Critical exponents at the metal-insulator transition in AlPdRe quasicrystals

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Insulating icosahedral AlPdRe with resistance ratios $R[=\rho(4.2\text{ K})/\rho(295\text{ K})]$ from 40 to 220 has been studied by magnetoresistance and conductivity measurements. Consistent results for the localization length ξ and the characteristic temperature T_o were determined. $(R-R_c)$ increases from the metal insulator transition (MIT) into the insulator as the inverse of the shrinking volume of the electron wave functions $\sim\xi^{-3}$. Localization is driven by disorder. Evidence is discussed that the MITs in *i*-AlPdRe and doped semiconductors belong to the same universality class.

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Quasicrystals have continued to reveal new and surprising properties in a number of different subfields of physics (see e.g., Ref. 1) ever since their discovery 20 years ago. One challenging case is electronic transport in icosahedral (*i*) AlPdRe. Phase pure samples of the same nominal composition can be prepared by different annealing conditions in states of strongly varying electrical properties, and with resistance ratios $R\,[=\rho(4.2\text{ K})/\rho(295\text{ K})]$ in the range 2–300. An explanation for this phenomenon has not been found. Studies of the magnetoresistance (MR) have shown insulating behavior with variable range hopping (VRH) at large R,[2,3] and weak localization contributions at low R, characteristic for weakly disordered metals.[4] A scaling approach to the zero-field metallic conductivity $\sigma(T)$ above 400 mK has also indicated a metal-insulator transition (MIT).[5] Different estimates of the MIT as a function of R all indicate that it occurs in the range 20–30.[2,4–6]

Little is known about this MIT however. In spite of intensive efforts, results from $\sigma(T)$ have remained inconclusive,[7–13] with a remarkably wide range of estimates of relevant parameters such as the localization length, ξ, and the characteristic temperature T_o of the hopping process. This is likely related to another unresolved difficulty in *i*-AlPdRe: the zero temperature conductivity $\sigma(0)$ is finite also in insulating samples and decreases exponentially with increasing R into the insulator.[14,15] The results indicate that a finite $\sigma(0)$ is an intrinsic effect in insulating *i*-AlPdRe. We speculated that it may be due to quantum tunneling of residual critical states. A generalized form for variable range hopping is then[7]

$$\sigma(T) = \sigma(0) + \sigma_o e^{[-(T_o/T)^{\nu}]}. \tag{1}$$

σ_o is a prefactor usually taken to be temperature independent, and ν is $1/4$ or $1/2$ for Mott and Efros Shklovskii (ES) VRH respectively. T_o also depends on the type of hopping. Equation (1) is numerically flexible. Results may depend on methods of analyses and temperature ranges studied, and well founded results for T_o and ξ in a series of *i*-AlPdRe alloys have not been obtained. A further problem is that quantitative analyses of the insulating MR have been limited to the temperature interval 1–10 K.[2,3] Below 1 K the MR changes character.[14] This MR is not understood.

i-AlPdRe thus shows an unusual and poorly explored metal-insulator transition. It is not clear how previous knowledge of MITs is relevant. In the on-going problem of the MIT, studies of new systems are important, and can allow determination of the universal character of parameters describing the MIT. In this paper we set out to describe the MIT of *i*-AlPdRe by an approach which circumvents some of the difficulties mentioned. It relies on three ingredients: (i) measurements at very low T to find $\sigma(0)$; (ii) confirmation from the MR of the type of hopping mechanism; and (iii) analyses in comparable temperature ranges of σ and the MR to find T_o and ξ. From these results, descriptions are found for the vanishing T_o and the diverging ξ at the MIT.

The MR was analyzed as recently described.[2] At magnetic fields above the minimum in $\Delta\rho/\rho$, the MR increases as $+B^2$ in a field range which increases with increasing temperature,

$$\frac{\Delta\rho(T,B)}{\rho(T,0)} = \frac{B^2}{B_o^2(T)}. \tag{2}$$

The temperature dependence of $B_o^{-2}(T)$ is characteristic for the type of hopping,

$$B_o^{-2}(T) \simeq \beta T^{\mu}, \quad \mu = 0, -3/4, -3/2. \tag{3}$$

The values of μ refer to nearest neighbor hopping, Mott VRH, and ES VRH, respectively. β also depends on the type of hopping. For ES VRH one has[16]

$$\beta = 0.0015(e/\hbar)^2\xi^4 T_o^{3/2}, \tag{4}$$

giving a relation between β, ξ, and T_o for each value of the resistance ratio R.

