# Vortex motion induced by lattice defects in two-dimensional easy-plane magnets

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In this work we investigate the dynamics of planar vortices in the presence of nonmagnetic impurities in a two-dimensional magnet with *XY* symmetry. We use analytical as well spin dynamics simulation methods. Our results indicate that vortices are attracted by defects in the lattice. Trapped by the nonmagnetic impurities they execute nonharmonic oscillations. In the regime of small oscillations our analytical results predict that the frequency is proportional to the inverse of the linear size of the system, in quite good agreement with numerical calculations. Although such oscillations barely can be observed in thermodynamic limit, it should be easy to be detected in nanoscale systems. We also consider the vortex motion near a line of nonmagnetic impurities. In such case, the vortex oscillates back and forth along the impurity line.

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## I. INTRODUCTION

The idea that nonlinear excitations arise in real physical systems had a strong impact on modern physics. Particularly, they are believed to play an important role in magnetic systems.[1–6] Recently, the direct experimental visualization of vortices in magnetic nanodots by magnetic force and Lorentz microscopy measurements[7–9] has given a new impulse to investigations in topological excitations in magnetic materials. Intensive artificial two-dimensional (2D) lattices of magnetic nanodots in vortex configurations[10] lead to potential technological applications of these structures in magnetic memory devices. Now even a delicate phenomenon like a shift of the vortex position from the center of a nanodot in an external magnetic field can be observed experimentally.[11] However, many particular features of vortex dynamics in layered magnets are still a matter of controversy. For example, below and above the Berezinskii-Kosterlitz-Thouless transition temperature there is no theory that can fully explain the central peaks observed both in inelastic neutron scattering experiments[12–14] and in combined Monte Carlo and spin dynamics simulations.[1,2,15–18] A qualitative agreement is achieved by a phenomenological vortex and vortex-pair gas theory,[1–3] but one of its assumptions—namely, ballistic vortex motion—is questionable. In fact, simulations indicate that a vortex has only local motion.[17–21]

The purpose of this paper is to discuss the effects due to lattice defects such as nonmagnetic impurities in magnetic two-dimensional *XY*-like materials. Lattice defects can induce and control vortex motion, playing a crucial role in disrupting order in solids.[22] Pinning of vortices on defects of magnetic structures has been considered recently by many authors.[23–26] Here, we show that a vortex oscillates around a nonmagnetic impurity. Particularly, it is also shown that a line of nonmagnetic atoms artificially built on a magnetic lattice by substitutions of the magnetic atoms by nonmagnetic ones (doping) will make a vortex to move in an oscillatory motion along the line of impurities. This phenomenon indicates that the vortex motion may be artificially controlled. Although the problem proposed here (with planar vortices) may have some connections with nanomagnetism, we note that in experiments with nanodots the vortices are in out-of-plane configuration[8] with just different dynamics.

We consider a classical magnetic system in a square lattice described by the *XY* model, which, for the case of orderly array of spins, can be written as

$$H = -J\sum_{\langle m,n\rangle}(S_m^xS_n^x + S_m^yS_n^y), \qquad (1)$$

where $J$ is a coupling constant, the classical spin vector has three components $\vec{S}=(S^x,S^y,S^z)$, and the summation is taken over the nearest-neighbor square lattice sites. This model should not be confused with the planar rotator model that, although it has a similar Hamiltonian, the spin vector $\vec{S}$ has only two components and has no true dynamics. The introduction of a low nonmagnetic impurity density modifies the array of spins and also the Hamiltonian. In earlier papers,[22,27] the authors studied the interaction between a single vortex and a static nonmagnetic impurity using analytical calculations based on the continuum limit of the *XY* Hamiltonian and also using simulation methods on a discrete lattice. Their conclusion was that the vortex is attracted and pinned by a nearby spin vacancy. In the continuum limit, the effective attractive potential vortex vacancy was obtained supposing that a nonmagnetic impurity could be represented by removing a disk of radius $a$ from the plane,[22] where $a$ is the lattice constant. Taking $\vec{S}^2=1$ and using the value of pinning energy equal to[22] $-3.54J$, this potential can be trivially expanded in a Taylor series around the bottom of the well, giving

