{D_{pN}^a D_{p'N}^b I_{NpNp'}(\mathbf{Q}_0)}{1+g\pi_0^{\omega_n}(\nu_n,Q_0)}. \tag{65}$$

Although results (59) and (65) allow us to compute the ac conductivity tensor $\sigma_{ab}(\omega)$ from now onwards we limit ourselves to the case of dc conductivity tensor $\sigma_{ab}\equiv\sigma_{ab}(\omega=0)$.

C. DC conductivity tensor σ_{ab} of the UCDW state

The conductivity tensor of the isotropic state is given as[1]

$$\hat{\sigma}^{(0)} = \frac{2N}{\pi}\hat{s}_0 - Ni\hat{s}_y, \tag{66}$$

where $\hat{s}_0$ denotes the unit matrix and matrices $\hat{s}_a$ with $a=x,y,z$ stand for the Pauli matrices.

Performing evaluation of frequency sums in Eqs. (59) and (65), we obtain the following result for the dc conductivity tensor of the UCDW state:

$$\hat{\sigma} = \hat{\sigma}^{(0)} - 4\pi N\Delta^2\mathcal{J}_1^2(r_0)h\left(\frac{1}{4\pi T_c\tau}\right)\cos 2\phi\hat{s}_z - 4\pi N\Delta^2 h_{xx}\left(\frac{1}{4\pi T_c\tau}\right)\hat{s}_0 - 4\pi N\Delta^2\left[2\pi\frac{T_c}{\omega_H}h_{xy}\left(\frac{1}{4\pi T_c\tau}\right) - \mathcal{J}_1^2(r_0)h\left(\frac{1}{4\pi T_c\tau}\right)\sin 2\phi\right]i\hat{s}_y. \tag{67}$$

Equation (67) constitutes one of the main results of the present paper. We remind that the result (67) describes the dc conductivity tensor near the isotropic-to-UCDW transition, $T_c-T\ll T_c$.

In order to simplify the discussion we put $\phi=0$, i.e., chose the x axis along the direction of the UCDW vector $\mathbf{Q}_0$. Then the term proportional to $\hat{s}_z$ corresponds to the anisotropic part of the dc conductivity tensor, the term proportional to $\hat{s}_0$ corresponds to isotropic part and the term proportional to $\hat{s}_y$ to Hall conductivity. We mention that anisotropic, isotropic, and Hall conductivity corrections are of the order of N as the conductivity of the isotropic state is itself [see Eq. (66)]. The anisotropic, isotropic, and Hall conductivity corrections are proportional to $(T_c-T)/T_c\ll 1$ and are much smaller therefore than the conductivity of isotropic state (66). We mention that the Hall conductivity correction contains additional small factor $\max\{T_c,\tau^{-1}\}/\omega_H$ compared to anisotropic and isotropic ones. The presence of anisotropic part in the conductivity tensor leads to the result that conductivity along the electron density modulation (σ_{xx}) is less than conductivity across the modulation (σ_{yy}).

The dependence of corrections to the conductivity tensor on the Landau level broadening τ^{-1} is encoded in functions $h(z)$, $h_{xy}(z)$, $h_{xy}(z)$ as well as in Δ^2 [see Eq. (55)]. The anisotropic part of the conductivity tensor involves the function $h(z)$ that appears as a result of summation over Landau level indices with $p=p'=N\pm 1$ in Eq. (65) and is given by

$$h(z) = 5z^3\frac{\zeta\left(6,\frac{1}{2}+z\right)}{\left[\zeta\left(2,\frac{1}{2}+z\right)\right]^2} = \begin{cases}\dfrac{4\pi^2}{3}z^3, & z\ll 1,\\ 1-\dfrac{3}{z}, & z\gg 1.\end{cases} \tag{68}$$

The $h(z)$ increases monotonically from 0 to 1, as it is shown in Fig. 4. The isotropic part of conductivity tensor is described by the function $h_{xx}(z)$ that contains Eq. (59) and terms with $p=N\pm 1$ and $p'=N\mp 1$ in the sum over Landau level indices in Eq. (65),

TABLE I. Expressions for quantities $I_{p_1p_2p_3p_4}(\mathbf{Q}_0)$ involved in Eqs. (59), (62), and (65).

$I_{N,N-1,N-1,N}$	=	$gn_L\mathcal{J}_0^2(r_0)$	$I_{N,N+1,N+1,N}$	=	$gn_L\mathcal{J}_0^2(r_0)$
$I_{N,N-1,N,N+1}$	=	$gn_L\mathcal{J}_1^2(r_0)$	$I_{N,N+1,N,N-1}$	=	$gn_L\mathcal{J}_1^2(r_0)$
$I_{N,N,N-1,N}$	=	$gn_Le^{i\phi}\mathcal{J}_0(r_0)\mathcal{J}_1(r_0)$	$I_{N,N+1,N,N}$	=	$gn_Le^{-i\phi}\mathcal{J}_0(r_0)\mathcal{J}_1(r_0)$
$I_{N,N-1,N,N}$	=	$-gn_Le^{i\phi}\mathcal{J}_0(r_0)\mathcal{J}_1(r_0)$	$I_{N,N,N+1,N}$	=	$-gn_Le^{-i\phi}\mathcal{J}_0(r_0)\mathcal{J}_1(r_0)$
$I_{N,N-1,N,N-1}$	=	$gn_Le^{2i\phi}\mathcal{J}_1^2(r_0)$	$I_{N,N+1,N,N+1}$	=	$gn_Le^{-2i\phi}\mathcal{J}_1^2(r_0)$
$I_{N,N-1,N+1,N}$	=	$gn_Le^{2i\phi}\mathcal{J}_1(r_0)\mathcal{J}_0(r_0)$	$I_{N,N+1,N-1,N}$	=	$gn_Le^{-2i\phi}\mathcal{J}_1(r_0)\mathcal{J}_0(r_0)$

