

# Alternative mechanism for the sign-reversal effect in superconductor-ferromagnet-superconductor Josephson junctions

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We consider a simple model of a multidomain superconductor-ferromagnet-superconductor (SFS) Josephson junction. Sign-alternating magnetization  $M$  in domains leads to a spatial modulation of the phase difference  $\phi(x)$ . Due to this modulation the Josephson critical current  $I_c$  may have a different sign depending on the ratio of the magnetic flux in a domain,  $4\pi M a(2d_F)$ , to the magnetic flux quantum. This phase modulation, but not a nonmonotonic dependence of the local critical current density  $j_c$ , may be the reason for oscillations of the current  $I_c$  as a function of the  $F$  layer thickness  $2d_F$  or temperature, observed in experiments.

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## I. INTRODUCTION

New states have been and observed in Josephson junctions in recent years. These states are characterized by a negative Josephson energy  $E_J = (I_c \hbar / 2e)(1 - \cos \phi)$ , that is, under certain conditions the Josephson critical current  $I_c$  changes its sign becoming negative. This means that the ground state corresponds to the phase difference  $\phi$  equal to  $\pi$  ( $\pi$  state), but not to zero as it takes place in ordinary Josephson junctions. Such states have been observed in Josephson junctions of different types: 1) in multiterminal SNS Josephson junctions,<sup>1</sup> (2) in junctions consisting of two  $d$ -wave superconductors (see references in the review,<sup>2</sup> (3) in SFS junctions<sup>3-5</sup> (see also the review<sup>7</sup>), where  $S$ ,  $N$ , and  $F$  stand for a superconductor, a normal metal and a ferromagnet, respectively. The  $\pi$ -state in SNS junctions is created by passing a dissipative current through the  $N$  layer. This current leads to a nonequilibrium distribution function of quasiparticles in the  $N$  wire with respect to the equilibrium distribution function in the superconductors. In the  $d$ -wave superconductors (high  $T_c$  superconductors) the sign of the order parameter  $\Delta$  depends on the direction in space with respect to crystallographic axes. Therefore if two  $S/N$  (or  $S/I$ , where  $I$  is an isolator) interfaces have different, properly chosen orientations, then the critical current  $I_c$ , which is proportional to the product  $\Delta_1 \Delta_2$ , may be negative. This occurs provided that the order parameter in one superconductor ( $\Delta_1$  or  $\Delta_2$ ) is negative. In SFS junctions the critical current  $I_c$  may be negative because the condensate (Gor'kov's) Green's function, which determines the current  $I_c$  (or to be more exact, the critical current density  $j_c$ ), oscillates in space changing sign. The sign reversal of  $j_c$  in SFS junctions was predicted a long time ago by Bulaevskii, Kuzii, and Sobyenin,<sup>8</sup> who considered electron tunneling between two superconductors via a magnetic impurity. Later this effect was studied in SFS junctions by Buzdin, Bulaevskii, and Panyukov<sup>9</sup> (references to other theoretical papers on this subject are given in the review<sup>7</sup>). It was shown that the current density  $j_c$  decays with increasing the thickness of the  $F$  layer  $2d_F$  and changes sign (damped oscillations of  $j_c$ ). Such behavior of the critical current has been observed experimentally in Refs. 3-6; the current  $I_c$  decays with increasing the thickness  $2d_F$

or temperature  $T$  in a nonmonotonic way changing sign. In the recent paper<sup>6</sup> a spontaneously circulating current in a superconducting ring with a SFS  $\pi$  junction has been observed.

