Spin-Hall effect in a disordered two-dimensional electron system

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We calculate the spin-Hall conductivity for a two-dimensional electron gas within the self-consistent Born approximation, varying the strength and type of disorder. In the weak disorder limit we find both analytically and numerically a vanishing spin-Hall conductivity even when we allow a momentum dependent scattering. Separating the reactive from the dissipative current response, we find the universal value $\sigma_{sH}^{R}=e/8\pi$ for the reactive response, which cancels however with the dissipative part $\sigma_{sH}^{D}=-e/8\pi$.

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Spin-orbit coupling in two-dimensional electron systems allows a number of unconventional transport phenomena, since charge current and the spin degrees of freedom are coupled.[1] In particular the spin-Hall effect in two-dimensional electron systems, i.e., a spin current which flows in the plane but perpendicular to the electrical current, and which is polarized perpendicular to the plane, has been discussed intensively[2–16,21] over the last year. The spin-Hall conductivity connects the spin current with an electric field $j_y^z=\sigma_{sH}E_x$, where j_y^z denotes a current in the y direction with spin polarization in the z direction. In a clean two-dimensional electron gas, the spin-Hall conductivity was predicted to have a universal value $\sigma_{sH}=e/8\pi$, independent of the strength of the spin-orbit scattering.[3] Several publications have addressed the issue of whether this result is modified in the presence of impurity scattering. Murakami[12] analyzed the Luttinger Hamiltonian,[17] which applies to two-dimensional hole gases, and concluded that the spin-Hall conductivity in the limit of weak impurity scattering reproduces the intrinsic value (at least when restricting to s-wave impurity scattering). For the Rashba model,[18] which applies to two-dimensional electron gases, conflicting results exist in the literature: By applying the standard Green's function techniques Inoue *et al.*[11] and Mishchenko *et al.*[15] concluded that s-wave impurities suppress the spin-Hall effect in bulk samples even when the disorder broadening of the energy levels is small compared to the spin-orbit splitting. On the other hand, Dimitrova[13] and Chalaev and Loss,[14] starting from the same model Hamiltonian and applying similar methods, found a nonzero spin-Hall conductivity. Even direct numerical evaluations of the effect do not fully agree with each other: Xiong and Xie[9] found within a scattering matrix approach the universal value of the spin-Hall conductance $G_{sH}=e/8\pi$ over a large parameter range. Nomura *et al.*[10] on the other hand found a spin-Hall conductivity of the order of but not identical to the universal value.

In this paper we calculate the spin-Hall conductivity for a bulk sample within the self-consistent Born approximation. We confirm Refs. 11 and 15, i.e., we find that even a weak disorder suppresses the spin-Hall conductivity. For s-wave scatterers we calculate the impurity self-energy and the dressed current vertex numerically. This allows us to obtain results beyond the limit $\epsilon_F\tau\to\infty$, which is accessible analytically. We find that a nonzero spin-Hall conductivity is, in principle, possible although it remains much smaller than $e/8\pi$.

Our calculations are based on our previous work,[19,20] where a number of technical details can be found. In the following we sketch the derivation of the spin-Hall conductivity. The starting point is the Hamiltonian

$$H=\frac{p^2}{2m}+\alpha\sigma\cdot\mathbf{p}\times\mathbf{e}_z, \tag{1}$$

where the parameter α describes the strength of the spin-orbit coupling, σ is a vector of Pauli matrices, and $\mathbf{e}_z$ is a unit vector perpendicular to the two dimensional system. The spin-Hall conductivity is obtained by the standard linear response theory as

$$\sigma_{sH}=\lim_{\omega\to 0}\frac{e}{\omega}\int\frac{d\epsilon}{2\pi}\,\mathrm{Tr}[\overline{j_s^y G^<(\epsilon) j_c^x G^A(\epsilon-\omega)} + \overline{j_s^y G^R(\epsilon) j_c^x G^<(\epsilon-\omega)}], \tag{2}$$