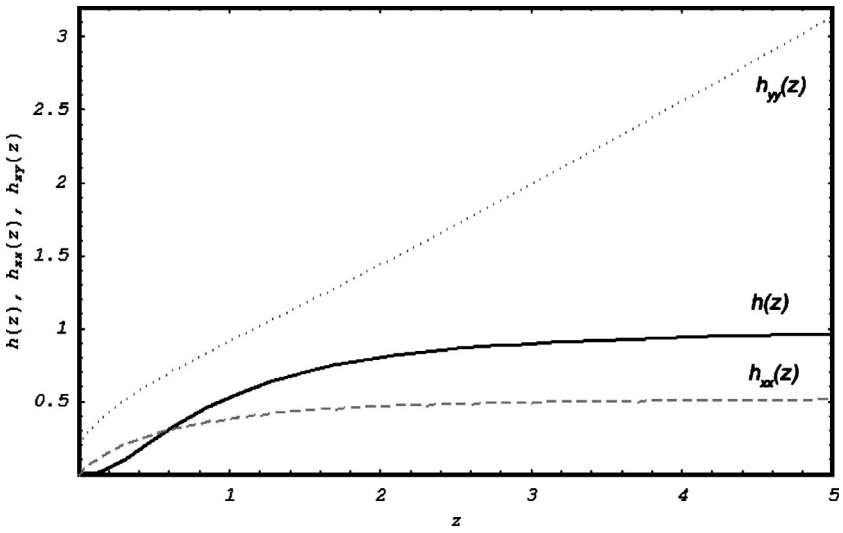

FIG. 4. Functions $h(z)$, $h_{xx}(z)$ and $h_{xy}(z)$.

$$h_{xx}(z) = \mathcal{J}_1^2(r_0)h(z) + \frac{1}{2z\zeta\left(2,\frac{1}{2}+z\right)} \times \left[1 - \frac{\mathrm{Re}\,\psi'\left(\frac{1}{2}+(1-i)z\right)}{\zeta\left(2,\frac{1}{2}+z\right)}\right] = \begin{cases} \frac{2}{3}z, & z \ll 1, \\ \mathcal{J}_1^2(r_0) + \frac{1}{4} - \frac{3\mathcal{J}_1^2(r_0)-\frac{1}{4}}{z}, & z \gg 1. \end{cases} \quad (69)$$

The function $h_{xx}(z)$ increases monotonically from 0 to 0.52 (see Fig. 4). The Hall conductivity correction involves the function $h_{xy}(z)$ given by

$$h_{xy}(z) = \left[\mathcal{J}_1^2(r_0)\left[4z^4\zeta\left(5,\frac{1}{2}+z\right) + z^3\zeta\left(4,\frac{1}{2}+z\right)\right] - 2h_{xx}(z) + z\,\mathrm{Im}\,\psi'\left(\frac{1}{2}+(1-i)z\right) - \mathrm{Im}\,\psi\left(\frac{1}{2}+(1-i)z\right)\right]\frac{1}{z\left[\zeta\left(2,\frac{1}{2}+z\right)\right]^2} = \begin{cases} \frac{2}{\pi^2}\left(1 - \frac{\pi^2 - 2\psi''\left(\frac{1}{2}\right)}{\pi^2}z\right), & z \ll 1, \\ \left(\frac{\pi}{4} - \frac{2\mathcal{J}_1^2(r_0)}{3}\right)z, & z \gg 1. \end{cases} \quad (70)$$

As one can see from Eq. (68) the absolute value of the anisotropic part of the conductivity tensor increases monotonically with growing of the Landau level broadening τ^{-1}. According to Eq. (69), the isotropic part of the conductivity tensor decreases as the disorder becomes stronger. The Hall conductivity increases with growing of the Landau level broadening [see Eq. (70)]. The enhancement of the anisotropic and Hall conductivity by disorder is counterintuitive, however, it is the effect of the magnetic field.[1] Also we notice that in the case of small Landau level broadening $\tau^{-1} \ll T_c$, the correction to the isotropic part of the conductivity tensor is much larger than the anisotropic part. In the opposite limit of large Landau level broadening $\tau^{-1} \gg T_c$, they are of the same order.

IV. EFFECT OF THE ORDER PARAMETER FLUCTUATIONS ON THE CONDUCTIVITY TENSOR

A. Order parameter fluctuations

The order parameter $\Delta(\mathbf{r})$ has meaning of the saddle-point solution for a plasmon field that appears in the Hubbard-Stratonovich transformation[33] of the screened electron-electron interaction. The expansion of such physical quantities as free energy and linear response in the order parameter series can be justified if fluctuations of the order parameter can be neglected. As it was shown,[16] fluctuations of the order parameter results in the first order transition from the isotropic state to the UCDW state at temperature $T_c - \delta T_c$ where $\delta T_c / T_c \propto N^{-2/3} \ll 1$. In the present section we investigate the effect of the fluctuations on the conductivity tensor above and below the mean-field transition.