By using Eq. (4) one can avoid a frequently used relation between T_o, ξ, and the density of states, $D(\epsilon_F)$,

$$T_o = \frac{\text{const}}{k_B\xi^3 D(\epsilon_F)}, \tag{5}$$

valid close to the MIT for both Mott and ES VRH.[17] $D(\epsilon_F)$ may vary with R, which complicates the use of Eq. (5). Further, since the MR is analyzed only between 1.5–10 K,

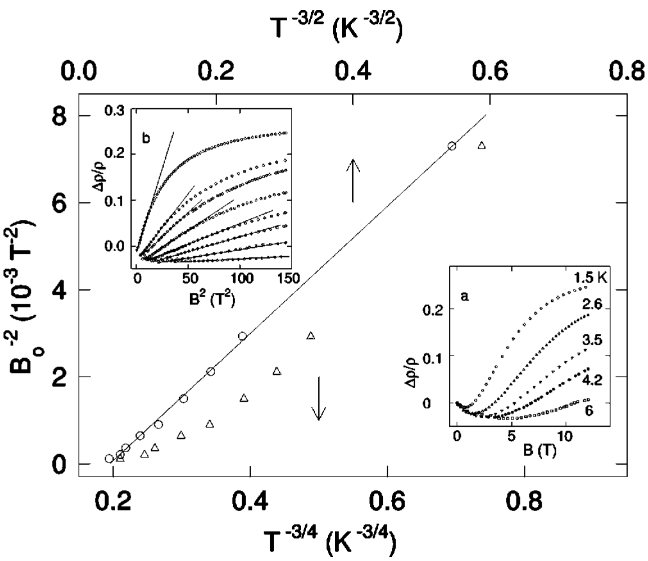

FIG. 1. $\Delta\rho(B,T)/\rho(0,T)$ for the $R=220$ sample. Inset a: Primary data at selected temperatures. Inset b: Analyses in terms of Eq. (2). Temperatures from top to bottom: 1.5, 2.6, 3, 3.5, 4.2, 5, 6, and 8 K. Main panel: Eq. (3) for $\mu=-3/2$ (top scale, ES VRH), and $\mu=-3/4$ (bottom scale, Mott VRH). Data confirm ES VRH.

analyses of σ should be limited to a similar interval.

Four samples were studied, all with nominal composition $Al_{70.5}Pd_{21}Re_{8.5}$. Three of them, with $R=40$, 60, and 220, were ingots, prepared by successive annealings of arc-melted ingots. One sample, $R=120$, had been melt-spun and annealed at various temperatures. Details of techniques and sample characterization have been published.[4] MR was measured in a cryostat equipped with a 12 T superconducting magnet. Low temperature measurements were made in a dilution refrigerator to 15 mK in Stockholm, or to 8 mK at the EC Ultra Low Temperature Facility in Bayreuth. $R=220$ is the largest R value so far for which detailed studies of electrical transport properties of quasicrystals have been made.

The MR can clearly distinguish between different types of hopping as shown by the analysis in Fig 1. The straight line in Fig. 1 (top scale), shows that $B_o^{-2}(T)\sim T^{-3/2}$, in agreement with ES VRH, while Mott VRH ($\sim T^{-3/4}$, open triangles) is not fulfilled. Above the B^2 region the MR follows $\sim B^{2/3}$, in further agreement with ES VRH.[16] Similar results were obtained for the $R=60$ sample. For $R=40$ and 120, results were taken from samples studied previously.[2] In total the MR has been studied in this scheme for six *i*-AlPdRe samples with R in the range 40–220. ES VRH was observed in all cases suggesting a general property for insulating *i*-AlPdRe. In particular, there is no sign of a crossover to Mott VRH in the range studied of B, T, and R.

To estimate $\sigma(0)$ it is necessary, for high-R samples to access temperatures <20 mK.[14] Our results for the $R=40$, 60, and 220 samples were found to be in good agreement with the previously obtained relation. At $R=120$ however, $\sigma(0)$ was 0.8 $(\Omega\ \mathrm{cm})^{-1}$ while ~ 0.3 $(\Omega\ \mathrm{cm})^{-1}$ was expected. This difference is not understood. The measured values were used in the analyses.