In this paper we show that the damped  $I_c$  oscillations are not necessarily related to such a dependence of the local critical current density  $j_c$  as it was anticipated previously. The total Josephson current  $I_J$ , which is measured in experiments, is an integral from the local current density  $j_J = j_c \sin \phi$  over the whole area of a SFS junction [we choose a simple, sinusoidal form of the dependence  $j_J(\phi)$ , but the conclusions we make are valid qualitatively in a general case]. It is important to have in mind that the phase difference  $\phi$  varies in space in the presence of a magnetic field, and in multidomain SFS junctions a spatial dependence of  $\phi$  arises even in the absence of an external magnetic field  $H_{\text{ext}}$ . It was already mentioned in Refs. 3 and 4 that the maximum value of  $I_c$  corresponds to zero external magnetic field  $H_{\text{ext}}$ . One can assume that this may be related to a multidomain structure of the  $F$  film. Otherwise the maximum value of  $I_c$  would be shifted by a certain value of  $H_{\text{ext}}$  for which the induction  $H_{\text{ext}} + 4\pi M$  is zero, where  $M$  is the magnetization in a one-domain  $F$  film. We consider a simple model of a multidomain structure of the  $F$  film and show that even if the local current density  $j_c$  is always positive, the critical current  $I_c$  changes sign when the in-plane magnetic flux in a domain  $\Phi_a = 4\pi M_0(2d_F a)$  is an integer of the flux quantum  $\Phi_0$ , where  $a$  is the domain width. The domain width  $a$  depends both on the thickness  $d_F$  and temperature  $T$  if the screening of stray magnetic fields by the Meissner currents is taken into account.<sup>10-12</sup> Therefore, the magnetic flux is changed with increasing  $d_F$  and temperature leading to damped oscillations of the critical current  $I_c$ . It will be shown that the dependence  $I_c(\Phi_a/\Phi_0)$  may be described by a Fraunhofer-like pattern:

$$I_c(\Phi_a/\Phi_0) = I_{c0} \sin(\pi\Phi_a/\Phi_0)/(\pi\Phi_a/\Phi_0), \quad (1)$$

where  $I_{c0} = j_c L_x$  is the critical current for a uniform (one-domain) SFS junction per unit length in the  $y$  direction (we assume that all quantities depend only on  $x$  and  $z$ ; see Fig. 1). This dependence describes the critical current for the case of

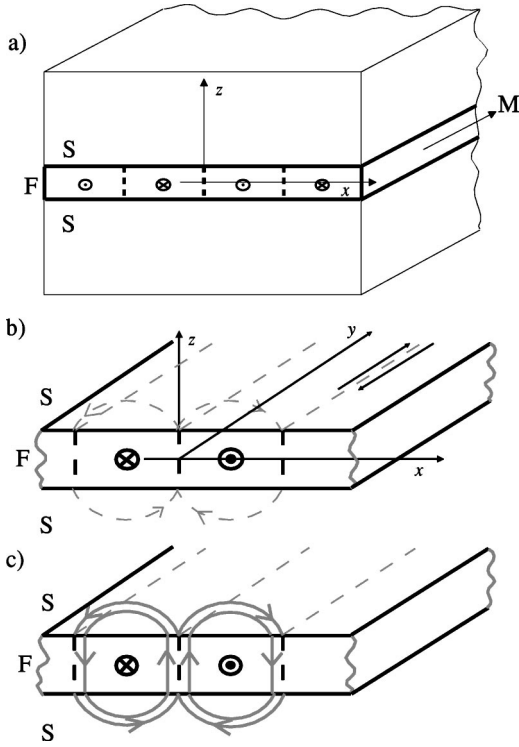


FIG. 1. Schematic image of the considered SFS junction with magnetic domains (a); (b) shows the supercurrents (solid lines with arrows) in the upper superconductor screening the normal component of a magnetic field near domain walls. These currents in the lower superconductor (not shown) have the same direction. The domain boundaries are represented by the dashed lines. The dashed lines with arrows illustrate the stray magnetic field related to rotation of the magnetization  $\mathbf{M}$  in the domain walls. The cross and dot in the circles show the directions of the magnetization vector in neighboring domains; (c) the solid lines with arrows show the screening supercurrents in the presence of the Josephson coupling.

domains with an alternating magnetization which takes constant values  $\pm M_0$  in domains.

## II. MODEL AND BASIC EQUATIONS

Consider a SFS Josephson junction in which the  $\mathbf{F}$  layer consists of stripe domains parallel to the  $y$  axis (see Fig. 1).