In the previous section we assumed that the direction of the CDW vector $\mathbf{Q}_0$ is fixed by intrinsic anisotropy of crystal or by applied parallel to 2DEG small magnetic field. However, the functional dependence of anisotropy term in the hamiltonian was insignificant for mean-field results obtained above. Now it should be concretized. Experimental research of the anisotropy that determines the direction along which the UCDW creates has been performed in a number of papers.[28–32] The results obtained can be explained if one suggests that the Hartree-Fock potential $U(\mathbf{Q})$ involves terms proportional to $\cos 2\phi$ and $\cos 4\phi$. We mention that the term $\cos 2\phi$ can be derived when small magnetic field parallel to 2DEG applied.[34] However, without parallel magnetic field

the term $\cos 2\phi$ is restricted by the symmetry of bulk GaAs crystal. To date its physical origin is unknown.[30] As experimentally proven,[32] coefficient of the $\cos 2\phi$ term depends on the density n of electrons. Moreover, at some certain value n_* of the electron density it vanishes and next term proportional to $\cos 4\phi$ becomes important. Below we restrict our discussion to the general case $n \neq n_*$. We note that the typical value of the anisotropy energy E_A is of the order of 1 mK per electron as it is obtained from experiment.[30] In order to take into account the anisotropy quantitatively we perform the following substitution [see Eq. (40)]

$$T_0(Q) \to T_0(Q) + E_A \frac{1-\cos 2\phi}{2} \tag{71}$$

near $Q=Q_0$. We note that the expression above has minimum at $\phi=0$.

At $T>T_c$ the UCDW order parameter is zero in average $\langle\Delta\rangle=0$ but the average of its square is nonzero $\langle\Delta^2\rangle\neq 0$. It results in the additional contribution to the conductivity tensor of the isotropic state. It is worthwhile to mention that the contribution discussed above is analogous to one for normal metal due to superconducting paring above critical temperature.[18]

The additional contribution to the conductivity tensor due to the order parameter fluctuations can be found with a help of the substitution $\langle\Delta(\mathbf{Q})\Delta(-\mathbf{Q})\rangle$ for Δ^2 in Eq. (67) and averaging over all possible vectors $\mathbf{Q}$. The Green's function of the order parameter fluctuations is as follows (see Ref. 16)

$$\langle\Delta(\mathbf{Q})\Delta(-\mathbf{Q})\rangle = \frac{T_c}{4T_0(Q_0)n_L}\left[\frac{T-T_c}{T_c}+\gamma\left(\frac{1}{4\pi T_c\tau}\right)\times(Q-Q_0)^2R_c^2+\eta\sin^2\phi\right]^{-1}, \tag{72}$$

where dimensionless parameter $\eta=E_A/T_0$ and we have introduced

$$\gamma(z)=\beta_1+\mathcal{J}_1^2(r_0)z^2\frac{\zeta\left(4,\frac{1}{2}+z\right)}{\zeta\left(2,\frac{1}{2}+z\right)}. \tag{73}$$

Here $\beta_1=\partial\ln T_0/\partial r_0\approx 2.58$. After integration over the absolute value of vector $\mathbf{Q}$ we find that in Eq. (67) the following substitution should be used:

$$f(\phi)\Delta^2 \to \frac{r_0}{4\pi N}\zeta\left(2,\frac{1}{2}+\frac{1}{4\pi T_c\tau}\right)\left[\gamma\left(\frac{1}{4\pi T_c\tau}\right)\right]^{-1/2}\times\sqrt{\frac{T_c}{T-T_c}}\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{f(\phi)}{\sqrt{1+\frac{\eta T_c}{T-T_c}\sin^2\phi}}. \tag{74}$$

It is worthwhile to mention that in order to obtain the result for $T<T_c$ from the known result for $T>T_c$ we should substitute $2(T_c-T)/T_c$ for $(T-T_c)/T_c$ as usual. For convenience, let us define therefore

$$t=\begin{cases}\dfrac{T-T_c}{T_c} & T>T_c,\\ 2\dfrac{T_c-T}{T_c} & T<T_c.\end{cases} \tag{75}$$

B. Fluctuational correction to the mean-field DC conductivity tensor

Integrating over angle ϕ in Eq. (74), we obtain the following fluctuational corrections to the mean-field dc conductivity tensor discussed in the previous section:

$$\delta\hat{\sigma}^{(\mathrm{fluc})} = -\frac{r_0}{\sqrt{t}}\mathcal{J}_1^2(r_0)H\left(\frac{1}{4\pi T_c\tau}\right)F_A\left(\frac{\eta}{t}\right)\hat{s}_z - \frac{r_0}{\sqrt{t}}H_{xx}\left(\frac{1}{4\pi T_c\tau}\right)F_I\left(\frac{\eta}{t}\right)\hat{s}_0. \tag{76}$$

$$-\frac{2\pi T_c}{\omega_H}\frac{r_0}{\sqrt{t}}H_{xy}\left(\frac{1}{4\pi T_c\tau}\right)F_I\left(\frac{\eta}{t}\right)i\hat{s}_y.$$

It is worthwhile to emphasize that Eq. (76) constitutes the principal result of the present paper.

