Absence of weak localization in *d*-density-wave conductors: A diagrammatic approach to impurity scattering

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Within the self-consistent *T*-matrix approximation, a diagrammatic approach is presented to describe the effect of nonmagnetic-impurity scattering in *d*-density-wave (DDW) conductors. We show that the presence of DDW order alters dramatically usual diagrammatic rules. As a result, the disorder effect in the DDW state is qualitatively different from that in the *d*-wave superconducting state. It is found that the usual weak-localization effect is absent in the DDW state due to time-reversal symmetry breaking.

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The *d*-density-wave (DDW) state is characterized by a condensate of electron-hole pairs.[1] The key feature of such a state is staggered orbital magnetic moments (i.e., staggered currents) which break parity and time-reversal symmetry as well as the translation invariance by one lattice constant and rotation by $\pi/2$. It was suggested that the typical characteristics of the psudogap of high-T_c cuprates as observed in many experiments, including photomission,[2] tunneling,[3] and muon spin relaxation,[4] can be explained by the DDW order model. Although the debate about the mechanism for the psudogap is far from settled, theoretical investigations for the features of the two-dimensional (2D) DDW state are interesting. Recently, the effect of impurity scattering in the DDW state has been received much attention. Several authors studied the quasiparticle transport in disordered DDW conductors.[5–8] The resonant state around a single impurity was also investigated.[9]

Generally speaking, weak localization (WL) has a significant effect on transport properties of disordered 2D electronic systems.[10] It describes the quantum interference effect that results from constructive interference between the closed electron paths and their time-reversal counterparts. In the diagrammatic language, the WL effect is captured by the cooperon propagator with a diffusive pole. It is well known that the WL effect leads to a logarithmic correction to the conductivity in 2D disordered normal metals. Recent investigations[11–13] revealed that both the density of states and transport coefficients of quasiparticles in the *d*-wave-superconducting (DWS) state are also subject to WL corrections due to impurity scatterings. However, the issue whether the WL effect has any manifestation in a disordered DDW conductor has not been yet studied. This is the very motivation of the present work.

In this paper, by the diagrammatic techniques, we study the effect of nonmagnetic-impurity scattering in 2D DDW conductors with the impurities randomly distributed on a small fraction of the sites. The Fermi surface is assumed to be nearly nested, so that the low-energy quasiparticles can be considered as Dirac fermions. We treat the impurity scattering within the self-consistent *T*-matrix approximation (SCTMA), which has been used in the studies of heavy-fermion superconductors[14] and the DWS state.[11–13] It is found that the presence of DDW order alters dramatically the usual diagrammatic rules. We show that the cooperon propagator in the DDW state does not contain any diffusive pole, and thus the usual WL effect is absent in DDW metals. This feature is qualitatively different from the disorder effect in the DWS state.