Using these estimates of $\sigma(0)$ and the result from MR that ES VRH is obeyed, Eq. (1) can be displayed as straight lines of the form $\ln[\sigma(T)-\sigma(0)]$ vs $T^{-1/2}$, with T_o as the only free parameter. Figure 2 shows that T_o is fairly well determined by this analysis. The measured β, then gives ξ from Eq. (4). β was found to be almost independent of R, with variations within $\sim 10\%$ for the present samples. Results for T_o and ξ are given in Table I.

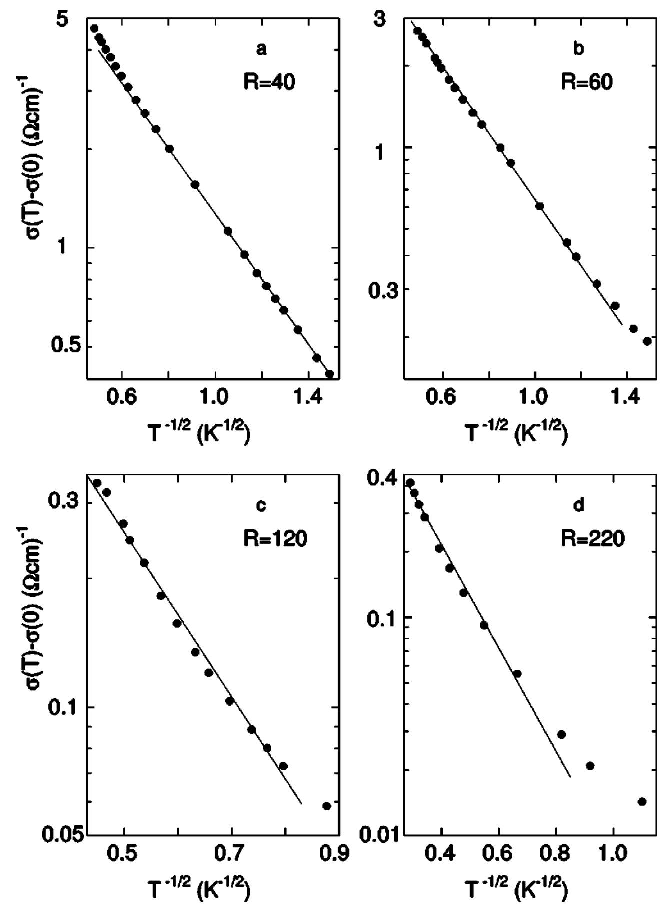

FIG. 2. Analyses of the temperature dependence of $\sigma(T)$ according to Eq. (1) with $\nu=1/2$ (ES VRH).

In Fig. 2, data at high temperatures deviate from linear fits above a temperature which increases with increasing R. At $R=40$ such deviations start at $T_o^{-1/2}\approx 0.6$, i.e., above 2.8 K, while at $R=220$ a straight line fit persists to ~ 12 K. This trend is expected from the results for T_o and the condition $T\ll T_o$ in the VRH range. At low temperatures in Fig. 2, there are deviations for the high R samples below ~ 1.5 K, in qualitative agreement with the progressively anomalous MR at large R.[14]

We now discuss ξ and T_o. According to scaling theories the localization length diverges when the MIT is approached. ξ as a function of R is investigated in Fig. 3 for three different choices of the value R_c of R at the MIT. A first conclusion

TABLE I. Results for T_o and ξ.

R	40	60	120	220
T_o (K)	5.23	7.85	19.6	29.7
ξ (Å)	253	220	153	128

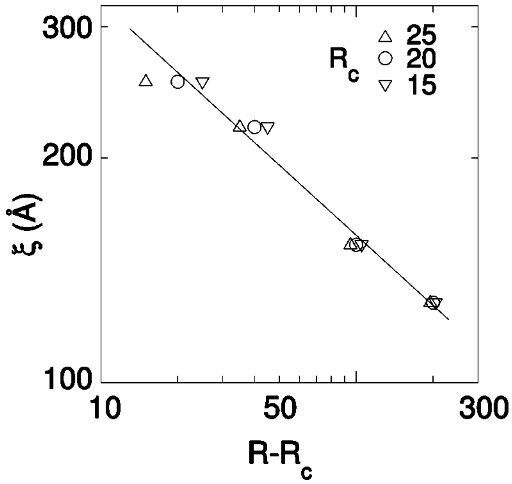

FIG. 3. ξ vs $R-R_c$ for three choices of R_c. Data were analyzed in the form $\xi \sim (R-R_c)^{-\gamma_R}$. For $R_c=20$, the straight line indicates $\gamma_R=0.31$.