To be specific, we consider the Bloch domains, that is, the magnetization vector  $\mathbf{M}$  lies in the  $y$ - $z$  plane changing its direction in domain walls of the width  $w$ . In the case of a narrow domain width the magnetization vector  $M$  is parallel to the  $y$  axis, constant in each domain and rotates in the domain wall. The magnitude  $|M|$  is assumed to be constant everywhere in the  $F$  film. The orientation of the magnetization  $M$  in the  $F$  film (in-plane or out-of-plane) depends on many factors such as the Curie temperature, the constant of magnetic anisotropy, the film thickness, etc. (see for example,<sup>13,14</sup> and references therein). In ultrathin  $F$  films (a few atomic layers) a transition between the in-plane to out-of-plane  $M$  orientation may occur.<sup>15</sup> We consider much more thicker  $F$  films, used in experiments,<sup>3-6</sup> where the magnetization orientation is determined by the direction of easy

magnetization. Therefore, it is assumed that the axis of easy magnetization is parallel to the plane of the  $F$  film. Even in the absence of the Josephson coupling between the superconductors  $S$ , the Meissner currents along the  $y$  direction are induced by the stray magnetic field  $\mathbf{H}=(H_x, 0, H_z)$  [dashed lines with arrows in Fig. 1(a)]. The magnetic field components  $H_{x,z}$  can be easily found from equations

$$\partial^2 H_{x,z}(k_n, z) / \partial z^2 - \kappa_n^2 H_{x,z}(k_n, z) = 0, \quad (2)$$

where  $H_{x,z}(k_n, z) = \int_{-a}^a (dx/2a) H_{x,z}(x, z) \exp(ik_n x)$  is the Fourier component of  $H_{x,z}(x, z)$ ,  $k_n = \pi n/a$ ,  $n=0, \pm 1, \dots$ ;  $\kappa_n^2 = k_n^2 + \kappa_L^2$ ,  $\kappa_L^{-1} = \lambda_L$  is the London penetration depth (in the ferromagnet the penetration depth  $\lambda_{L,F}$  may be taken infinite because the amplitude of the condensate in the  $F$  layer is small and therefore the screening is weak). This equation is supplemented by the boundary conditions<sup>16</sup>

$$[H_x] = 0, [H_z] = 4\pi M_z(k_n, z), \quad (3)$$

where the square brackets mean a difference:  $[H_x] = H_x(k_n, d_F+0) - H_x(k_n, d_F-0)$ . The component  $M_z$  is not zero in the domain walls. This implies that the in-plane component of  $H$  is continuous across the  $S/F$  boundary and the normal component of  $H$ , which exists near the domain walls, experiences a jump. One can easily solve Eq. (2) and find the fields  $H_{x,z}$ , which are connected with the vector potential  $\mathbf{A}: H_x = -\partial A_y / \partial z$ ,  $H_z = \partial A_y / \partial x$ . The expression for  $A_y$  in the superconductor is

$$A_y(k_n, z) = -4\pi M_z(k_n) (f_n i k_n)^{-1} \exp[-\kappa_n(z - d_F)]; \quad z > d_F, \quad (4)$$

where  $f_n = 1 + \kappa_n / (k_n \tanh \theta_n)$ ,  $\theta_n = k_n d_F$ . The vector potential  $A_y$  in the lower superconductor ( $z < d_F$ ) has the same sign. This means that the stray fields  $H_{x,z}$  lead to the screening currents

$$j_y(k_n, z) = -(c/4\pi) \kappa_L^2 A_y(k_n, z), \quad (5)$$

which have the same direction in the upper and lower superconductors, but the opposite directions in neighboring domains [see Fig. 1(a)]. Because the Meissner currents flow in the same direction in both superconducting electrodes, they do not lead to a phase difference between the superconductors and do not influence the Josephson current essentially. These currents may locally reduce the amplitude of the order parameter and therefore decrease the critical current  $I_c$  if the magnetization  $M_z$  is strong enough. We will not discuss this simple effect.