$$V(X) \simeq -3.54J + \frac{1}{2!}\left(\frac{7.63}{a^2}\pi J\right)X^2 - \frac{1}{4!}\left(\frac{431.6}{a^4}\pi J\right)X^4 + \cdots, \tag{2}$$

where $X$ is the distance between the centers of the vortex and the vacancy. Although it is an approximation, expression (2) suggests that, in the small-amplitude regime $X \ll a$, the vortex center may oscillate around the spinless site under a force $\vec{F}=-k\vec{X}$, $k=7.63\pi J/a^2$ being the effective force constant. Alike motion may be possible for solitons in isotropic magnets.[6,28] If we consider the vortex as an ordinary particle of mass $M$, the resulting motion is nothing but a two-dimensional harmonic oscillator. However, to calculate the oscillation frequency, an important question related to the vortex effective mass is still open. It follows from a theory that this mass is proportional to $J\ln(L/a)$, where $L$ is the size of the system.[29,30] More recently, for a planar vortex, Wysin[31] has shown that the coordinate $\vec{X}$ of the vortex center satisfies the Newton equation $\vec{F}=M\ddot{\vec{X}}$ with a mass that increases with increasing system size as $M \propto L^2$. However, it may not be the vortex mass but the mass of coherent spin waves in the presence of vortex which really is proportional to $L^2$. If the results of Ref. 30 could be applied to the potential hole of defect, the oscillation frequency should be approximated by $\omega=\sqrt{7.63\pi J/Ma^2}$—i.e., proportional to $1/L$. This vortex oscillatory mode may be very difficult to observe in the thermodynamic limit since its frequency should be very small. Therefore, keeping the differences in mind, it may be possible to observe it experimentally in magnetic nanostructures like nanodots. The fabrication of small ferromagnetic particles containing artificial point defects is already possible.[32] At high temperature we expect the oscillation amplitude to grow and nonharmonic terms in Eq. (2) to become more important. Beside that, corrections due to discreteness effects and the development of out-of-plane components should play an important role. In the following we use Monte Carlo simulations in order to compare the simulations with the simple analytical results commented here.

## II. SIMULATIONS

To study the dynamical behavior of a vortex close to a spin vacancy we use a Monte Carlo numerical simulation. Our simulations were carried out using the standard Metropolis algorithm and lattices of size $L \times L$ with $L$ from $L=20$ up to 500. The dynamics is obtained through the equations of motion for each spin:

$$\frac{d\vec{S}_{i,j}}{dt} = \vec{S}_{i,j} \times \vec{H}_{eff}, \tag{3}$$

$$\vec{H}_{eff} = J\sum_{\alpha} (S^{\alpha}_{i-1,j} + S^{\alpha}_{i+1,j} + S^{\alpha}_{i,j-1} + S^{\alpha}_{i,j+1})\hat{e}_{\alpha}, \tag{4}$$

where $\alpha=x,y$ and $(i,j)$ stands for the spin position in the lattice. Equation (3) represents a set of coupled equations to be integrated numerically. To integrate the equations we have used a fourth-order Runge-Kutta scheme with size step $\delta t=0.04J^{-1}$. To obtain the dynamical behavior of the vortex near a spin vacancy, we have to have some control over the vortex properties. If we use open or periodic boundary conditions, it is impossible to get an isolated vortex due to the geometric constraint imposed by the boundary conditions: they always appear as bounded vortex-antivortex pairs. We can artificially create an isolated vortex somewhere in the system, but that is not an equilibrium state of the system such that the vortex excitation disappears very quickly generating spin waves. Our strategy is to use diagonally antiperiodic boundary conditions,[17,22,33] whose ground state has a single vortex or antivortex. We do not need to equilibrate the system at any specific temperature our only exigence being that there is only one vortex in the system. For possible applications of our results in magnetic nanodots, we would have to consider also the magnetodipole interactions. The magnetostatic energy contribution is related to the surface magnetic charges (due to dipoles) appearing along the envelop of a nanodisk. In order to minimize stray field energy, the magnetization of a nanodisk is aligned along the edge to close the magnetic flux. However, it is not the interest of the present paper and will be studied in a future work. Considering only the exchange interaction, we proceed as following. We create a vacancy at a position out of the center of the system, starting the Monte Carlo procedure at a very low temperature. Analyzing the system we observe that a vortex is created close to the vacancy. We take this as the initial spin conditions to be used in the equations of motion. The vortex motion can be followed by analyzing the phases of the spins close to vortex center. If the vortex center oscillates with frequency $\omega$, the time-dependent spin component at site $n$ is given by[6]