is that the result for the exponent γ_R, of order 1/3, is fairly robust for different choices of R_c. Closer inspection reveals that the best straight line fit, found for $R_c=20$, is significantly better than for $R_c=25$ and slightly better than for $R_c=15$. The latter value is outside the range of previous results.[2,4–6] $R_c=20$ was taken to be the critical value.

The result in Fig. 3 suggests, roughly, that $(R-R_c) \sim \xi^{-3}$, i.e., $R-R_c$ is inversely proportional to the correlation volume. One can view the increase of $R-R_c$ when moving into the insulator as a reflection of a shrinking volume of the electrons that hop, resulting in progressive localization. Of course, at $R=R_c$, ξ becomes infinite, i.e. metallic, this relation breaks down, and the question of the interpretation of R on the metallic side is left open.

The apparent absence of R dependence in β of Eq. (4) can be compared with published results. From data for the ES VRH magnetoresistance in the B^2 region in doped Ge:As with n/n_c of about 0.3 and 0.8,[18] it can be calculated that β was ~50% larger for the more insulating sample. Here n is the charge density, and n_c its value at the transition. In Si:(P,Si) with n/n_c of 0.3 and 0.6, and in magnetic fields comparable to ours, ES VRH was observed, and β was a factor ~2 larger for the more insulating sample.[19] These comparisons suggest that a near constant β may be specific to the quasicrystal.

When β is constant, Eq. (4) gives $T_o \sim \xi^{-8/3}$ at all R. This simplifies description of parameters near the MIT. Equation (4) with an average value of $\beta=0.0167\ T^{-2}K^{3/2}$ is compared in the inset of Fig. 4 with T_o from Fig. 2, and with Eq. (5) for a constant $D(\epsilon_F)$. Equation (5) (top scale) has a more restricted validity. The differences between T_o calculated from an average or from the observed β are small as illustrated by an error bar. The curve for T_o vs R in the main panel was obtained as described in the legend. This gives a convenient description of $T_o(R)$.

It is of interest to compare the diverging ξ at the MIT with results in doped semiconductors. To achieve this we assumed that variations of a relevant charge density can be estimated from σ(295 K), taking $n/n_c=\sigma(295\text{ K},R)/\sigma(295\text{ K},R=20)$.

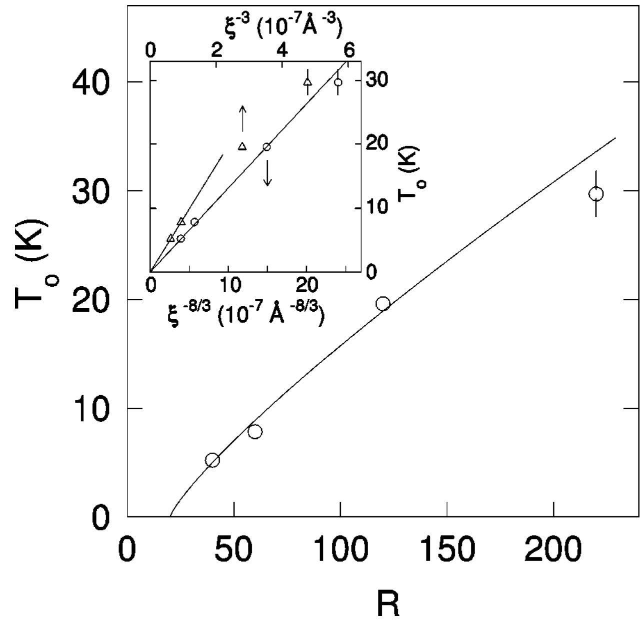

FIG. 4. T_o vs R. The curve, $T_o=0.42(R-20)^{0.83}$, was calculated from Eq. (4) with a constant β, and the result for ξ in Fig. 3. Inset: T_o vs two functions of ξ. Bottom scale: Eq. (4) with constant β. Top scale: Eq. (5) with constant $D(\epsilon_F)$.