with $G^<(\epsilon)=f(\epsilon)(G^R-G^A)$, $f(\epsilon)$ being the Fermi function. In Eq. (2), the spin- and charge-current operators are given by $\mathbf{j}_s=(1/4)\{\sigma_z\mathbf{v}+\mathbf{v}\sigma_z\}$ and $\mathbf{j}_c=\mathbf{v}$, respectively. The velocity operator $\mathbf{v}$ is obtained from the Hamiltonian (1) and reads $v^{x,y}=p^{x,y}/m\mp\alpha\sigma_{y,x}$. We choose the electron charge as $-e$ ($e>0$). The trace in Eq. (2) is over the eigenstates of the Hamiltonian, and the bar indicates that the expression must be averaged over the disorder configurations.

When performing the disorder average, we rely on the self-consistent Born approximation. To begin with, we consider pointlike, i.e., pure s-wave scatterers. The retarded/advanced impurity self-energy is then given by

$$\Sigma^{R,A}=\frac{1}{2\pi N_0\tau}\sum_{\mathbf{p}}G^{R,A}(\mathbf{p}). \tag{3}$$

Due to the spin-orbit coupling, the Green's functions have a nontrivial structure in the spin space, although the self-energy remains diagonal. Explicitly, one finds that $\Sigma_{ss'} = \Sigma_0\delta_{ss'}$, $G_{ss'} = G_0\delta_{ss'} + G_1\sigma^x_{ss'} + G_2\sigma^y_{ss'}$ with

$$G_0(\mathbf{p}) = \frac{1}{2}(G_+ + G_-), \tag{4}$$

$$G_1(\mathbf{p}) = \frac{1}{2}\frac{p_y}{p}(G_+ - G_-), \tag{5}$$

$$G_2(\mathbf{p}) = -\frac{1}{2}\frac{p_x}{p}(G_+ - G_-), \tag{6}$$

$$G_\pm = \left(\epsilon + \mu - \frac{p^2}{2m} \mp \alpha p - \Sigma_0\right)^{-1}. \tag{7}$$

By taking the zero frequency limit of Eq. (2), the spin-Hall conductivity reads

$$\sigma_{sH} = -\frac{e}{4\pi}\sum_{\mathbf{p}} \mathrm{Tr}_\sigma[2j_s^y G^R(\mathbf{p}) J_c^x G^A(\mathbf{p})], \tag{8}$$

since terms of the type $G^R G^R$ and $G^A G^A$ contribute only in the order $(1/\epsilon_F\tau)(\alpha/v_F)^2$ and can be safely neglected in the limit $\alpha p_F \ll \epsilon_F$ and/or for weak disorder $\epsilon_F\tau \gg 1$. The charge current J_c^x has to be calculated including the vertex corrections, $J_c^x = p^x/m + \Gamma^x$, compare Eq. (33) of Ref. 20. In the case of s-wave impurity scattering, the momentum dependent part of the current vertex is not renormalized, while the momentum independent, but spin-dependent part, Γ^x, is obtained by solving the set of equations

$$\Gamma^x_{ss'} = \gamma^x_{ss'} + \frac{1}{2\pi N_0\tau}\sum_{\mathbf{p}}\sum_{ab} G^R_{sa}\Gamma^x_{ab}G^A_{bs'}, \tag{9}$$

with the *effective* bare vertex given by

$$\gamma^x_{ss'} = -\alpha\sigma^y_{ss'} + \frac{1}{2\pi N_0\tau}\sum_{\mathbf{p},a} G^R_{sa}(\mathbf{p})\frac{p_x}{m}G^A_{as'}(\mathbf{p}). \tag{10}$$