For a perfect 2D square lattice, the electronic spectrum in the tight-binding approximation reads $\xi_{\boldsymbol{k}}=-2t(\cos k_x a+\cos k_y a)$, where t is the nearest-neighbor hopping integral, and a is the lattice constant. The $d_{x^2-y^2}$-density-wave order parameter is given by $\Delta_{\boldsymbol{k}}=\Delta(\cos k_x a-\cos k_y a)$. The Hamiltonian describing the DDW state of a clean system can be expressed as $H^{(0)}=\Sigma_{\boldsymbol{k}\sigma}(\epsilon_{\boldsymbol{k}}C^{\dagger}_{\boldsymbol{k}\sigma}C_{\boldsymbol{k}\sigma}+i\Delta_{\boldsymbol{k}}C^{\dagger}_{\boldsymbol{k}\sigma}C_{\boldsymbol{k}+\boldsymbol{Q}\sigma})$, where $\epsilon_{\boldsymbol{k}}=\xi_{\boldsymbol{k}}-\mu^{(0)}$ with $\mu^{(0)}$ the chemical potential, $C_{\boldsymbol{k}\sigma}$ and $C^{\dagger}_{\boldsymbol{k}\sigma}$ represent, respectively, the annihilation and creation operators of electrons with σ the spin index, $\Sigma_{\boldsymbol{k}}$ denotes the summation of $\boldsymbol{k}$ over the first Brillouin zone, and $\boldsymbol{Q}$ is one of the four nesting vectors $(\pm\pi/a,\pm\pi/a)$ with $\boldsymbol{k}+\boldsymbol{Q}$ located also within the first Brillouin zone. As shown in Fig. 1, if $\boldsymbol{k}$ is located within (out of) the *reduced* Brillouin zone, then $\boldsymbol{k}+\boldsymbol{Q}$ is situated out of (within) the *reduced* Brillouin zone. Therefore, the Hamiltonian can be also expressed as $H^{(0)}=\Sigma'_{\boldsymbol{k}\sigma}(\epsilon_{\boldsymbol{k}}C^{\dagger}_{\boldsymbol{k}\sigma}C_{\boldsymbol{k}\sigma}+\epsilon_{\boldsymbol{k}+\boldsymbol{Q}}C^{\dagger}_{\boldsymbol{k}+\boldsymbol{Q}\sigma}C_{\boldsymbol{k}+\boldsymbol{Q}\sigma}+i\Delta_{\boldsymbol{k}}C^{\dagger}_{\boldsymbol{k}\sigma}C_{\boldsymbol{k}+\boldsymbol{Q}\sigma}+i\Delta_{\boldsymbol{k}+\boldsymbol{Q}}C^{\dagger}_{\boldsymbol{k}+\boldsymbol{Q}\sigma}C_{\boldsymbol{k}\sigma})$, where $\Sigma'_{\boldsymbol{k}}$ represents the summation of $\boldsymbol{k}$ over the *reduced* Brillouin zone. Using the nesting symmetry $\xi_{\boldsymbol{k}+\boldsymbol{Q}}=-\xi_{\boldsymbol{k}}$ and $\Delta_{\boldsymbol{k}+\boldsymbol{Q}}=-\Delta_{\boldsymbol{k}}$, one can express the Hamiltonian in the spinor representation as $H^{(0)}=\Sigma'_{\boldsymbol{k}\sigma}\Psi^{\dagger}_{\boldsymbol{k}\sigma}\mathcal{E}_{\boldsymbol{k}}\Psi_{\boldsymbol{k}\sigma}$, with $\Psi^{\dagger}_{\boldsymbol{k}\sigma}$

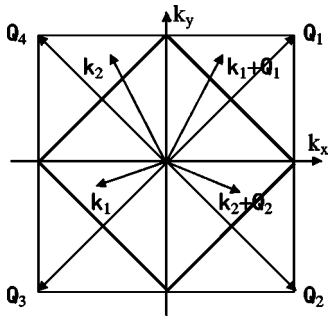

FIG. 1. Brillouin zone for the DDW state. The area enclosed by the heavy lines is the *reduced* Brillouin zone. $\boldsymbol{Q}_i(i=1,2,3,4)$ denote the nesting vectors.

$=(C^{\dagger}_{k\sigma}, -iC^{\dagger}_{k+Q\sigma})$ and $\mathcal{E}_k=\Delta_k\tau_1+\xi_k\tau_3-\mu^{(0)}\tau_0$, where τ_0 is the 2×2 unity matrix, and τ_1, τ_2, and τ_3 stand for the Pauli matrices. The spinor operators satisfy $\{\Psi_{k\sigma}, \Psi^{\dagger}_{k'\sigma'}\}_+ = \delta_{kk'}\delta_{\sigma\sigma'}\tau_0$. The Green's function matrix is defined by $G^{(0)}_k(\tau)=-\langle\mathcal{T}[\Psi_{k\sigma}(\tau)\Psi^{\dagger}_{k\sigma}(0)]\rangle$. One can easily show that $G^{(0)}_k(i\epsilon_n)=(i\epsilon_n\tau_0-\mathcal{E}_k)^{-1}$, yielding

$$G^{(0)}_k(i\epsilon_n)=\frac{[i\epsilon_n+\mu^{(0)}]\tau_0+\Delta_k\tau_1+\xi_k\tau_3}{[i\epsilon_n+\mu^{(0)}]^2-E_k^2}, \tag{1}$$