n is likely smaller than the electron density of the quasicrystal, and may, e.g., reflect the number of electrons that localize at low T and participate in hopping.[2] The only present assumption, however, is that the changing charge can be estimated by the change of a normalized room temperature σ. This transformation from R to n/n_c as a driving parameter is somewhat related to the frequent use of a relation between R and n in doped semiconductors, e.g., for calibration of the n scale in Si:P and Si:B.[20,21] At $R_c=20$ we estimated $\sigma(295\text{ K})=250\ (\Omega\text{ cm})^{-1}$.[22] n/n_c was found to decrease from 0.85 to 0.3 when R increased from 40 to 220. When analyzed in the form $\xi \sim |1-n/n_c|^{-\gamma_n}$ as illustrated in Fig. 5, the result was $\gamma_n=0.46$.

In three-dimensional Wegner-scaling the critical exponent of the conductivity is the same as for the localization

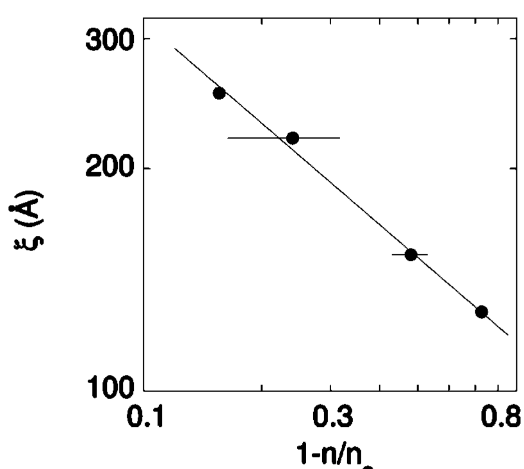

FIG. 5. ξ vs $(1-n/n_c)$ on logarithmic scales. The slope $-\gamma_n$ is close to −0.5. A 10% error in n is marked on some points.

length,[23] and furthermore, a critical exponent is identical on both sides of a phase transition. One can thus compare ξ of insulating *i*-AlPdRe with the conductivity of doped semiconductors on the metallic side. This approach circumvents the problem with a finite $\sigma(0)$ in the quasicrystal. The vanishing conductivity at the MIT of doped semiconductors has been frequently studied. At least for a large class of uncompensated semiconductors in zero magnetic field, the conductivity exponent has been experimentally found and theoretically confirmed to be 0.5.[20,24,25] Our result for γ_n is close to this value. This finding suggests that the MIT in *i*-AlPdRe belongs to the same universality class as for doped semiconductors. In that case the MIT is disorder driven, and this has been expected also from results for metallic *i*-AlPdRe.[4,5] Within the framework of a common universality class this result emerges naturally.

One would like to compare ξ with corresponding results on the metallic side. ξ has been estimated for metallic *i*-AlPdRe,[5] using a quantum critical scaling formalism at the MIT,[26] with $\xi(R) \approx [D\tau_{ie}(4\text{ K})]^{1/2}\sigma(R_c,4\text{ K})/\sigma(R,4\text{ K})$. D is the diffusion coefficient and τ_{ie} the inelastic scattering time. Unfortunately the square root, corresponding to the inelastic scattering length, was assumed to be constant. This overestimates the increase of ξ towards the MIT since $\tau_{ie}(4\text{ K})$ on the metallic side decreases strongly with increasing R,[4] and D is also expected to decrease with increasing R.[27] However, the approach is interesting and merits further investigation.

In summary, from the analyses of the MR, $\sigma(T)$, and $\sigma(0)$, we have obtained consistent results for ξ and T_o in *i*-AlPdRe for a broad range of insulating samples. $(R-R_c)$ was found to increase into the insulator as the decrease of the volume of hopping electrons. The parameter β, obtained from the T-dependence of the MR in the B^2 region, was found to be almost R independent, in contrast to doped semiconductors where β appears to increase into the insulating side. The present result gives convenient descriptions of parameters at the MIT in the form of simple power laws. When describing the MIT in terms of variations of an effective charge density as estimated from $\sigma(295\text{ K})$, comparison could be made with the conductivity exponent in doped semiconductors. The result suggests that the MIT in *i*-AlPdRe and doped semiconductors belong to the same universality class.