If the Josephson coupling between the superconductors is negligible (this case was considered in Ref. 10), the magnetic field has only the components  $H_{x,z}$  which are determined by the vector potential  $A_y(k_n, z)$ . The component  $H_y$  is zero. However the components  $A_{x,z}(k_n, z)$  of the vector potential are not zero. For example in one-domain case

$$A_x(z) = 4\pi M_y z \text{ in } F, \quad (6)$$

$$A_x(z) = \pm 4\pi M_y d_F \text{ in } S. \quad (7)$$

In the absence of the Josephson current the components  $A_{x,z}$  in a multidomain SFS structure are found from the equa-

tions:  $\mathbf{H} = \nabla \times \mathbf{A} = 0$  and  $\nabla \cdot \mathbf{A} = 0$ . However, in this case the presence of components  $A_{x,z}$  does not lead to the superconducting currents and therefore to the appearance of an additional magnetic field  $H$ . The term proportional to  $A_x$  in the expression for the supercurrent is compensated by the term proportional to gradient of the phase [see Eq. (9)]. If the Josephson current  $j_J$  is not zero, additional screening currents  $j_{x,z}$  and therefore the component  $H_y$  arise in the system [see Fig. 1(b)]. The component  $A_x$  affects the phase difference and therefore the critical current  $I_c$  in the junction under consideration.

In order to calculate the critical current  $I_c$ , one needs to derive an equation governing the phase difference  $\phi$  in a multidomain SFS Josephson junction. For simplicity we assume that the thickness of the F layer is small:  $2d_F \ll a, w$ , where  $2a$  is the period of the domain structure and  $w$  is the width of the domain wall (one can analyze a more general case, but the calculations become more cumbersome). We need an equation for the component  $H_y$  which is related to the local Josephson current density

$$(\nabla \times \mathbf{H})_z = \partial H_y / \partial x|_{z=0} = (4\pi/c)j_c \sin \phi. \quad (8)$$

We assumed the simplest form of the relationship between the Josephson current density  $j_J$  and the phase difference  $\phi$ , but this assumption is not essential. We also dropped a contribution to the gauge-invariant ‘‘phase difference’’ which stems from the vector potential and has the form  $\int_{-d_F}^{d_F} A_z dz$ . One can easily show that this part is smaller than  $\phi$  by the parameter  $(a/d_F)$  (we choose a gauge in which  $\nabla \cdot \mathbf{A} = 0$ ). We write down the  $x$  component of one of the Maxwell equations for the current density in the superconductor at  $z = \pm d_F$ :

$$-\partial H_y / \partial z = (4\pi/c)j_x = \kappa_L^2[-A_x + (\Phi_0/2\pi) \partial \chi / \partial x], \quad (9)$$

where  $\kappa_L^2$  is defined in Eq. (2) and  $\chi$  is the phase of the order parameter. Subtract Eq. (9) taken at  $z = +d_F$  and  $z = -d_F$  from one another (a similar method was used by one of the authors in Ref. 18 in the study of collective modes in layered superconductors), we get

$$-[\partial H_y / \partial z] = \kappa_L^2\{-[A_x] + (\Phi_0/2\pi) \partial \phi / \partial x\}, \quad (10)$$

where the square brackets means, as before, a jump across the F layer and  $\phi = \chi(d_F) - \chi(-d_F)$  is the phase difference. The jump  $[A_x]$  is found from the equation

$$4\pi M_y = \partial A_x / \partial z - \partial A_z / \partial x \quad (11)$$

and is equal to:  $[A_x(k_n)] = 4\pi M_y(k_n)d_F$  in accordance with Eq. (6) (see Ref. 19). The contribution of the second term is smaller by the parameter  $(a/d_F)$ . The component of the magnetic field  $H_y$  can be found from an equation similar to Eq. (2). With account for the boundary condition [Eq. (10)] this equation acquires the form

$$\partial^2 H_y(k_n, z) / \partial z^2 - \kappa_n^2 H_y(k_n, z) = \delta(z) \kappa_L^2 \{[A_{x,k}] + (\Phi_0/2\pi) i k_n \phi_k\}. \quad (12)$$