The temperature dependence of the fluctuational correction (76) is encoded into a function $F_A(x)$ for the anisotropic part and a function $F_I(x)$ for the isotropic part and the Hall conductivity. Function $F_A(x)$ involves complete elliptic functions of the first (K) and second (E) kind

$$F_A(x)=\frac{2}{\pi}\left[\left(1+\frac{2}{x}\right)K(i\sqrt{x})-\frac{2}{x}E(i\sqrt{x})\right]=\begin{cases}\dfrac{x}{8}, & x\ll 1,\\ \dfrac{\pi}{\sqrt{x}}\ln 16e^{-4}x, & x\gg 1,\end{cases} \tag{77}$$

whereas function $F_I(x)$ is determined as

$$F_I(x)=\frac{2}{\pi}K(i\sqrt{x})=\begin{cases}1, & x\ll 1,\\ \dfrac{1}{\pi\sqrt{x}}\ln 16x, & x\gg 1.\end{cases} \tag{78}$$

We emphasize that with respect to the mean-field result (67) the fluctuational correction (76) is of the order of N^{-1}. However, because of the singular behavior at $T=T_c$ it cannot be neglected.

The dependence of the fluctuational correction (76) on the Landau level broadening τ^{-1} is determined by function $H(z)$ for the anisotropic part, $H_{xx}(z)$ for the isotropic part and $H_{xy}(z)$ for the Hall conductivity. They are given as

$$H(z)=\frac{\zeta\left(2,\frac{1}{2}+z\right)h(z)}{\sqrt{\gamma(z)}}=\begin{cases}\dfrac{2\pi^4}{3\sqrt{\beta_1}}z^3, & z\ll 1,\\ \dfrac{\sqrt{3}}{z\sqrt{\mathcal{J}_1^2(r_0)+3\beta_1}}, & z\gg 1.\end{cases} \tag{79}$$

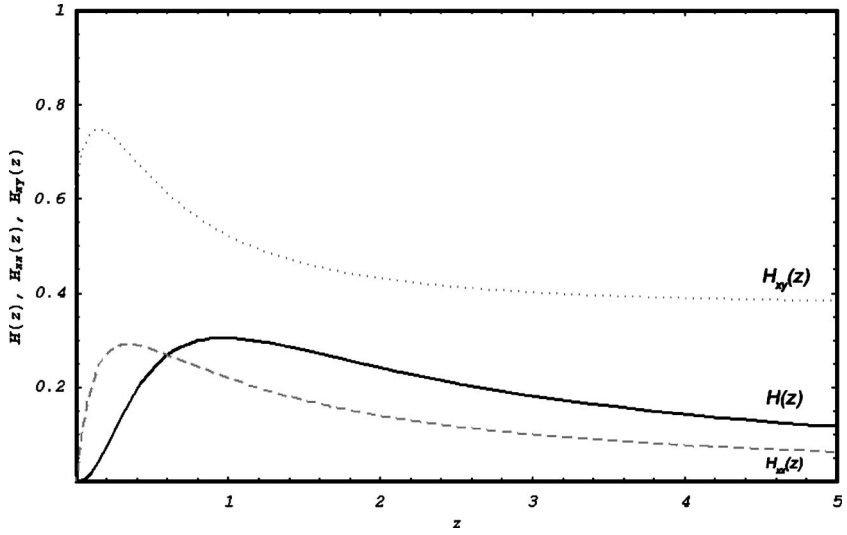

FIG. 5. Functions $H(z)$, $H_{xx}(z)$, and $H_{xy}(z)$.

$$H_{xx}(z)=\frac{\zeta\left(2,\frac{1}{2}+z\right)h_{xx}(z)}{\sqrt{\gamma(z)}}=\begin{cases}\dfrac{\pi^2}{3\sqrt{\beta_1}}z, & z\ll 1,\\ \dfrac{\sqrt{3}\{1+4\mathcal{J}_1^2(r_0)\}}{4\sqrt{\mathcal{J}_1^2(r_0)+3\beta_1}}\dfrac{1}{z}, & z\gg 1,\end{cases} \tag{80}$$

$$H_{xy}(z)=\frac{\zeta\left(2,\frac{1}{2}+z\right)h_{xy}(z)}{\sqrt{\gamma(z)}}=\begin{cases}\dfrac{1}{\sqrt{\beta_1}}\left(1-\dfrac{\pi^2-2\psi''\left(\frac{1}{2}\right)}{\pi^2}z\right), & z\ll 1,\\ \dfrac{3\pi-8\mathcal{J}_1^2(r_0)}{4\sqrt{3}\sqrt{\mathcal{J}_1^2(r_0)+3\beta_1}}, & z\gg 1.\end{cases} \tag{81}$$

We note that the function $H(z)$ has maximum at $z\approx 0.97$, $H_{xx}(z)$ at $z\approx 0.34$ and $H_{xy}(z)$ at $z\approx 0.16$, as it is shown in Fig. 5. In the case of small Landau level broadening, $\tau^{-1}\ll T_c$, the correction to the isotropic part of the conductivity tensor is much larger than the anisotropic part as one can see from Fig. 5. In the opposite limit of large Landau level broadening, $\tau^{-1}\gg T_c$, they are of the same order. The fluctuational correction to the Hall conductivity contains additional small factor $T_c/\omega_H\ll 1$.