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[1] *Proceedings of the 8th International Conference on Quasicrystals*, edited by J. H. Simmons and K. F. Kelton [J. Non-Cryst. Solids, Vol. 334 (2004)].
[2] V. Srinivas, M. Rodmar, S. J. Poon, and Ö. Rapp, Phys. Rev. B **63**, 172202 (2001).
[3] T. I. Su, C. R. Wang, S. T. Lin, and R. Rosenbaum, Phys. Rev. B **66**, 054438 (2002).
[4] M. Rodmar, M. Ahlgren, D. Oberschmidt, C. Gignoux, J. Delahaye, C. Berger, S. J. Poon, and Ö. Rapp, Phys. Rev. B **61**, 3936 (2000).
[5] J. Delahaye and C. Berger, Phys. Rev. B **64**, 094203 (2001).
[6] Y. Y. Lay, J. C. Jan, J. W. Chiou, H. M. Tsai, W. F. Pong, M. H. Tsai, T. W. Pi, J. F. Lee, C. I. Ma, K. L. Tseng, C. R. Wang, and S. T. Lin, Appl. Phys. Lett. **82**, 2035 (2003).
[7] Q. Guo and S. J. Poon, Phys. Rev. B **54**, 12 793 (1996).
[8] C. R. Wang, H. S. Kuan, S. T. Lin, and Y. Y. Chen, J. Phys. Soc. Jpn. **67**, 2383 (1998).
[9] J. Delahaye, J. P. Brisson, and C. Berger, Phys. Rev. Lett. **81**, 4204 (1998).
[10] M. Rodmar, F. Zavaliche, S. J. Poon, and Ö. Rapp, Phys. Rev. B **60**, 10 807 (1999).
[11] C. R. Wang and S. T. Lin, J. Phys. Soc. Jpn. **68**, 3988 (1999).
[12] R. Rosenbaum, R. Haberkern, P. Häussler, E. Palm, T. Murphy, S. Hannah, and B. Brandt, J. Phys.: Condens. Matter **12**, 9735 (2000).
[13] R. Rosenbaum, S. T. Lin, and T. I. Su, J. Phys.: Condens. Matter **15**, 4169 (2003).
[14] V. Srinivas, M. Rodmar, R. König, S. J. Poon, and Ö. Rapp, Phys. Rev. B **65**, 094206 (2002).
[15] Ö. Rapp, V. Srinivas, P. Nordblad, and S. J. Poon, J. Non-Cryst. Solids **334**, 356 (2004).
[16] B. I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors* (Springer Verlag, Berlin, 1984).
[17] T. G. Castner, in *Hopping Transport in Solids*, edited by M. Pollak and B. Shklovskii (North-Holland, Amsterdam, 1991), Chap. 1.
[18] A. N. Ionov, I. S. Shlimak, and M. N. Matveev, Solid State Commun. **47**, 763 (1983).
[19] M. Iqbal, J. Galibert, J. Leotin, S. Waffenschmidt, and H. v. Löhneysen, Philos. Mag. B **79**, 1591 (1999).
[20] H. Stupp, M. Hornung, M. Lakner, O. Madel, and H. v. Löhneysen, Phys. Rev. Lett. **71**, 2634 (1993).
[21] S. Bogdanovich, P. Dai, M. P. Sarachik, and V. Dobrosavljevic, Phys. Rev. Lett. **74**, 2543 (1995).
[22] M. Ahlgren, C. Gignoux, M. Rodmar, C. Berger, and Ö. Rapp, Phys. Rev. B **55**, R11 915 (1997).
[23] F. J. Wegner, Z. Phys. B **25**, 327 (1976).
[24] P. Dai, Y. Zhang, S. Bogdanovich, and M. P. Sarachik, Phys. Rev. B **48**, 4941 (1993).
[25] J. C. Philips, Phys. Rev. B **45**, 5863 (1992).
[26] H. L. Lee, J. P. Carini, D. V. Baxter, W. Henderson, and G. Grüner, Science **287**, 633 (2000).
[27] Ö. Rapp, in *Physical Properties of Quasicrystals*, edited by Z. Stadnik (Springer-Verlag, Berlin, 1999), Chap. 5.