Solving this equation for  $H_y$  and substituting the solution into Eq. (8), we obtain an equation for Fourier components of the phase difference  $\phi_k = \phi(k_n, z)$ :

$$\kappa_n^2 \phi_k + \kappa_J^2 (\kappa_n / \kappa_L) [\sin \phi(x)]_k = i k_n 4\pi M_y(k_n) (2d_F) (2\pi / \Phi_0), \quad (13)$$

where  $\kappa_J = \sqrt{16\pi^2 j_c / c \kappa_L \Phi_0}$  is the inverse Josephson length. In the coordinate representation Eq. (13) has the form

$$-\partial^2 \phi / \partial x^2 + \kappa_J^2 \int dx_1 K(x-x_1) \sin \phi(x_1) = -4\pi d_F (2\pi / \Phi_0) \partial M_y(x) / \partial x, \quad (14)$$

where the kernel  $K(x-x_1)$  is defined as follows

$$K(x-x_1) = (1/2a) \sum_n (\kappa_n / \kappa_L) \exp[-i k_n (x-x_1)], \quad (15)$$

Eq. (14) describes the dc Josephson effect in a simple model of multidomain SFS junctions with a thin F layer.

### III. TWO TYPES OF DOMAIN STRUCTURES

If the London penetration depth is small compared to the domain size  $a$  and the width of the domain wall ( $\lambda_L \ll a, w$ ), then Eq. (14) is simplified. This condition means that the characteristic  $k_n$  values are much smaller than  $\kappa_L$ . In this case  $K(x-x_1) \approx \delta(x-x_1)$  and Eq. (14) acquires the form

$$-\partial^2 \phi / \partial x^2 + \kappa_J^2 \sin \phi(x) = -4\pi (2d_F) (2\pi / \Phi_0) \partial M_y(x) / \partial x. \quad (16)$$

This equation differs from the standard Josephson equation only by the term on the right-hand side. One can study various properties of the SFS junctions described by Eq. (16) or by Eq. (14), but in this paper we analyze only the critical current and its dependence on different parameters (external magnetic field, the thickness  $d_F$  etc). First we consider the case of a periodic  $M_y(x)$  dependence. If the period  $2a$  of this dependence is much less than the long Josephson length  $\kappa_J^{-1}$ , then a solution for Eq. (14) (the relation between  $\lambda_L$  and  $a$ ,  $w$  may be arbitrary) is

$$\phi = \phi_M + \phi_0, \quad \phi_M = (2\pi / \Phi_0) 4\pi (2d_F) \int_0^x dx_1 M_y(x_1) \quad (17)$$

where  $\phi_M(x)$  is a function fast varying in space,  $\phi_0$  is a constant (or a function smoothly varying over the period  $a$ ). The total Josephson current (per unit length in  $y$  direction) is

$$I_J = j_c \int_0^{L_x} dx_1 \sin[\phi_0 + \phi_M(x_1)] \quad (18)$$

where  $\phi_M$  is given by Eq. (17). Note that the weak Josephson coupling does not affect the domain structure, and therefore this structure can be studied in the absence of the Josephson effect. As we noted, the domain structure was analyzed theoretically in Ref. 10 for arbitrary  $d_F$  and in Refs. 11 and 12 for thick F layers ( $d_F \gg a$ ). It was shown in Ref. 10 that the period  $a$  depends on  $d_F$  in a nonmonotonic way and the width of the domain walls  $w$  may be much less than or comparable with the domain width  $a$ . One has a step-like struc-

ture  $M_y(x)$  in the first case and an oscillatory structure in the second case. Consider two limiting cases.