where $\epsilon_n=(2n+1)\pi k_BT$ is the Matsubara frequency, and $E_k=\sqrt{\xi_k^2+\Delta_k^2}$. Equation (1) indicates that the quasiparticle energies are given by $\pm E_k$. The retarded and advanced Green's functions are obtained by the analytic continuation of $i\epsilon_n \rightarrow \epsilon\pm i0^+$ in Eq. (1). Throughout this paper, the Fermi surface is assumed to be nearly nested [$|\mu^{(0)}|$ is very small], so that the low-lying quasiparticles are excited around the four nodes $k_i=(\pm\pi/2a, \pm\pi/2a)(i=1,2,3,4)$. Then the dispersion relation of the nodal quasiparticles can be linearized as $E_k \approx \sqrt{(v_f\cdot\tilde{k})^2+(v_g\cdot\tilde{k})^2}$, where $v_f=\partial\xi_k/\partial k$, $v_g=\partial\Delta_k/\partial k$, and $\tilde{k}$ is the momentum measured from the node k_i. A simple calculation yields $v_f=2\sqrt{2}ta$ and $v_g=\sqrt{2}\Delta a$. The directions of the Fermi velocity v_f and "gap velocity" v_g at the four nodes can be described by Fig. 1 in Ref. 13. A more realistic band structure would include the next-nearest neighbor hopping energy $\zeta_k=4t'\cos(k_xa)\cos(k_ya)$ in the single-particle dispersion relation. As pointed out in Ref. 5, however, the effect of t' can be neglected within the nodal-quasiparticle approximation, due to the fact that both ζ_k and $\partial\zeta_k/\partial k$ are vanishing at the nodes. If the system is far from half filling so that the nodal-quasiparticle approximation is not suitable, the effect of t' should be considered.

The nonmagnetic impurities are assumed to be pointlike, and randomly substituted on a small fraction of host atoms. The Hamiltonian for the impurity scattering reads $H_{\rm imp}=V\Sigma_{j=1}^{N_i}\Sigma_\sigma c^{\dagger}_{j\sigma}c_{j\sigma}=V\Sigma_{j=1}^{N_i}\Sigma_\sigma\Sigma_{kk'}c^{\dagger}_{k\sigma}c_{k'\sigma}e^{i(k-k')\cdot R_j}$, where V is the impurity potential, N_i is the total number of impurities, and R_j is the position of the impurity on site j. One can also write $H_{\rm imp}=V\Sigma_{j=1}^{N_i}\Sigma_\sigma\Sigma'_{kk'}[c^{\dagger}_{k\sigma}c_{k'\sigma}+c^{\dagger}_{k+Q\sigma}c_{k'+Q'\sigma}e^{i(Q-Q')\cdot R_j}+c^{\dagger}_{k+Q\sigma}c_{k'\sigma}e^{iQ\cdot R_j}+c^{\dagger}_{k\sigma}c_{k'+Q'\sigma}e^{-iQ'\cdot R_j}]e^{i(k-k')\cdot R_j}$, with k and k' limited within the reduced Brillouin zone. Noting that $e^{i(Q-Q')\cdot R_j}=1$ for nesting vectors Q and Q', we can reexpress $H_{\rm imp}$ as

$$H_{\rm imp}=\sum_{kk'\sigma}{}'\Psi^{\dagger}_{k\sigma}[U_0(k,k')\tau_0+U_2(k,k')\tau_2]\Psi_{k'\sigma}, \tag{2}$$

with $U_0(k,k')=V\Sigma_{j=1}^{N_i}e^{i(k-k')\cdot R_j}$ and $U_2(k,k')=V\Sigma_{j=1}^{N_i}e^{i(k-k'+Q)\cdot R_j}$.