Equation (76) has singularity at $T\to T_c$ that indicates nonapplicability of it near T_c. The fluctuational correction should be small as compared to the mean-field conductivity of the isotropic state (66). Hence we obtain the range of applicability of Eq. (76) as

$$1\gg\frac{|T_c-T|}{T_c}\gg N^{-2}. \tag{82}$$

V. DISCUSSION

In the previous sections we have derived a number of results for the conductivity tensor of two-dimensional interacting electrons on a half-filled Nth Landau level with $N\gg 1$. Our results are applicable for samples of small size $L\ll\xi$ *only* where ξ stands for the minimal lengthscale among the translational order breaking length and Larkin length such that the fluctuations do not destroy the mean-field UCDW state on these lengthscales. We have demonstrated that below temperature T_c of isotropic-to-UCDW transition the anisotropic part of the conductivity tensor emerges in the mean-field approximation. At $(T_c-T)/T_c\ll 1$ the anisotropic part is proportional to deviation of temperature from T_c. As it is shown in Fig. 6, it results in a cusp of temperature dependence of the conductivity at $T=T_c$. According to results (76), fluctuations of the UCDW order parameter above and below transition temperature T_c smooth out the cusp (see Fig. 6).

The theory developed above involves several important assumptions such that the electron-electron interaction is weak (1), an applied magnetic field is weak (4) and random potential is white-noise potential that produces small Landau level broadening compared to Δ_{ex} [see Eq. (3)]. All the assumptions are failed for the experimental samples. In Refs. 2 and 3 the electron-electron interaction was relatively strong, $r_s\sim 1$, the applied magnetic field was moderate with $N=2,3,4$ and the random potential had correlation length $d\sim l_H$. Therefore, the comparison of our theory with experiments can be *qualitative* only.

The results (67) and (76) for the anisotropic part of the conductivity tensor are in qualitative agreement with experimental ones.[2,3,15] If a current runs in the direction of UCDW modulation, the conductivity (resistance) is less (more) than the conductivity (resistance) for the case of current running in the direction perpendicular to charge density modulation. Correction to the Hall conductivity is virtually nonexistent (it contains small parameter $\max\{T_c,\tau^{-1}\}/\omega_H\ll 1$) that is consistent with experimental data.[2,3,15] The temperature dependence of the conductivity tensor derived in the present paper describes the region $|T-T_c|\ll T_c$. It agrees qualitatively with experimental dependence near the temperature where strong deviation between R_{xx} and R_{yy} appears. In our theory this temperature should be of the order of T_c. Estimation of T_c from Eqs. (53) and (54) are in reasonable agreement[16] with experimental result.

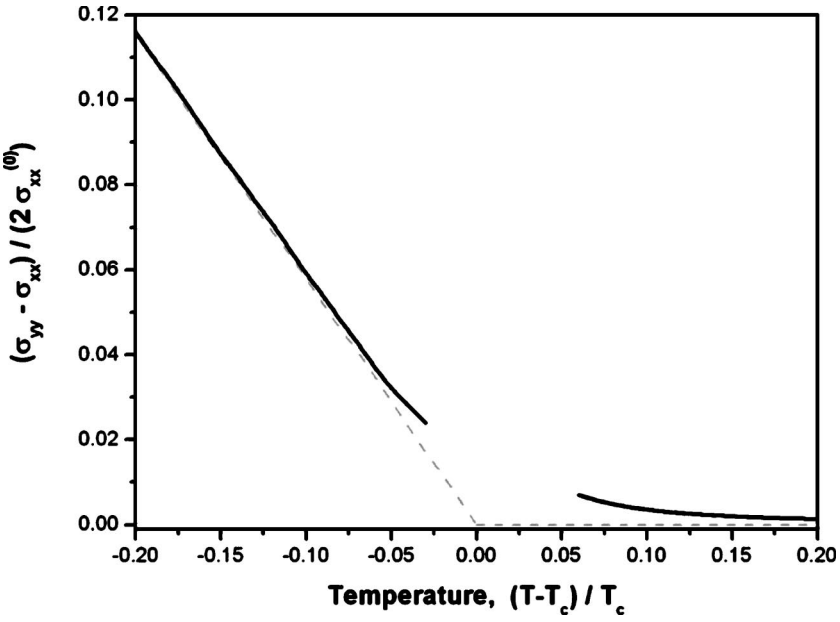

FIG. 6. The dependence of anisotropic part of conductivity $(\sigma_{yy}-\sigma_{xx})/2\sigma_{xx}^{(0)}$ on temperature for $1/4\pi T_c\tau=0.24$ and $\eta=0.01$. Dashed line corresponds to mean-field result (67).

The UCDW state exists also near half-filled high Landau level.[6,16] If chemical potential $\mu_N \neq 0$, the functions h_{ab} and H_{ab} are dependent not only on parameter $1/4\pi T_c\tau$ but on ratio μ_N/T_c too. Increasing the chemical potential μ_N we transform the UCDW state to the triangular CDW state characterized by vectors $\mathbf{Q}_1$, $\mathbf{Q}_2$, and $\mathbf{Q}_3$ directed at angles ϕ, $\phi+2\pi/3$, and $\phi+4\pi/3$ with respect to the x axis, respectively. However, due to the identity

$$1 + e^{2\pi i/3} + e^{-2\pi i/3} = 0 \tag{83}$$

the contribution from three vectors $\mathbf{Q}_i$ to the anisotropic part of the conductivity tensor vanishes. Thus, the conductivity tensor of the triangular CDW state does not contain the anisotropic part in the approximation which is of the second order in the CDW order parameter. For reasons to be explained shortly, the conductivity tensor of the triangular CDW state should not have anisotropic component in general.