$$S^k_n \simeq S^k_{0,n} + A(n)\exp(-i\omega t), \tag{5}$$

obtained in a continuum approximation. Here the subscript 0 refers to the static vortex, $A(n) \ll 1$ is a small amplitude and $k=x,y,z$. By monitoring a particular spin or a group of spins belonging to the vortex structure (spins were chosen near the vacancy), we note that their components execute a rather more complicated oscillation than that described by Eq. (5), as can be seen in Fig. 1. On the other hand, to confirm that this motion is due to the vortex-vacancy interaction, we also considered the vortex in a lattice with no vacancies. It was not observed any spin oscillation for pure systems, as can also be seen in Fig. 1. The Fourier transforms for the oscillations of the spin components are presented in Fig. 2. Because the spin motion observed in Fig. 1 is not so simple, the center of a vortex must not execute a simple harmonic oscillation. The discrepancy is due to many different causes. First, for a vortex in an easy-plane magnet the spin-wave spectrum is gapless. It implies that for oscillations of the vortex under the influence of the vacancy force with a finite frequency $\omega$, which frequency inevitably falls in a spin-wave continuum. As a result, vortex motion excites spin-wave modes, and that will lead to fundamentally different consequences for vortex motion in an unbounded medium and in a finite sample. For a magnet of finite size, one expects that the radiation of spin

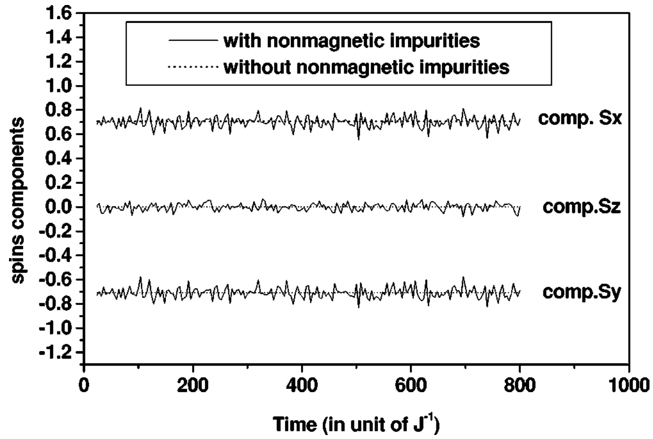


FIG. 1. Oscillation of the components $S_x$, $S_y$, and $S_z$ of a spin vector belonging to the vortex structure in the presence of a vacancy localized at the vortex center (solid lines). The dashed lines show that these same spin components practically do not oscillate if there is no vacancy placed at the vortex center. Here $L=100a$.

waves, their reflection off the boundary, and their effect back on the vortex will result in the establishment of a dynamical state of the magnet which includes both the moving vortex and the coherent magnetization oscillations matched to the vortex motion. This is a complex picture and really, in this case, an adequate description of the dynamics of a magnetic vortex in easy-plane magnets may be obtained only with the use of a complicated hierarchy of equations of motion, containing all the higher-order time derivatives.[34] Second, discreteness effects and nonharmonic terms present in expansion (2) may also be relevant in modifying Eq. (5). In fact, our simulations indicate that the modulus of $X$ can be as large as one lattice spacing $a$, suggesting that the third term in Eq. (2) is important.

Besides, in the continuum theory, the nonmagnetic impurity was represented by a circular disk cut out from the plane leading to a central potential of interaction between the vortex and the vacancy. However, in a real square lattice, the plaquette around the impurity has a form of a square, and of