In the SCTMA, the impurity-averaged self-energy is given by $\Sigma(i\epsilon_n)=n_iT(i\epsilon_n)$, where the impurity concentration n_i is very small. The impurity-averaging approach for normal metals is detailed in Ref. 15. However, it should be noted that the impurity-averaging procedure is very subtle for the DDW state. Since k and k' are located within the reduced Brillouin zone, we obtain the following results of averaging over the impurity positions: $\overline{U_0(k,k')}=(VN_i/N)\Sigma_R e^{i(k-k')\cdot R}=VN_i\delta_{kk'}$, and $\overline{U_2(k,k')}=(VN_i/N)\Sigma_R e^{i(k-k'+Q)\cdot R}=0$, where N is the total number of sites. Then the first-order contribution to the T matrix is given by $T^{(1)}=V\tau_0$. For the nth contribution, we have $\overline{U_{i_1}(k,k_1)U_{i_2}(k_1,k_2)\cdots U_{i_n}(k_{n-1},k')}=V^nN_i\delta_{kk'}$ (if there are even number of U_2), or 0 (if there are odd number of U_2), where $i_1, i_2, \ldots, i_n$ can be equal to 0 or 2. This implies that only the terms with an *even* number of τ_2 appear in the perturbative series of the T matrix, as shown by Fig. 2(a). For examples, the second- and third-order contributions can be expressed, respectively, as $T^{(2)}=V^2(\tau_0G\tau_0+\tau_2G\tau_2)$ and $T^{(3)}=V^3(\tau_0G\tau_0G\tau_0+\tau_0G\tau_2G\tau_2+\tau_2G\tau_0G\tau_2+\tau_2G\tau_2G\tau_0)$ with $G(i\epsilon_n)=\Sigma'_kG_k(i\epsilon_n)$, where $G_k(i\epsilon_n)$ is the impurity-averaged Green's function matrix. As indicated below [see Eq. (4)], $G(i\epsilon_n)$ has the expression as $G=g_0\tau_0+g_3\tau_3$ with $g_0(i\epsilon_n)$ and $g_3(i\epsilon_n)$ the expansion coefficients. Thus we get $T^{(2)}=2V^2g_0\tau_0$ and $T^{(3)}=4V^3g_0^2\tau_0$. The above calculations indicate that the terms containing g_3 cancel each other. Such a law can be shown to remain in all orders of the perturbation theory. Generally, the nth contribution to the T matrix is shown to be $T^{(n)}=d_nV^ng_0^{n-1}\tau_0$ with $d_n=C_0^n+C_2^n+C_4^n+\cdots=2^{n-1}$. As a result, we get $\Sigma(i\epsilon_n)=\Sigma_0(i\epsilon_n)\tau_0$ and $T(i\epsilon_n)=T_0(i\epsilon_n)\tau_0$, where

$$\Sigma_0=n_iT_0=n_i\sum_{n=1}^{\infty}V^n(2g_0)^{n-1}=\frac{n_i}{V^{-1}-2g_0}. \tag{3}$$

A use of Dyson's equation, $G_k(i\epsilon_n)^{-1}=G^{(0)}_k(i\epsilon_n)^{-1}-\Sigma(i\epsilon_n)$, yields a formal result as

$$G_k(i\epsilon_n)=\frac{[i\epsilon_n+\mu^{(0)}-\Sigma_0(i\epsilon_n)]\tau_0+\Delta_k\tau_1+\xi_k\tau_3}{[i\epsilon_n+\mu^{(0)}-\Sigma_0(i\epsilon_n)]^2-E_k^2}. \tag{4}$$

Similarly with the DWS case,[11–13] the self-energy for low-energy quasiparticles has the form as $\Sigma_0^{R(A)}(\epsilon)\approx(\eta\mp i)\gamma+\lambda_\pm\epsilon$ for $|\epsilon|\ll\gamma$, where γ is the impurity-induced relaxation rate, $\lambda_\pm$ is the mass renormalization factor, and η is a dimensionless parameter. Thus one obtains

$$G_k^{R(A)}(\epsilon)=\frac{[(1-\lambda_\pm)\epsilon+z_\pm]\tau_0+\Delta_k\tau_1+\xi_k\tau_3}{[(1-\lambda_\pm)\epsilon+z_\pm]^2-E_k^2}, \tag{5}$$

where $z_\pm=\mu\pm i\gamma$, with $\mu=\mu^{(0)}-\eta\gamma$ the modified chemical potential. The density of states at the Fermi surface is calculated as $\rho_f=-(1/\pi)\mathrm{Im}\,\Sigma'_{k\sigma}\,\mathrm{Tr}\,G^R_k(0)=4(l\gamma+\theta\mu)/(\pi^2v_fv_g)$ for $\gamma\ll\epsilon_c$ and $|\mu|\ll\epsilon_c$, where $\theta=\arctan(\mu/\gamma)$, l

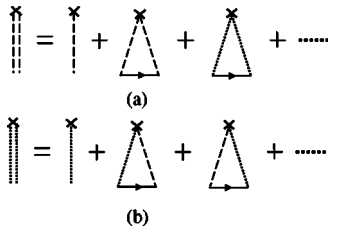

FIG. 2. Diagrams for (a) the T matrix (double dashed line) and (b) the $\tilde{T}$ matrix (double dotted line). The single-dashed and single-dotted lines with crosses represent, respectively, $V\tau_0$ and $V\tau_2$. The solid lines with arrows denote the impurity-averaged Green's function matrices.