It is worthwhile to mention that from the physical point of view the anisotropic part of the mean-field conductivity tensor appears due to the existence of the UCDW induced anisotropic potential $\lambda(\mathbf{r})$ in action (49). Anisotropic resistance of two-dimensional electrons in a weak magnetic field in the presence of unidirectional periodic potential has been measured at several kelvins in heterostructures with mobility $\mu_0 \sim 10^6$ cm^2/V s by Weiss, von Klitzing, Ploog, and Weimann fifteen years ago.[35] Theoretically, the effect of unidirectional periodic potential on the conductivity tensor has been investigated with the help of both the kinetic equation[36,37] and the diagrammatic technique.[38–40] However, the case of separate Landau levels considered in the present paper has not been analyzed. The important difference of the induced potential $\lambda(\mathbf{r})$ from external periodic potential is that the $\lambda(\mathbf{r})$ exists on the Nth Landau level only whereas external potential scatters electrons on all Landau levels.

VI. CONCLUSION

We obtained the conductivity tensor of two-dimensional electrons in the presence of weak disorder and weak magnetic field at half-filled high Landau level where the UCDW state exists. In the framework of the order parameter expansion we derived that at $T_c - T \ll T_c$ anisotropic part of the conductivity tensor proportional to $(T_c-T)/T_c$ emerges. Also we demonstrated that the order parameter fluctuations result in additional anisotropic contribution near T_c to the conductivity tensor that wash out the mean-field cusp at $T=T_c$. The results obtained are in qualitative agreement with the experimental findings.

In spite of remarkable qualitative agreement of our theory with experimental results[2,3,15] there is the principal question of whether the enhancement of anisotropic part of the resistance around half-filled Landau levels should be interpreted in terms of the isotropic-to-UCDW transition or a Kosterlitz-Thouless-like isotropic-to-nematic transition rounded by crystal field.[9,10] Since the results for the lengthscale ξ at which the fluctuations destroy the UCDW order obtained in simple weak crystallization treatment[16] as well as in more elaborate treatments[12,14] suggest the power law dependence of ξ on N we conclude that the scenario with isotropic-to-nematic transition[9,10] for moderate values of N is more probable. On the one hand, the microscopic theory that involves real electron-electron interaction and long-ranged random potential is needed in order to determine the elastic parameters for nematic and stripe phases as well as transport coefficients. On the other hand, the real-space imaging[41] of the UCDW order (stripes) or polarized light scattering experi-

ments to detect orientational order in nematic phase would give the most definitive answer.

ACKNOWLEDGEMENTS

The author is grateful to M. A. Baranov, L. I. Glazman, M. V. Feigelman, S. V. Iordansky, P. M. Ostrovsky, M. A. Skvortsov for discussions and especially to R. R. Du for experimental plots. Financial support from Russian Foundation for Basic Research (RFBR), the Russian Ministry of Science, Dynasty Foundation, Forschungszentrum Jülich (Landau Scholarship), and Dutch Science Foundation (FOM) is acknowledged.

APPENDIX A: CALCULATION OF CHARACTERISTIC ENERGY T_1

According to Eqs. (21) and (40) the $T_1(q)$ is given by

$$T_1(q) = -n_L e^{i\phi}\Bigg[U_{\rm scr}(q)\mathcal{J}_0(qR_c)\mathcal{J}_1(qR_c)e^{-i\phi} - \sqrt{2N}\int \frac{d\boldsymbol{p}}{(2\pi)^2 n_L} e^{-i\mathbf{q}\mathbf{p}l_H^2} U_{\rm scr}(p)\frac{p_x - ip_y}{p^2 l_H} \times L_{NN}\left(\frac{p^2 l_H^2}{2}\right) L_{N,N-1}\left(\frac{p^2 l_H^2}{2}\right) e^{-p^2 l_H^2/2}\Bigg], \quad \text{(A1)}$$

where we use the following result:[1]

$$F_{N,N-1}(\mathbf{q}) = \sqrt{2N}\frac{q_x - iq_y}{q^2 l_H} e^{-q^2 l_H^2/4} L_{N,N-1}\left(\frac{q^2 l_H^2}{2}\right) \approx e^{-i\phi}\mathcal{J}_1(qR_c),\ qR_c \ll 2N. \quad \text{(A2)}$$

The characteristic energy $T_1 = T_1(Q_0)$ is given by

$$T_1 = \frac{r_s \omega_H}{4\sqrt{2}}\int_0^{4N} \frac{dx}{\tilde{\epsilon}(x)} \frac{\mathcal{J}_1(4Nx)\mathcal{J}_0(4Nx)}{\sqrt{1-x^2}} \mathcal{J}_1(2r_0 x), \quad \text{(A3)}$$

where

$$\tilde{\epsilon}(x) = 1 + \frac{r_s}{x\sqrt{2}}[1 - \mathcal{J}_0^2(4Nx)]. \quad \text{(A4)}$$

Performing calculation of the integral, we find

$$T_1 = \frac{r_s \omega_H}{16\pi N\sqrt{2}}\left[\frac{r_0}{2}\ln\left(1 + \frac{1}{\sqrt{2}r_0 r_s}\right) + \frac{c_1}{1 + \sqrt{2}r_0 r_s}\right], \quad \text{(A5)}$$

where constant c_1 equals

$$c_1 = \sqrt{\frac{r_0}{\pi}}\int_{1/2r_0}^1 \frac{dx}{x\sqrt{x(1-x^2)}} \sin\left(2r_0 x - \frac{\pi}{4}\right) \approx 1.097. \quad \text{(A6)}$$

As we can see from Eq. (A5) the characteristic energy $T_1 \sim T_0/N$ as we mentioned above.