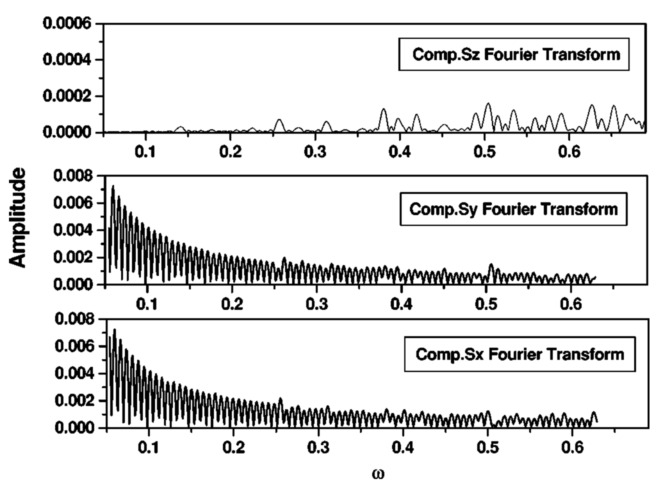


FIG. 2. The Fourier transforms for the oscillation of the components $S_x$, $S_y$, and $S_z$ shown in Fig. 1 for $L=100a$.

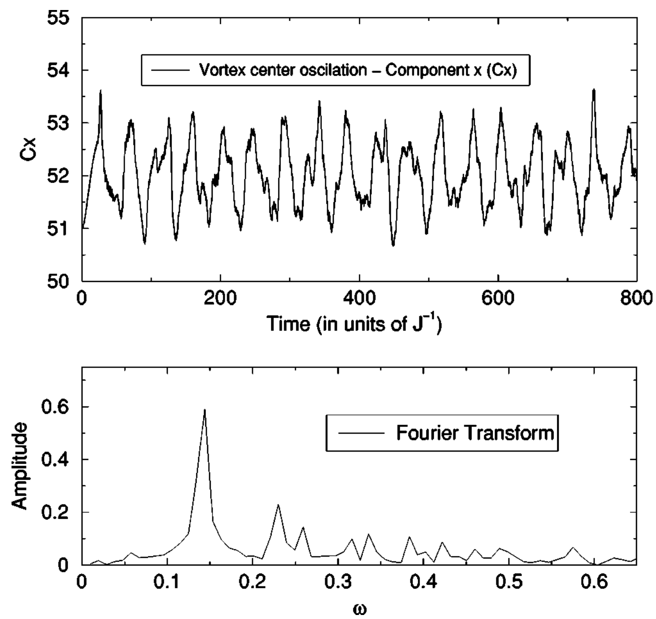


FIG. 3. Vortex center position as a function of time for $L=100a$ and its Fourier transform.

course, the vortex-vacancy potential must not be isotropic. We believe that the above considerations are responsible for disturbing the vortex movement around the vacancy, being the main causes of a more complicated motion than expected. Another effect associated with the vortex motion may also induce further complications. When a planar vortex starts moving it develops a small out-of-plane structure,[2] which in this case must be also time dependent, due to the changes in the direction of motion. Figures 1 and 2 show that this last contribution can be neglected, mainly for low frequencies, as it is expected for strongly anisotropic systems.[35]

The method used to identify the vortex center position is the following: consider a line of spins (belonging to the vortex structure) parallel to the $x$ axis. Following the spin orientation along this line, it is easy to see that the $y$ component of the spins changes the sign at the $x$ coordinate of the vortex center position. Then, we considered all possible lines parallel to the $x$ axis in a particular lattice, finding for each line the exact point in which the $y$ component of spins changed sign. The mean value of these points was used as the $x$ coordinate $(Cx)$ of the vortex center position. Similar procedure can be done to determine the $y$ coordinate of the vortex center position. In Fig. 3 we plot the $x$ coordinate of the vortex center position $(C_x)$ as a function of time for $L=100a$. Note that the amplitude of oscillation is of the order of one lattice spacing. Figure 3 also presents the Fourier transform of the vortex center oscillations. Many peaks at well-defined frequencies are observed in this graphic. These results imply in a complicated vortex motion. As can be seen in the Fourier transform (Fig. 3), there is a main peak representing a normal mode and many other small peaks representing other normal modes that would be also excited around the vortex structure. Here, we will consider only the three first modes. It is easy to observe that the first relevant mode contributes very little to the vortex oscillations and its frequency is $\omega_1 \approx 0.059J$. The second mode is the main peak (with frequency