$=\ln(\epsilon_c/\sqrt{\mu^2+\gamma^2})$, and $\epsilon_c \sim \sqrt{v_f v_g}/a$. From Eq. (5), one gets $g_0^{R(A)}(\epsilon) \approx \mp i\pi\rho_f/4-(l\mu-\theta\gamma)/\pi v_f v_g+(\lambda_\pm-1)(l-1\pm i\theta)\epsilon/\pi v_f v_g$. Substituting the expressions of $\Sigma_0^{R(A)}(\epsilon)$ and $g_0^{R(A)}(\epsilon)$ into Eq. (3), we obtain $\gamma=2n_i/[\pi\rho_f(1+\eta^2)]$, $\lambda_\pm^{-1}=1-(\eta\pm i)(l+\theta\mu/\gamma)/[(\eta\mp i)(l-1\pm i\theta)]$, and $\eta=2/\pi\rho_f U$ with $U^{-1}=V^{-1}+2(l\mu-\theta\gamma)/\pi v_f v_g$. The above self-consistent expressions of ρ_f, γ, $\lambda_\pm$, and η are suitable for nodal-quasiparticle approximation and low impurity concentration. These results will be used to investigate the quasiparticle transport. One notes that the Fermi surface is exactly nested for $|\mu|=0$, and thus the deviation from the nesting case is measured by the magnitude of $|\mu|$. In the case of $|\mu| \ll \gamma$, the results of Born ($|\eta| \gg l$) and unitary ($|\eta| \ll 1$) limits are readily obtained, given, respectively, by

$$\gamma=\frac{\pi}{2}n_i\rho_f U^2, \quad \rho_f=\frac{4l\gamma}{\pi^2 v_f v_g}, \quad \lambda_\pm=1-l,$$

and

$$\gamma=\frac{2n_i}{\pi\rho_f}, \quad \rho_f=\frac{4l\gamma}{\pi^2 v_f v_g}, \quad \lambda_\pm=\frac{l-1}{2l-1}.$$

These two limiting results have the same formulas with those of the DWS state.[13] In the case of $|\mu| \gg l\gamma$, we have

$$\gamma=\frac{\pi}{2}n_i\rho_f U^2, \quad \rho_f=\frac{2|\mu|}{\pi v_f v_g}, \quad \lambda_\pm=0,$$

for the Born limit ($|\eta| \gg l$), and

$$\gamma=\frac{2n_i}{\pi\rho_f}, \quad \rho_f=\frac{2|\mu|}{\pi v_f v_g}, \quad \lambda_\pm=0,$$

for the unitary limit ($|\eta| \ll 1$).

The operator for electric-current density can be derived via the charge conservation law $\boldsymbol{q}\cdot\boldsymbol{j}_{\boldsymbol{q}}=\{\rho_{\boldsymbol{q}},H\}_-$, where $\rho_{\boldsymbol{q}}=-e\Sigma'_{\boldsymbol{k}\sigma}\Psi^\dagger_{k\sigma}\Psi_{\boldsymbol{k}+\boldsymbol{q}\sigma}$ is the operator for the charge density. For $\boldsymbol{q}\to 0$, one gets $\boldsymbol{j}=\Sigma'_{\boldsymbol{k}\sigma}\Psi^\dagger_{\boldsymbol{k}\sigma}\boldsymbol{\Lambda}_{\boldsymbol{k}}\Psi_{\boldsymbol{k}\sigma}$ with $\boldsymbol{\Lambda}_{\boldsymbol{k}}=-e[\boldsymbol{v}_g(\boldsymbol{k})\tau_1+\boldsymbol{v}_f(\boldsymbol{k})\tau_3]$, which has been obtained in Ref. 8. The quasiparticle dc conductivity is calculated via $\sigma_d=(1/2\pi)\mathrm{Re}(\Pi^{\mathrm{RA}}-\Pi^{\mathrm{RR}})$, where Π^{RA} and Π^{RR} stand, respectively, for the current-current correlation functions in retarded-advanced (RA) and retarded-retarded (RR) channels.[13,15] The contribution of "bare bubble" diagram is given by $\Pi^{\mathrm{RA(RR)}}=(1/2)\Sigma'_{\boldsymbol{k}\sigma}\mathrm{Tr}[\boldsymbol{\Lambda}_{\boldsymbol{k}}G^R_{\boldsymbol{k}}(0)\cdot\boldsymbol{\Lambda}_{\boldsymbol{k}}G^{A(R)}_{\boldsymbol{k}}(0)]$, yielding $\Pi^{\mathrm{RA}}=\zeta\theta\alpha e^2/\pi$ and $\Pi^{\mathrm{RR}}=-\alpha e^2/\pi$, where $\alpha=(v_f^2+v_g^2)/v_f v_g$ and $\zeta=(\mu^2+\gamma^2)/\mu\gamma$. Thus we obtain the Drude conductivity