APPENDIX B: CALCULATION OF THE $I_{p_1 p_2 p_3 p_4}(\mathbf{Q}_0)$

Using definition (63) and Eq. (A2), we obtain in the limit $N \gg 1$

$$I_{N,N-1,N-1,N} = I_{N,N+1,N+1,N} = gn_L N\int_0^\infty \frac{dx}{x}e^{-x} \times [L_N^{-1}(x)]^2 \mathcal{J}_0(r_0\sqrt{2x}), \quad \text{(B1)}$$

$$I_{N,N-1,N,N+1} = I_{N,N+1,N,N-1} = gn_L N\int_0^\infty \frac{dx}{x}e^{-x} \times L_N^{-1}(x)L_{N+1}^{-1}(x)\mathcal{J}_0(r_0\sqrt{2x}),$$

$$I_{N,N,N,N-1} = -I_{N,N,N,N+1} = gn_L e^{i\phi}\sqrt{N}\int_0^\infty \frac{dx}{\sqrt{x}}e^{-x} \times L_N^{-1}(x)L_N(x)\mathcal{J}_1(r_0\sqrt{2x}),$$

$$I_{N,N+1,N,N} = -I_{N,N+1,N,N} = gn_L e^{-i\phi}\sqrt{N}\int_0^\infty \frac{dx}{\sqrt{x}} \times e^{-x}L_{N+1}^{-1}(x)L_N(x)\mathcal{J}_1(r_0\sqrt{2x}),$$

$$I_{N,N-1,N,N-1} = I_{N,N+1,N,N+1}^* = gn_L e^{2i\phi}N\int_0^\infty \frac{dx}{x}e^{-x} \times [L_N^{-1}(x)]^2 \mathcal{J}_2(r_0\sqrt{2x}),$$

$$I_{N,N-1,N+1,N} = I_{N,N+1,N-1,N}^* = gn_L e^{2i\phi}N\int_0^\infty \frac{dx}{x}e^{-x} \times L_N^{-1}(x)L_{N+1}^{-1}(x)\mathcal{J}_2(r_0\sqrt{2x}).$$

With a help of asymptotic expression for Laguerre polynomial[42]

$$L_N^\alpha(x) \simeq \frac{1}{\sqrt{\pi x}}e^{x/2}\left(\frac{N}{x}\right)^{(2\alpha-1)/4}\cos\left(2\sqrt{Nx} - \frac{\alpha\pi}{2} - \frac{\pi}{4}\right), \quad N \gg 1, \quad \text{(B2)}$$

we find

$$I_{N,N-1,N-1,N} = I_{N,N+1,N+1,N} = gn_L\frac{2}{\pi}\int_0^1 dx\frac{\mathcal{J}_0(2r_0x)}{\sqrt{1-x^2}}, \quad \text{(B3)}$$

$$I_{N,N-1,N,N+1} = I_{N,N+1,N,N-1} = gn_L\frac{2}{\pi}\int_0^1 dx(1-2x^2)\frac{\mathcal{J}_0(2r_0x)}{\sqrt{1-x^2}},$$

$$I_{N,N,N,N-1} = -I_{N,N,N,N+1} = gn_L e^{i\phi}\frac{2}{\pi}\int_0^1 dxx\frac{\mathcal{J}_1(2r_0x)}{\sqrt{1-x^2}},$$

$$I_{N,N+1,N,N} = -I_{N,N+1,N,N} = gn_L e^{-i\phi}\frac{2}{\pi}\int_0^1 dxx\frac{\mathcal{J}_1(2r_0x)}{\sqrt{1-x^2}},$$

$$I_{N,N-1,N,N-1} = I_{N,N+1,N,N+1}^* = gn_L e^{2i\phi}\frac{2}{\pi}\int_0^1 dx\frac{\mathcal{J}_2(2r_0x)}{\sqrt{1-x^2}},$$

$$I_{N,N-1,N+1,N} = I^{*}_{N,N+1,N-1,N} = gn_L e^{2i\phi}\frac{2}{\pi}\int_0^1 dx(1-2x^2)\frac{\mathcal{J}_2(2r_0x)}{\sqrt{1-x^2}}.$$

The integrals can be evaluated by using the following equality:[42]

$$\frac{2}{\pi}\int_0^{\pi/2} d\phi\cos(2\mu\phi)\mathcal{J}_{2\nu}(2r_0\cos\phi) = \mathcal{J}_{\nu+\mu}(r_0)\mathcal{J}_{\nu-\mu}(r_0). \tag{B4}$$

Finally, it yields the results presented in Table I.