(a) Step-like domain structure. The magnetization vector equals  $M=[0, M_y(x), 0]$ , where  $M_y(x)=+M_0$  for  $0 < x < a$ , and  $M_y(x)=-M_0$  for  $-a < x < 0$ . Outside this interval the dependence  $M_y(x)$  is periodically repeated. For this structure the Josephson current  $I_J$  is described by the formula

$$I_J = I_{c0} \sin \phi_0 \frac{\sin(\pi\Phi_a/\Phi_0)}{\pi\Phi_a/\Phi_0}, \quad (19)$$

where  $I_{c0}=j_c L_x$ ,  $\Phi_a=4\pi M_0(2d_F a)$  is the in-plane magnetic flux in one domain. Therefore the critical current  $I_c$ , which is given by Eq. (1) oscillates and decays with increasing  $\Phi_a$ . This implies that the amplitude of the critical current oscillations decreases with increasing  $d_F$  or temperature  $T$  because the period of the domain structure  $2a$  depends on  $\lambda_L(T)$  (see the theoretical papers<sup>10–12</sup> and the experimental paper,<sup>17</sup> where it was shown that the domain structure is changed with changing  $T$ ).

b) Oscillatory domain structure:  $M_y(x)=M_0 \sin(k_0 x)$ ,  $k_0 = \pi/a$ .

In this case the critical current is equal to

$$I_c = I_{c0} J_0[\phi_M(a)], \quad (20)$$

where  $J_0$  is the Bessel function of the zeroth order. In both cases the behavior of the critical current  $I_c$  as a function of  $\Phi_a$  is qualitatively the same: the current  $I_c$  decreases with increasing  $\phi_M(a)$  and changes sign.

In our model of a periodic domain structure the action of an in-plane external magnetic field  $H_{\text{ext}}$  on  $I_c$  can be easily analyzed. In the presence of the field  $H_{\text{ext}}$  the phase difference equals  $\phi(x)=\phi_0+\phi_H(x)+\phi_M(x)$ , where  $\phi_M(x)$  is given by Eq. (17),  $\phi_0$  is a constant and  $\phi_H(x)=2\lambda_L x H_{\text{ext}}(2\pi/\Phi_0)$ . If the domain size  $a$  is much less than the length of the junction  $L_x$ , the averaging over the period of the structure can be done as before at a fixed coordinate  $x$ , and we arrive at Eq. (19) in which one has to replace  $\phi_0 \Rightarrow \phi_0 + \phi_H(x)$ . The final averaging over  $L_x$  yields, for example, in the model of a step-like domain structure for  $I_c$ :

$$I_c(H_{\text{ext}}) = I_{c0} \frac{\sin \phi_H(L_x)}{\phi_H(L_x)} \frac{\sin(\pi\Phi_a/\Phi_0)}{\pi\Phi_a/\Phi_0}. \quad (21)$$

Thus the dependence  $I_c(H_{\text{ext}})$  is given by the usual Fraunhofer curve with an effective critical current  $I_{c0} \sin(\pi\Phi_a/\Phi_0)/(\pi\Phi_a/\Phi_0)$  the sign and value of which depends on  $d_F$ ,  $a$ , etc.

Although in the theoretical papers<sup>10–12</sup> only a regular domain structure is considered, in real samples the domain structure is not strictly periodic. It may be almost regular (see for example,<sup>21,22</sup>) or very irregular<sup>23,24</sup> with in-plane or out-of plane magnetizations. We study the effect of a possible irregularity of the domain structure on the basis of a simple model. We assume simply that the domain size  $a$  fluctuates around a mean value  $\bar{a}$  and fluctuations of  $a$  are described by the Gaussian distribution. Then the dependence of  $I_c(\delta)$  on the dispersion of the fluctuations is given by the integral (in the absence of  $H_{\text{ext}}$ )