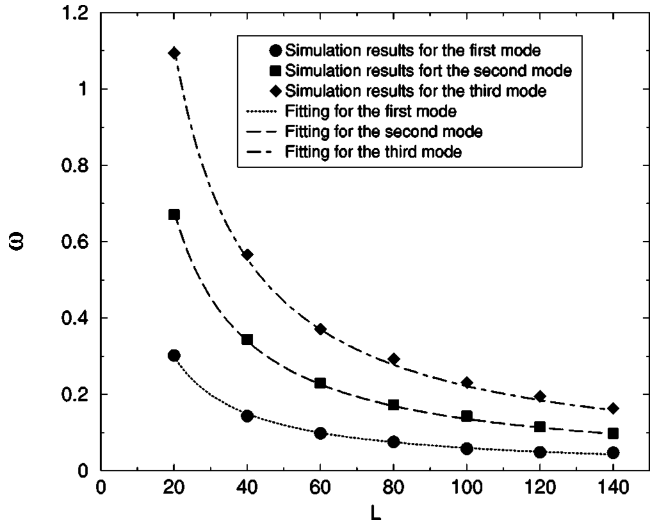


FIG. 4. Frequencies of the three lowest modes observed in the Fourier transform of the vortex position versus lattice size $L$. These frequencies can be fitted by $\omega_1 \approx 5.97/L$, $\omega_2 \approx 13.57/L$, and $\omega_3 \approx 22.2/L$.

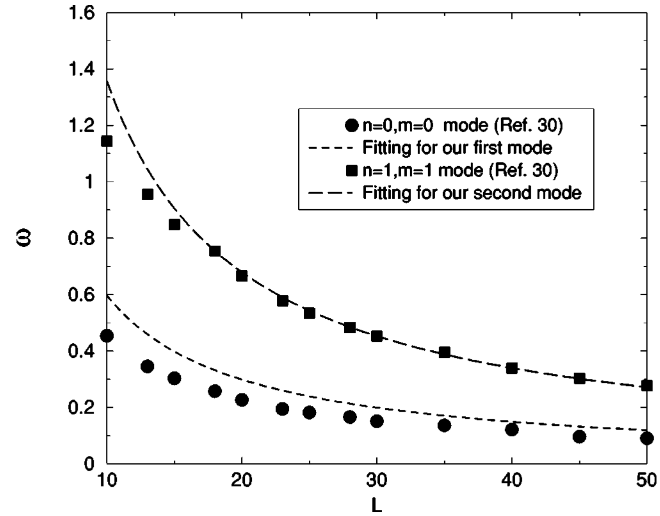


FIG. 5. Our first and second modes compared to the normal modes with $(n=0, l=0)$ and $(n=1, l=1)$ obtained by studying the vortex-magnon interactions in Ref. 31.

$\omega_2 \approx 0.14J$) and the third mode (with frequency $\omega_3 \approx 0.22J$) has the second more significative contribution for the vortex oscillations. Some of these peaks are also present in the oscillations of spin in the $z$ direction (for instance, see the small peak at frequency close to $0.14J$ in Fig. 2). Of course, such a peak in the Fourier transform for the oscillation of the component $S_z$ is connected with the vortex motion. We also plotted the frequencies of these three normal modes as a function of the lattice size $L$ (see Fig. 4). These frequencies can be fitted by $\omega_1 \approx 5.97/L$, $\omega_2 \approx 13.57/L$, and $\omega_3 \approx 22.2/L$. We notice that some of these modes can be associated with normal modes that result from the vortex-magnon interactions.[31]

In fact, Wysin[31] has studied the vortex-magnon spectra (in 2D systems without vacancies) considering the dependence of the eigenfrequencies $\omega_{nl}$ on the system radius. These modes are classified according to azimuthal quantum number $l$ (due to the fact that the wave function must not be multiple valued) and a principal quantum number $n$, defined as the number of nodes in a radial direction in the wave function. In Ref. 31, only the frequencies of some of the lowest modes versus the system size were calculated. As can be seen in Fig. 5, our first and second modes are associated with the modes with $(n=0, l=0)$ and $(n=1, l=1)$ obtained by Wysin,[31] respectively. Our third mode cannot be compared with any $(n,l)$ mode because its frequency is bigger than the ones shown in Ref. 31. Besides these modes, other higher modes are also present in the Fourier transform in Fig. 3 contributing to the observed vortex behavior. These results suggest that the vortex oscillations may have an interesting and different interpretation. Instead of thinking about an oscillating vortex, we can imagine a static vortex centered on the impurity surrounded by spin waves. These modes could be identified by viewing the motions of spins that result when some given spin-wave eigenfunctions are added to the original vortex structure. Then, the spin movements would be interpreted as normal modes on the vortex structure and not as being due to the vortex oscillation. However, the Fourier transform of spin oscillations is not in accordance with the Fourier transform of the vortex center oscillations. Note that the spin oscillations are very complicated without any visible periodicity (see Fig. 1) while the vortex center oscillations are evidently periodical (Fig. 3). Thus, it is not all clear that the observed modes are really due to vortex-magnon interactions alone.