By expanding $\Gamma^x_{ss'} = \Sigma_\mu \Gamma^x_\mu \sigma^\mu_{ss'}$ in Pauli matrices we obtain the spin-Hall conductivity as

$$\sigma_{sH} = -\frac{e}{\pi}\Gamma^x_2 \,\mathrm{Im}\sum_{\mathbf{p}}\frac{p_y}{m}G^R_0(\mathbf{p})G^A_1(\mathbf{p}). \tag{11}$$

Performing the momentum integration Eq. (11) under the restriction that $\alpha p_F \ll \epsilon_F$ and $\epsilon_F\tau \gg 1$ leads to

$$\sigma_{sH} = -\frac{e}{\pi}\Gamma^x_2 \pi N_0\tau\frac{\alpha p_F v_F\tau}{1 + 4\alpha^2 p_F^2\tau^2}. \tag{12}$$

Apparently σ_{sH} goes to zero when the spin-splitting αp_F is small compared to the disorder broadening of the levels $1/\tau$. If we neglect vertex corrections, i.e., if we insert in Eq. (11) the bare vertex $\Gamma^x_2 = -\alpha$, we find

$$\sigma_{sH}\big|_{\text{bare vertex}} = \frac{e}{8\pi}\frac{4\alpha^2 p_F^2\tau^2}{1 + 4\alpha^2 p_F^2\tau^2}, \tag{13}$$

i.e., the universal value $\sigma_{sH} = e/8\pi$ is recovered in the weak disorder limit. On the other hand, by inserting the dressed vertex $\Gamma^x_2 \approx 0$ as calculated in Refs. 19 and 20 one finds that $\sigma_{sH} \approx 0$. Our result then agrees with that found in Refs. 11 and 15.

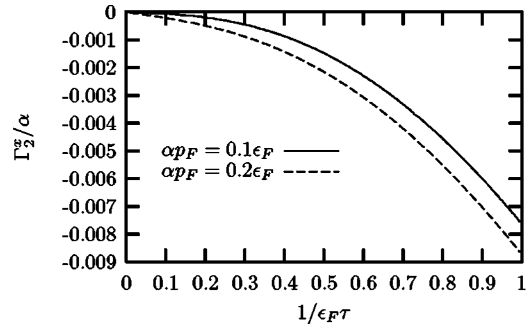

FIG. 1. The dressed vertex Γ^x_2 in units of α as function of disorder strength $1/(\epsilon_F\tau)$. Γ^x_2 enters the dressed charge current, $J_c^x = p_x/m + \Gamma^x_2\sigma_y$ and thus the spin-Hall conductivity, Eq. (12). In comparison with the bare chare current, $j_c^x = p_x/m - \alpha\sigma_y$, the spin dependence is strongly reduced.

As explained in Ref. 20, the vanishing of the dressed vertex Γ^x_2 is due to the fact that the integral on the right-hand side of Eq. (10) gives $\approx \alpha\sigma^y$, making the effective bare vertex γ itself to vanish. A more careful numerical evaluation of the integral for arbitrary disorder strength actually shows that the compensation of the two terms in Eq. (10) is exact only in the weak disorder limit, i.e., $\epsilon_F\tau \gg 1$. In Fig. 1 we show the dressed vertex as a function of disorder. Γ^x_2 goes to zero as $1/\epsilon_F\tau \to 0$, nothing special is observed as $\alpha p_F \sim 1/\tau$, and even in the strong disorder limit Γ^x_2 remains much smaller than its bare value $(-\alpha)$. We conclude that although, in principle, a nonzero spin-Hall conductivity may be obtained, one expects a much smaller value than the universal one. For a quantitative theory of the spin-Hall conductivity in the strong disorder limit on the other hand one has to go beyond the Born approximation and ladder summation. This however is not the scope of the present paper.