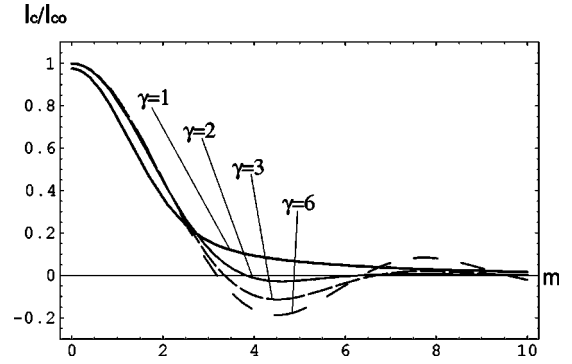


FIG. 2. Normalized critical Josephson current as a function of the normalized magnetic flux in a domain  $m = \pi\Phi_a(a)/\Phi_0$  for different parameters  $\gamma = \bar{a}/\delta$ , where  $\bar{a}$  is the averaged domain size and  $\delta/\sqrt{2}$  is the dispersion of the domain size fluctuations.

$$I_c(\delta) = I_{c0} c_1 \int_0^\infty da \frac{\sin[\pi\Phi_a(a)/\Phi_0]}{\pi\Phi_a(a)/\Phi_0} \exp[-(a-\bar{a})^2/\delta^2] \quad (22)$$

where  $c_1 = \{\int_0^\infty da \exp[-(a-\bar{a})^2/\delta^2]\}^{-1}$  is a normalization constant. In Fig. 2 we plot the dependence  $I_c(\pi\Phi_a(\bar{a})/\Phi_0)$  for different parameter  $\gamma = \bar{a}/\delta$ . One can see that for large  $\gamma$  this dependence coincides with a Fraunhofer pattern, but with decreasing  $\gamma$  the amplitude of oscillations of  $I_c$  decreases and finally the function  $I_c(\Phi_0)$  does not change sign (no  $\pi$ -states).

#### IV. CONCLUSION

In conclusion, using a simple model of a multidomain SFS Josephson junction, we have calculated the critical current  $I_c$ . It turns out that the current  $I_c$  changes signs when the in-plane magnetic flux  $\Phi_a=4\pi M a(2d_F)$  in each domain equals  $n\Phi_0$ . The magnetic flux  $\Phi_a$  is caused by the magnetization in the ferromagnetic domains and therefore exists even in the absence of an external magnetic field. The oscillations of  $I_c$  observed experimentally by varying thickness  $2d_F$  or temperature  $T$  may be related not to the sign reversal of the local critical current density  $j_c$ , but to a simple mechanism—the Fraunhofer-like oscillations of  $I_c$  caused by the internal magnetization  $M$  in domains. Almost nothing is known about the domain structure in real SFS junctions. For estimations we take  $4\pi M_0 \approx 1$  kOe,  $a \approx 1$  mkm,  $2d_F \approx 100$  Å. For these values we obtain  $\Phi_a \approx 10^{-7}$  Oe cm<sup>2</sup>. This means that the critical current  $I_c$  changes sign for the thickness  $2d_F$  about 100 Å. This value of thickness is close to that used in experiments, although, strictly speaking, the values of the magnetization  $M$  and of the domain structure period  $a$  are not known. In order to make more convincing conclusions about what is the mechanism of the sign reversal effect (whether it is caused by the sign reversal of the critical current density  $j_c$  or by the spatial phase modulation in a multidomain SFS structure), further theoretical and, especially,

experimental studies are needed. In particular, it would be interesting to study the influence of the domain structure on Shapiro steps in SFS junctions measured in a recent paper.<sup>20</sup>