$$\sigma_d=(\alpha e^2/2\pi^2)(1+\zeta\theta), \tag{6}$$

leading to $\sigma_d=\alpha e^2/\pi^2$ for $|\mu| \ll \gamma$, and $\sigma_d=\alpha e^2|\mu|/4\pi\gamma$ for $|\mu| \gg \gamma$. These two limiting results of σ_d agree with those obtained in Ref. 8. Using the Einstein relation, $\sigma_d=e^2\rho_f D$, one gets the diffusion coefficient $D=(v_f^2+v_g^2)(1+\zeta\theta)/8(l\gamma+\theta\mu)$. For $|\mu| \ll \gamma$, we obtain $D=(v_f^2+v_g^2)/4l\gamma$, which has the same form with that of DWS state.[11–13] In the case of $|\mu| \gg l\gamma$, one gets $D=(v_f^2+v_g^2)/8\gamma$.

In order to investigate the issue whether σ_d is subject to a WL correction, we consider the cooperon equation in RA channel[13] (see Fig. 3)

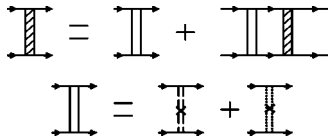

FIG. 3. Ladder diagrams for the cooperon propagator (shaded blocks) in the DDW state.

$$\mathcal{C}(\boldsymbol{q};\epsilon,\epsilon')=\mathcal{W}(\epsilon,\epsilon')+\mathcal{W}(\epsilon,\epsilon')\mathcal{H}(\boldsymbol{q};\epsilon,\epsilon')\mathcal{C}(\boldsymbol{q};\epsilon,\epsilon'), \tag{7}$$

where $\mathcal{H}(\boldsymbol{q};\epsilon,\epsilon')=\Sigma'_{\boldsymbol{k}}G^R_{\boldsymbol{q}-\boldsymbol{k}}(\epsilon)\otimes G^A_{\boldsymbol{k}}(\epsilon')$, and $\mathcal{W}(\epsilon,\epsilon')=n_i T^R(\epsilon)\otimes T^A(\epsilon')+n_i\widetilde{T}^R(\epsilon)\otimes\widetilde{T}^A(\epsilon')$. It is worthwhile to note that for the DDW state an additional $\widetilde{T}$ matrix appears in the expression of $\mathcal{W}(\epsilon,\epsilon')$, different from the case of DWS state.[11–13] The $\widetilde{T}$ matrix is diagrammatically represented by Fig. 2(b), and expressed as $\widetilde{T}(i\epsilon_n)=V\tau_2+V^2(\tau_2 G\tau_0+\tau_0 G\tau_2)+V^3(\tau_2 G\tau_0 G\tau_0+\tau_0 G\tau_2 G\tau_0+\tau_0 G\tau_0 G\tau_2+\tau_2 G\tau_2 G\tau_2)+\cdots$, yielding $\widetilde{T}(i\epsilon_n)=\tau_2/(V^{-1}-2g_0)$. Similar to the case of the one-particle Green's function, each term in the perturbative series of $\mathcal{W}(\epsilon,\epsilon')$ should contain *even* number of τ_2 due to the impurity-averaging procedure. Although only the terms with *odd* number of τ_2 appear in the perturbative series of the $\widetilde{T}$ matrix, each term in the perturbative series of the direct product $n_i\widetilde{T}^R(\epsilon)\otimes\widetilde{T}^A(\epsilon')$ contains *even* number of τ_2. One notes that both $\mathcal{H}(0;0,0)$ and $\mathcal{W}(0,0)$ are finite. Let us *assume* that the cooperon $\mathcal{C}(\boldsymbol{q};\epsilon,\epsilon')$ has a diffusive pole at $\boldsymbol{q}=0$ and $\epsilon=\epsilon'=0$, then Eq. (7) yields