Next we address the question whether the spin-Hall effect is sensitive to the type of disorder potential. Inoue *et al.*[11] argued that σ_{sH} may be nonzero for long-range defect potentials, although an explicit result has not been given. In our calculation we follow again closely[20] where all the details can be found. Here we assume weak disorder, so that the inequalities $\epsilon_F \gg \alpha p_F \gg 1/\tau$ hold. In the following, we work in the eigenstate basis of the Hamiltonian (1)

$$|\mathbf{p}\pm\rangle = \frac{1}{\sqrt{2}}\{\pm i\exp(-i\varphi)|\mathbf{p}\uparrow\rangle + |\mathbf{p}\downarrow\rangle\}, \tag{14}$$

where $\tan(\varphi) = p_y/p_x$ and the corresponding eigenvalues are $E_\pm = p^2/2m \pm \alpha p$. In this basis, the matrix elements of the current operators read

$$\langle\mathbf{p}\pm|j_s^y|\mathbf{p}\pm\rangle = 0, \tag{15}$$

$$\langle\mathbf{p}\pm|j_s^y|\mathbf{p}\mp\rangle = -\frac{1}{2}\frac{p}{m}\sin(\varphi), \tag{16}$$

$$\langle \mathbf{p} \pm | j_c^x | \mathbf{p} \pm \rangle = \left(\frac{p}{m} \pm \alpha \right) \cos(\varphi), \tag{17}$$

$$\langle \mathbf{p} \mp | j_c^x | \mathbf{p} \pm \rangle = \mp i\alpha \sin(\varphi). \tag{18}$$

To use Eq. (8) we need the dressed charge operator J_c^x. Since, as seen from the above equations, the spin-current operator is off diagonal in the eigenstate basis we get the spin-Hall conductivity in the form

$$\sigma_{sH} = -\frac{e}{\pi} \sum_{\mathbf{p}} \mathrm{Re}[\langle \mathbf{p} + | j_s^y | \mathbf{p} - \rangle \langle \mathbf{p} - | J_c^x | \mathbf{p} + \rangle G_-^R(\mathbf{p}) G_+^A(\mathbf{p})]. \tag{19}$$

To calculate the dressed current operator we make use of the assumption that the spin orbit splitting is large compared to the impurity broadening of the levels, $\alpha p_F \gg 1/\tau$. The off-diagonal matrix elements of the current operator are then obtained in terms of the diagonal ones

$$\begin{aligned} \langle \mathbf{p} \mp | J_c^x | \mathbf{p} \pm \rangle &= \langle \mathbf{p} \mp | j_c^x | \mathbf{p} \pm \rangle \\ &\quad + \sum_{\mathbf{p}', m} [\langle \mathbf{p} \mp | V | \mathbf{p}' m \rangle \langle \mathbf{p}' m | V | \mathbf{p} \pm \rangle \\ &\quad \times G_m^R(\mathbf{p}') G_m^A(\mathbf{p}') \langle \mathbf{p}' m | J_c^x | \mathbf{p}' m \rangle]. \end{aligned} \tag{20}$$

The diagonal matrix elements on the other hand were already considered in Ref. 20, and are obtained from the equation

$$\begin{aligned} \langle \mathbf{p} \pm | J_c^x | \mathbf{p} \pm \rangle &= \langle \mathbf{p} \pm | j_c^x | \mathbf{p} \pm \rangle \\ &\quad + \sum_{\mathbf{p}', m} |\langle \mathbf{p} \pm | V | \mathbf{p}' m \rangle|^2 G_m^R(\mathbf{p}') G_m^A(\mathbf{p}') \\ &\quad \times \langle \mathbf{p}' m | J_c^x | \mathbf{p}' m \rangle. \end{aligned} \tag{21}$$