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- <sup>1</sup>J. J. A. Baselmans, A. F. Morpurgo, B. J. van Wees, and T. M. Klapwijk, *Nature* (London) **397**, 43 (1999).
- <sup>2</sup>C. C. Tsuei and J. R. Kirtley, *Rev. Mod. Phys.* **72**, 969 (2000).
- <sup>3</sup>V. V. Ryazanov, V. A. Oboznov, A. Yu. Rusanov, A. V. Veretennikov, A. A. Golubov, and J. Aarts, *Phys. Rev. Lett.* **86**, 2427 (2001).
- <sup>4</sup>T. Kontos, M. Aprili, J. Lesueur, F. Genêt, B. Stephanidis, and R. Boursier, *Phys. Rev. Lett.* **89**, 137007 (2002).
- <sup>5</sup>Y. Blum, A. Tsukernik, M. Karpovski, and A. Palevski, *Phys. Rev. Lett.* **89**, 187004 (2002).
- <sup>6</sup>A. Bauer, J. Bentner, M. Aprili, M. L. Della-Rocca, M. Reinwald, W. Wegscheider, and C. Strunk, *Phys. Rev. Lett.* **92**, 217001 (2004).
- <sup>7</sup>A. A. Golubov, M. Yu. Kupriyanov, and E. Il'ichev, *Rev. Mod. Phys.* **76**, 411 (2004).
- <sup>8</sup>L. N. Bulaevskii, V. V. Kuzii, and A. A. Sobyenin, *JETP Lett.* **25**, 299 (1977).
- <sup>9</sup>A. I. Buzdin, L. N. Bulaevskii, and S. V. Panyukov, *JETP Lett.* **35**, 178 (1982).
- <sup>10</sup>A. Stankiewicz, S. J. Robinson, G. A. Gehring, and V. V. Tarasenko, *J. Phys.: Condens. Matter* **9**, 1019 (1997).
- <sup>11</sup>L. N. Bulaevskii and E. M. Chudnovsky, *Phys. Rev. B* **63**, 012502 (2001).
- <sup>12</sup>E. B. Sonin, *Phys. Rev. B* **66**, 136501 (2002).
- <sup>13</sup>A. P. Levanyuk and N. Garcia, *Phys. Rev. Lett.* **70**, 1184 (1993).
- <sup>14</sup>A. B. Kashuba and V. L. Pokrovsky, *Phys. Rev. B* **48**, 10 335 (1993).
- <sup>15</sup>D. P. Pappas, K. P. Kämper, and H. Hopster, *Phys. Rev. Lett.* **64**, 3179 (1990).
- <sup>16</sup>L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Butterworth-Heinemann, Oxford, 1982), p. 107.
- <sup>17</sup>S. V. Dubonos, A. K. Geim, K. S. Novoselov, and I. V. Grigorieva, *Phys. Rev. B* **65**, 220513(R) (2002).
- <sup>18</sup>A. F. Volkov, *Phys. Lett. A* **138**, 213 (1989).
- <sup>19</sup>Note that an induced magnetization arises also in the superconductors. As already shown [F. S. Bergeret, A. F. Volkov, and K. B. Efetov, *Europhys. Lett.* **69**, 174504 (2004); *Europhys. Lett.* **66**, 111 (2004)], the magnetization penetrates from the ferromagnet into the superconductor (with the opposite sign) over the correlation length  $\xi_S$  if the proximity effect is essential. In the case of an itinerant ferromagnet (the magnetization is mainly due to free electrons) the total magnetic moment in the ferromagnet may be completely compensated by the magnetic moment induced in the superconductor. We do not take into account this inverse proximity effect assuming that  $M_F$  is produced mainly by localized magnetic moments. Account for the inverse proximity effect leads to replacement of  $M_y$  by an effective magnetization  $M_{y,\text{eff}}$ .
- <sup>20</sup>H. Sellier, C. Baraduc, F. Lefloch, and R. Calemczuk, *Phys. Rev. Lett.* **92**, 257005 (2004).
- <sup>21</sup>M. Feigenson, L. Klein, J. W. Reiner, and M. R. Beasley, *Phys. Rev. B* **67**, 134436 (2003).
- <sup>22</sup>E. Y. Vedmedenko, A. Kubetzka, K. von Bergmann, O. Pietzsch, M. Bode, J. Kirschner, H. P. Oepen, and R. Wiesendanger, *Phys. Rev. Lett.* **92**, 077207 (2004).
- <sup>23</sup>M. Speckmann, H. P. Oepen, and H. Ibach, *Phys. Rev. Lett.* **75**, 2035 (1995).
- <sup>24</sup>M. Lange, M. J. Van Bael, and V. V. Moshchalkov, *Phys. Rev. B* **68**, 174522 (2003).