$$\mathcal{A}\mathcal{C}(\boldsymbol{q};\epsilon,\epsilon')=0, \tag{8}$$

for small $\boldsymbol{q}$, ϵ, and ϵ', where $\mathcal{A}=\tau_0\otimes\tau_0-\mathcal{W}(0,0)\mathcal{H}(0;0,0)$. Using the decomposition of $X=\Sigma_{ij}X_{ij}\tau_i\otimes\tau_j$ for $\mathcal{C}$, $\mathcal{W}$, $\mathcal{H}$, and $\mathcal{A}$, one can show that the nonvanishing components of $\mathcal{A}$ satisfy $A_{00}=1+A_{22}$, $A_{21}=iA_{03}$, and $A_{12}=iA_{30}$. It then follows from Eq. (8) that

$$\begin{aligned} C_{00}-C_{22}=C_{11}+C_{33}=C_{03}+iC_{21}=C_{30}+iC_{12} \\ =(1+2A_{22})C_{00}=(1+2A_{22})C_{11} \\ =(1+2A_{22})C_{03}=(1+2A_{22})C_{30}=0. \end{aligned} \tag{9}$$

Equations (7)–(9) are shown to be also suitable for the RR channel. A direct calculation yields $A_{22}^{\mathrm{RA}}=-(\mu^2+\gamma^2)/4\mu(l\gamma+\theta\mu)\neq -1/2$ and $A_{22}^{\mathrm{RR}}=\gamma(\eta-i)/4(l\gamma+\theta\mu)(\eta+i)\neq -1/2$, indicating that all components of $\mathcal{C}$ are vanishing. This implies that the above assumption is incorrect, and the cooperon $\mathcal{C}(\boldsymbol{q};\epsilon,\epsilon')$ is actually of no diffusive pole. Therefore, the usual WL effect is in fact absent in the DDW state. As shown above, it is the appearance of $\widetilde{T}$ matrix in the expression of $\mathcal{W}(\epsilon,\epsilon')$ that leads to the absence of diffusive pole in the cooperon. The physical origin of the absence of WL effect is the breaking of the time-reversal symmetry by the DDW order. It has been shown that the WL effect in normal metals is also suppressed by perturbations such as magnetic-impurity scattering[16] and applied magnetic field,[17] which break the time-reversal symmetry. If these perturbations are strong enough, the WL effect will be killed completely. In the present work, the disordered DDW conductor is shown to

be also a typical 2D Fermion system without the WL effect. We wish to point out that the present theory is not suitable for the systems in which the DDW order is very small. This is because in such DDW conductors the low-energy quasiparticles are excited near the whole Fermi surface, and thus the nodal-quasiparticle approximation is not valid for these systems.

It is worth while to compare our results with those of the DWS state. The WL effect has been shown to be prominent in the DWS state.[11–13] With a nonperturbative approach, Pépin and Lee[18] obtained a logarithmic correction to the low-energy density of states as $\delta\rho(\epsilon)\approx n_i/(2|\epsilon|\ln^2|\epsilon/\Delta_0|)$ (Δ_0 is the superconducting gap), in the limit of strong, dilute scatters in the DWS state. The results of Ref. 18 are unlikely to hold for the DDW state, which is essentially different from the DWS state. While a superconducting state is of a condensate of particle-particle pairs, in which the time-reversal symmetry is preserved, the DDW order results from particle-hole pairing with the time-reversal symmetry broken. It is believed that the absence of WL effect is a generic property of the pure DDW state. Thus the density of states at the Fermi surface remains a finite value, having no WL correction. The present theory is suitable for a pure DDW state, which was suggested to be a possible candidate phase in the pseudogap regime of the cuprates, i.e., at the temperature region of $T_c<T<T^*$ with T_c the superconducting transition temperature. For $T<T_c$, there coexist the DWS and DDW orders. In such a coexisting state, the WL effect is expected to be partially destroyed (not completely killed), as the DDW order is considered to be a subdominant one.