We consider impurity scattering which conserves spin, but allow the scattering amplitude to be momentum-transfer dependent. Such a dependence appears as a product of two contributions. The first is due to the type of disorder potential one considers $V_{\mathbf{p},\mathbf{p}'}$, while the second is induced by the transformation to the eigenstate basis. The latter gives rise to the following matrix elements:

$$\langle \mathbf{p} - | V | \mathbf{p}' \pm \rangle \langle \mathbf{p}' \pm | V | \mathbf{p} + \rangle = \mp \tfrac{i}{2} \sin(\varphi - \varphi') |V_{\mathbf{p},\mathbf{p}'}|^2 \tag{22}$$

and

$$|\langle \mathbf{p} \pm | V | \mathbf{p}' \pm \rangle|^2 = \tfrac{1}{2} |V_{\mathbf{p},\mathbf{p}'}|^2 [1 + \cos(\varphi - \varphi')], \tag{23}$$

$$|\langle \mathbf{p} \pm | V | \mathbf{p}' \mp \rangle|^2 = \tfrac{1}{2} |V_{\mathbf{p},\mathbf{p}'}|^2 [1 - \cos(\varphi - \varphi')]. \tag{24}$$

We assume that the scattering amplitude $V_{\mathbf{p},\mathbf{p}'}$ depends on the (modulus of the) momentum transfer, $|\mathbf{p} - \mathbf{p}'|$, so that the scattering probability can be expressed in terms of the angle between the incoming and scattered particle. Under this condition we can expand the scattering probability as

$$|V_{\mathbf{p},\mathbf{p}'}|^2 = V_0 + 2V_1 \cos(\varphi - \varphi') + 2V_2 \cos(2\varphi - 2\varphi') + \ \ldots\ , \tag{25}$$

where the harmonics $V_0, V_1, \ldots$, are functions of $|\mathbf{p}|$ and $|\mathbf{p}'|$. In the following we will ignore this dependence. This is justified when $V_{\mathbf{p},\mathbf{p}'}$ depends on the momentum transfer only on a scale $\Delta\mathbf{p}$ which is larger than the momentum scale defined by the spin-orbit coupling $\Delta\mathbf{p} \sim m\alpha$ or disorder broadening, $\Delta\mathbf{p} \sim 1/v_F\tau$. In the limit we consider here, namely, $1/\tau \ll \alpha p_F \ll \epsilon_F$, all relevant momentum integrations are restricted to a narrow region near the Fermi momenta of the two subbands, $p \approx p_F$, so that the harmonics of the scattering probability have to be determined at the Fermi momentum, e.g., $V_0 = V_0(p_F, p_F)$. Given the scattering probability it is straightforward to calculate the self-energy. Whereas for s-wave scatterers the impurity self-energy has a trivial structure in spin space, cf. Eq. (3), in the present situation the self-energy is diagonal in the eigenstate basis of the Hamiltonian with different lifetimes in the two bands, $\mathrm{Im}\,\Sigma_\pm^{R,A} = \mp i/2\tau_\pm$.[20]

To obtain the spin-Hall conductivity the two momentum integrations over $\mathbf{p}$ and $\mathbf{p}'$ have to be performed. We split the momentum integration in an integral over the energy $\xi = p^2/2m - \mu$ and the angular variable φ

$$\sum_{\mathbf{p}} \to N_0 \int \mathrm{d}\xi \int \frac{\mathrm{d}\varphi}{2\pi}, \tag{26}$$

and find

$$N_0 \int \mathrm{d}\xi G_-^R G_+^A = \frac{2\pi i N_0}{2\alpha p_F + i/\tau} \approx \frac{i\pi N_0}{\alpha p_F}, \tag{27}$$

$$N_0 \int \mathrm{d}\xi G_\pm^R G_\pm^A = 2\pi N_\pm \tau_\pm, \tag{28}$$

where $N_\pm$ and $\tau_\pm$ are the density of states and the lifetime in the two subbands. Notice that the first of the two integrals is correct only to the lowest order in the (small) parameter $\alpha p_F/\epsilon_F$, whereas the second integration is valid beyond that limit. Finally the spin-Hall conductivity is determined as

$$\sigma_{sH} = \frac{e}{8\pi} \left[1 - \frac{1}{4\alpha} \frac{N_+\tau_+ J_+^x - N_-\tau_- J_-^x}{N_0 \tau} \frac{V_2 - V_0}{V_0} \right], \tag{29}$$