To explore in more details the above question, it would be interesting to follow the vortex motion with larger amplitude. It can be done by studying the vortex behavior along a line of vacancies. Thus, we consider now a line of $p$ neighbor nonmagnetic impurities, implying in $p$ neighbor holes of radius $a$, which the centers are distributed along a line in the $x$ axis. For simplicity, we use $p$ odd $(p=1,3,5,\ldots)$. Thus, if the central impurity is localized at origin and the vortex center is placed at distance $r=\sqrt{x^2+y^2}$ away, the results of Ref. 22 can be generalized leading to the potential experienced by a vortex due to the defect line:

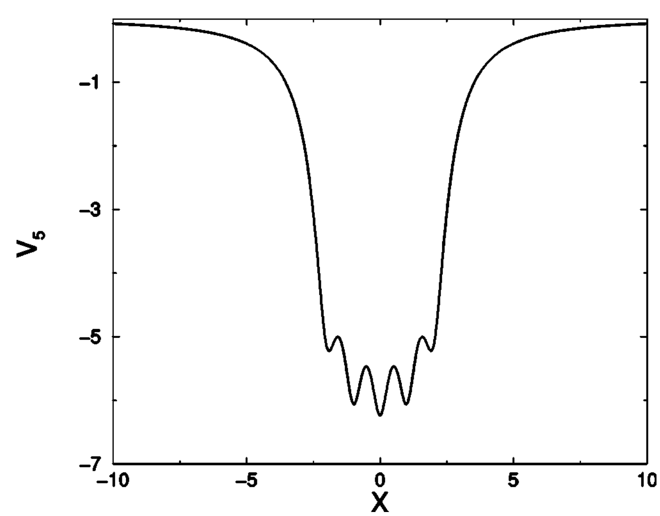


FIG. 6. Interaction potential between a vortex and a line of five impurities. Here the vacancies are placed along the axis $y=0$. The central impurity is located at (0, 0).

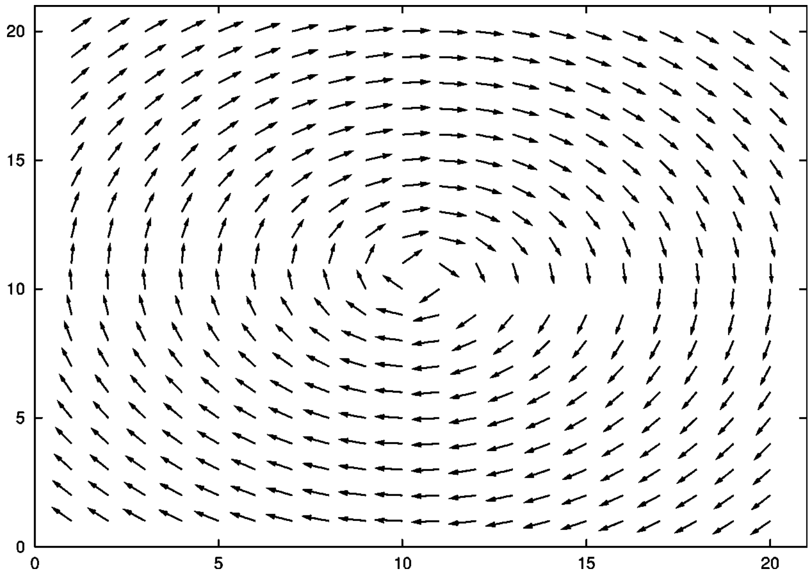


FIG. 7. A typical initial configuration of a vortex initially around the impurity line containing five vacancies.