Generally, the WL effect may manifest itself in low-temperature electronic transport of disordered systems.[10] The inelastic scattering (electron-electron or electron-phonon interaction) introduces a temperature-dependent dephasing time for the quantum interference effect. As a result, the WL correction to the dc conductivity is also dependent of the temperature through the dephasing time. Therefore, the WL effect can be revealed by the experimental measurements on temperature behavior of the electronic conductivity (as in a 2D normal conductor[10]), or thermal conductivity (as was expected for the DWS state[12]). According to the present theoretical prediction, the WL behavior of the conductivity should be absent in the experimental result made on the DDW phase.

In summary, we have developed a SCTMA theory for the effects of nonmagnetic-impurity scattering in disordered DDW conductors. The appearance of DDW order makes the diagrammatic rules quite different from the cases of DWS state and normal metals. The dc conductivity is obtained within the SCTMA. The WL effect, which was found to be prominent in the DWS state, is shown to be absent in DDW conductors due to the breaking of time-reversal symmetry. Such a qualitative difference between the disorder effects in DDW and DWS states might be helpful to settle the problem whether the psudogap behavior of cuprates results from the development of DDW order.

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[1] C. Nayak, Phys. Rev. B **62**, 4880 (2000); **62**, R6135 (2000); S. Chakravarty, R. B. Laughlin, D. K. Morr, and C. Nayak, *ibid.* **63**, 094503 (2001).

[2] M. R. Norman, H. Ding, M. Randeria, J. C. Campuzano, T. Yokoya, T. Takeuchi, T. Takahashi, T. Mochiku, K. Kadowaki, P. Guptasarma, and D. G. Hinks, Nature (London) **392**, 157 (1998).

[3] V. M. Krasnov, A. Yurgens, D. Winkler, P. Delsing, and T. Claeson, Phys. Rev. Lett. **84**, 5860 (2000).

[4] J. E. Sonier, J H. Brewer, R. F. Kiefl, R. I. Miller, G. D. Morris, C. E. Stronach, J. S. Gardner, S. R. Dunsiger, D. A. Bonn, W. N. Hardy, R. Liang, and R. H. Heffner, Science **292**, 1692 (2001).

[5] X. Yang and C. Nayak, Phys. Rev. B **65**, 064523 (2002).

[6] W. Kim and J. P. Carbotte, Phys. Rev. B **66**, 033104 (2002).

[7] B. Dóra, A. Virosztek, and K. Maki, Phys. Rev. B **66**, 115112 (2002).

[8] S. G. Sharapov, V. P. Gusynin, and H. Beck, Phys. Rev. B **67**, 144509 (2003).

[9] J.-X. Zhu, W. Kim, C. S. Ting, and J. P. Carbotte, Phys. Rev. Lett. **87**, 197001 (2001); Q.-H. Wang, *ibid.* **88**, 057002 (2002); D. K. Morr, *ibid.* **89**, 106401 (2002); C. P. Moca, Phys. Rev. B **66**, 132509 (2002).

[10] G. Bergmann, Phys. Rep. **107**, 1 (1984); P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985); B. Kramer and A. MacKinnon, Rep. Prog. Phys. **56**, 1969 (1993).

[11] A. G. Yashenkin, W. A. Atkinson, I. V. Gornyi, P. J. Hirschfeld, and D. V. Khveshchenko, Phys. Rev. Lett. **86**, 5982 (2001).

[12] Y. H. Yang, D. Y. Xing, M. Liu, and Y. G. Wang, Europhys. Lett. **63**, 114 (2003).

[13] Y. H. Yang, D. Y. Xing, M. Liu, and Min-Fong Yang, Phys. Rev. B **69**, 144517 (2004).

[14] P. J. Hirschfeld, P. Wölfle, and D. Einzel, Phys. Rev. B **37**, 83 (1988).

[15] G. D. Mahan, *Many-Particle Physics* (Plenum Press, New York, 2000).

[16] S. Hikami, A. I. Larkin, and Y. Nagaoka, Prog. Theor. Phys. **63**, 707 (1980).

[17] B. L. Altshuler, D. Khmel'nitzkii, A. I. Larkin, and P. A. Lee, Phys. Rev. B **22**, 5142 (1980).

[18] C. Pépin and P. A. Lee, Phys. Rev. B **63**, 054502 (2001).