[1] For a review, see T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).
[2] M. P. Lilly, K. B. Cooper, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **82**, 394 (1999).
[3] R. R. Du, D. C. Tsui, H. L. Stormer, L. N. Pfeiffer, and K. W. West, Solid State Commun. **109**, 389 (1999).
[4] A. A. Koulakov, M. M. Fogler, and B. I. Shklovskii, Phys. Rev. Lett. **76**, 499 (1996).
[5] M. M. Fogler, A. A. Koulakov, and B. I. Shklovskii, Phys. Rev. B **54**, 1853 (1996).
[6] R. Moessner and J. T. Chalker, Phys. Rev. B **54**, 5006 (1996).
[7] J. Toner and D. R. Nelson, Phys. Rev. B **23**, 316 (1981).
[8] E. Fradkin and S. A. Kivelson, Phys. Rev. B **59**, 8065 (1999).
[9] E. Fradkin, S. A. Kivelson, E. Manousakis, and K. Nho, Phys. Rev. Lett. **84**, 1982 (2000).
[10] K. B. Cooper, M. P. Lilly, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **65**, 241313 (2002).
[11] For a review, see M. M. Fogler, in *High Magnetic Fields: Applications in Condensed Matter Physics and Spectroscopy*, edited by C. Berthier, L.-P. Levy, and G. Martinez (Springer-Verlag, Berlin, 2002).
[12] D. G. Barci, E. Fradkin, S. A. Kivelson, and V. Oganesyan, Phys. Rev. B **65**, 245319 (2002); D. G. Barci and E. Fradkin, *ibid.* **65**, 245320 (2002).
[13] A. I. Larkin, Sov. Phys. JETP **31**, 784 (1970).
[14] S. Scheidl and F. von Oppen, Europhys. Lett. **55**, 260 (2001).
[15] J. P. Eisenstein, M. P. Lilly, K. B. Cooper, L. N. Pfeiffer, and K. W. West, Physica E (Amsterdam) **9**, 1 (2000).
[16] I. S. Burmistrov and M. A. Baranov, Phys. Rev. B **68**, 155328 (2003).
[17] I. S. Burmistrov, Ph.D. thesis (in Russian), L. D. Landau Institute for Theoretical Physics, Chernogolovka, Russia, 2004.
[18] L. G. Aslamasov and A. I. Larkin, Phys. Lett. **26**, 238 (1968).
[19] I. S. Burmistrov, Pis'ma Zh. Eksp. Teor. Fiz. **79**, 212 (2004); [JETP Lett. **79**, 177 (2004)].
[20] I. L. Aleiner and L. I. Glazman, Phys. Rev. B **52**, 11296 (1995).
[21] I. S. Burmistrov, Zh. Eksp. Teor. Fiz. **122**, 150 (2002) [JETP **95**, 132 (2002)].
[22] S. F. Edwards and P. W. Anderson, J. Phys. F: Met. Phys. **5**, 965 (1975).
[23] In the clean case ($\tau^{-1}\to 0$) the screened electron-electron interaction was found in Ref. 20.
[24] H. Fukuyama, P. M. Platzman, and P. W. Anderson, Phys. Rev. B **19**, 5211 (1979).
[25] K. B. Efetov, A. I. Larkin, and D. E. Khemel'nitzkii, Zh. Eksp. Teor. Fiz. **79**, 1120 (1980) [Sov. Phys. JETP **52**, 568 (1980)].
[26] A. M. M. Pruisken, Nucl. Phys. B **235**, 277 (1984).
[27] In the approximation quadratic in the order parameter $\Delta_{p_1p_2}(\mathbf{Q})$ the contributions to the thermodynamic potential and to the free energy are the same.
[28] W. Pan, R. R. Du, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. **83**, 820 (1999).
[29] M. P. Lilly, K. B. Cooper, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **83**, 824 (1999).
[30] K. B. Cooper, M. P. Lilly, J. P. Eisenstein, T. Jungwirth, L. N. Pfeiffer, and K. W. West, Solid State Commun. **119**, 89 (2001).
[31] K. B. Cooper, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **92**, 026806 (2004).
[32] J. Zhu, W. Pan, H. L. Stormer, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **88**, 116803 (2002).
[33] R. L. Stratonovich, Dokl. Akad. Nauk SSSR **115**, 1097 (1957) [Sov. Phys. Dokl. **2**, 416 (1957)]; J. Hubbard, Phys. Rev. Lett. **3**, 77 (1959).
[34] T. Jungwirth, A. H. MacDonald, L. Smrčka, and S. M. Girvin, Phys. Rev. B **60**, 15574 (1999).
[35] D. Weiss, K. von Klitzing, K. Ploog, and G. Weimann, Europhys. Lett. **8**, 179 (1989).
[36] C. W. J. Beenakker, Phys. Rev. Lett. **62**, 2020 (1989).
[37] A. D. Mirlin and P. Wölfle, Phys. Rev. B **58**, 12986 (1998).
[38] R. R. Gerhardts, D. Weiss, and K. von Klitzing, Phys. Rev. Lett. **62**, 1173 (1989).
[39] C. Zhang and R. R. Gerhardts, Phys. Rev. B **41**, 12850 (1990).
[40] F. M. Peeters and P. Vasilopoulos, Phys. Rev. B **46**, 4667 (1992).
[41] G. Finkelstein, P. I. Glicofridis, R. C. Ashoori, and M. Shayegan, Science **289**, 90 (2000); S. Ilani, A. Yacobi, D. Mahalu, and H. Shtrikman, Phys. Rev. Lett. **84**, 3133 (2000).
[42] I. S. Gradshteyn and I. M. Ryzik, *Mathematical Tables* (Academic, New York, 1980).