$$V_p(x,y) = \frac{\pi J}{2} \ln\Bigg\{\left[1 - \frac{a^2}{[x-(p-1)a/2]^2 + y^2 + b^2}\right] \times\left[1 - \frac{a^2}{[x-((p-1)/2-1)a]^2 + y^2 + b^2}\right]\cdots \left[1 - \frac{a^2}{[x+((p-1)/2-1)a]^2 + y^2 + b^2}\right] \times\left[1 - \frac{a^2}{[x+(p-1)a/2]^2 + y^2 + b^2}\right]\Bigg\}. \qquad (6)$$

This approximated potential is shown in Fig. 6 for $p=5$. Note that it contains four local minima and the fundamental state is obtained when the vortex center coincides with the center of the central impurity. The potential is attractive, and for $p>5a$, it is weak, becoming stronger as the vortex center approximates to the central impurity. For even $p$, the potential is alike that of Fig. 6, but it contains two minima at the two neighbor vacancies located at the middle of the line. These results indicate that an initially stopped vortex will start to move in direction to the line defect and after penetrating this line, it may start to oscillate along the spinless sites. In fact, after penetrating the line of impurities, the vortex center will feel a kind of periodic potential.

To go from one side to the other, small barriers must be overcome. The vortex will be trapped by the lattice defect, oscillating from one side to the other along the line of spinless sites, in a complicated movement that will also be influenced by the pinning potential due to the discreteness effects (not present in the analytical calculations). Figure 7 shows a typical spin configuration of a vortex initially around the impurity line containing five vacancies and Fig. 8 shows a spin configuration after 1000 time steps. The vortex center is already inside the defect line.

We also plotted the oscillations of the $x$ component of the vortex center position $(C_x)$ as a function of time for $L=100a$, as well as their Fourier transforms, in Fig. 9. As can be seen, for the vortex center, like the case of only one vacancy, our numerical simulations show that a vortex really

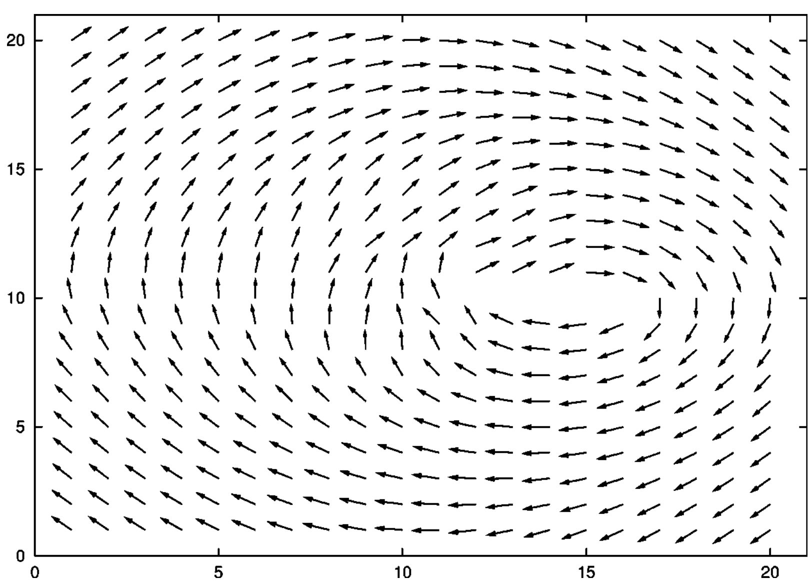


FIG. 8. Configuration after 1000 time steps.

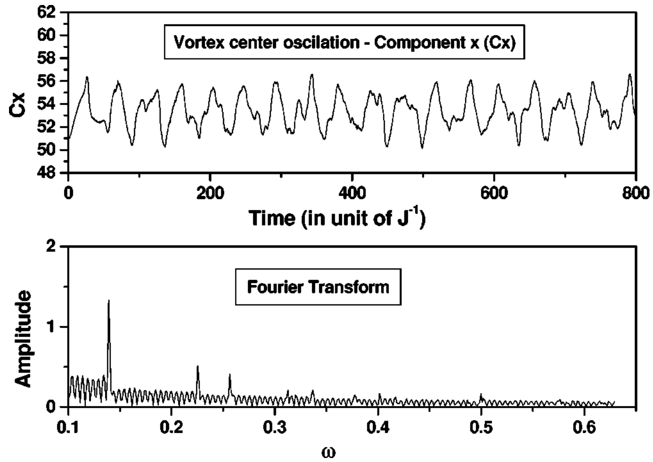


FIG. 9. Vortex center position as a function of time for $L=100a$ and its Fourier transform, in presence of the defect line containing five vacancies.