where the second term is due to vertex corrections. When calculating the product $N_\pm \tau_\pm J_\pm^x$ to zero order in $\alpha p_F/\epsilon_F$ the vertex corrections disappear. Expanding the density of states, the scattering time and to dressed current operator to first order yields

$$N_\pm \approx N_0 \left(1 \mp \frac{\alpha p_F}{2\epsilon_F} \right), \tag{30}$$

$$\tau_\pm \approx \tau \left(1 \pm \frac{V_1}{V_0} \frac{\alpha p_F}{2\epsilon_F} \right), \tag{31}$$

$$J_{\pm}^{x} \approx \frac{V_0}{V_0 - V_1}\frac{p_F}{m} \mp \alpha\frac{V_0 + V_2}{V_0 - V_2}. \qquad (32)$$

Notice that the dressed current operator, to the leading order in α, is of the familiar form $J = j\tau_{tr}/\tau$, where τ_{tr} is the transport scattering time. By combining all the terms, one then finds that the spin-Hall conductivity (29) vanishes as in the case of pure s-wave scattering, $\sigma_{sH}=0$.[22]

As a last useful observation, we separate the reactive and dissipative contributions to the current response, $\sigma_{sH} = \sigma_{sH}^{R} + \sigma_{sH}^{D}$ where

$$\sigma_{sH}^{R} = \lim_{\omega\to 0}\frac{e}{\omega}\int\frac{d\epsilon}{2\pi}\,\mathrm{Tr}[\overline{j_s^y G^{<}(\epsilon) j_c^x \,\mathrm{Re}\, G^{A}(\epsilon-\omega)} + \overline{j_s^y \mathrm{Re}\, G^{R}(\epsilon) j_c^x G^{<}(\epsilon-\omega)}], \qquad (33)$$

$$\sigma_{sH}^{D} = -\frac{e}{\pi}\,\mathrm{Tr}[\overline{j_s^y\,\mathrm{Im}\,G^{R} j_c^x\,\mathrm{Im}\,G^{R}}]. \qquad (34)$$

Since the zero frequency spin-Hall conductivity is real, the terms with imaginary (real) current matrix elements contribute to σ_{sH}^{R} (σ_{sH}^{D}), respectively. It then follows that the first term on the right-hand side of Eq. (29) corresponds to $\sigma_{sH}^{R} = e/8\pi$, whereas the second term (the vertex corrections) is the dissipative response with

$$\sigma_{sH}^{D} = -\frac{e}{8\pi}\frac{1}{4\alpha}\frac{N_+\tau_+J_+^x - N_-\tau_-J_-^x}{N_0\tau}\frac{V_2 - V_0}{V_0} = -\frac{e}{8\pi} \qquad (35)$$

and only the sum of the reactive and dissipative response is zero.

In summary, we calculated the spin-Hall conductivity in a two-dimensional electron gas within the self-consistent Born approximation, including the vertex corrections in the ladder approximation. We remark that, although a number of similar studies exist in the recent literature, the final conclusions are often contradictory. This may be due to the fact that the relevant integrals depend in a very subtle way on the type of the physical limit considered. For this reason in this work we evaluated all the relevant integrals both analytically and numerically. This allowed us to confirm the conclusions of Refs. 11 and 15. In particular, we find that the spin-Hall conductivity is strongly suppressed below the universal value of $e/8\pi$. Furthermore we have demonstrated that the result is not only valid for pure s-wave scattering, but is robust upon the inclusion of a weak momentum dependence of the scattering probability.[21]

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[22] For completeness we remark that an estimate of the effect of the dependence of the scattering amplitude on the distance from the Fermi surface may be obtained by considering the extreme forward scattering limit where intersubband scattering is not allowed, as analyzed in Ref. 20. It is not difficult to see that even in that case the spin-Hall conductivity vanishes. We then believe that $\sigma_{sH}=0$ holds quite generally.