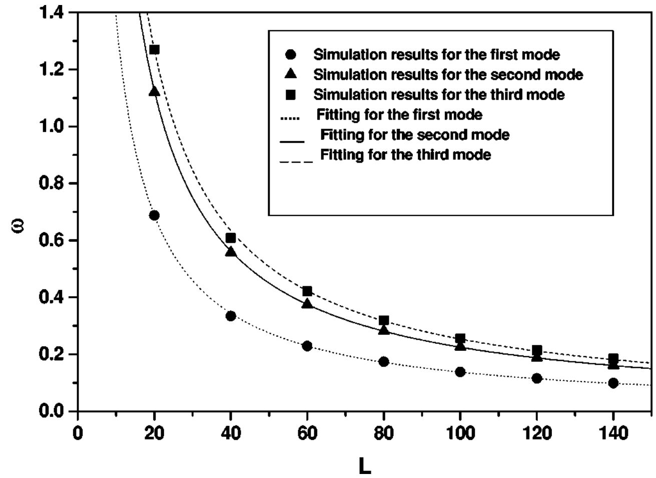


FIG. 10. Frequencies of the three lowest modes observed in the Fourier transform of the vortex position versus lattice size $L$, in the presence of the defect line containing five vacancies. These frequencies can be fitted by $\omega_1' \approx 13.763/L$, $\omega_2' \approx 22.503/L$, and $\omega_3' \approx 25.390/L$.

oscillates back and forth along the impurity line. One important thing to note in the Fourier transform for the vortex center position around the defect line is that some of the main peaks are too close to that observed in the case of only one vacancy (compare Figs. 3 and 9). In fact, for a defect line, there are essentially three peaks, which can be fitted as $\omega_1'=13.763/L$, $\omega_2'=22.503/L$, and $\omega_3'=25.390/L$, respectively, shown in Fig. 10. The fittings for the first and second modes observed for the defect line are related to the second and third modes observed in the case of one impurity—i.e., $\omega_1' \sim \omega_2$ and $\omega_2' \sim \omega_3$. It suggests that these modes are strongly associated with the vortex oscillations around vacancies (mainly the mode with frequency $\omega_3$ absent in Wysin results). Since the obtained relations for an impurity and a line of impurities decay as $1/L$ ($\omega_3 \approx 22.2/L$), this work also reinforces the idea that the vortex mass indeed may be proportional to $L^2$ as Wysin has shown.[31]

## III. CONCLUSIONS

In summary, we have investigated the magnetic vortex motion near spin vacancies. For small-amplitude oscillations, we developed a simple analytical model that predicts a harmonic motion for the vortex center around a nonmagnetic impurity center with a well defined frequency $\omega=\sqrt{7.63\pi J/Ma^2} \propto 1/L$. Our numerical simulations shows that the amplitude oscillation is not so small and hence the center of the vortex execute a rather complex motion. The vortex center should really move around the vacancy center, but following trajectories more difficulty to understand. In the case of a vortex in a magnet of finite size the effective equations turn out to be more complicated than Newton equations. Indeed, the vortex oscillations under the influence of the vacancy force has a finite frequency and falls in the continuum. Because of this, magnon modes are excited modifying the dynamical state of the magnet, which should include the coherent magnetization oscillations matched to the vortex motion. We also studied the vortex motion along a line of nonmagnetic impurities. Again, the vortex oscillates back and forth along the impurity line. For a long line, the vortex is able to dislocate large distances from one extremity to the other and later it comes back to the initial position and so on. Many properties of this movement (like direction, etc.) can be artificially controlled. The possibility of a control on the vortex motion may be of interest in connection with the proposal to use topological structures as information carriers in magnetic memory systems. As a final remark, we would like to mention that these complex motions of vortices in both situations (with one impurity or a line of impurities) should be reflected in the response functions of the magnet. It is already known that vacancies have a strong impact on the critical properties of these easy-plane magnetic materials.[36,37]

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