

## Quantum Hall fractions in ultracold fermionic vapors

N. Regnault and Th. Jolicœur

*Département de Physique, Laboratoire Pierre Aigrain, 24, rue Lhomond, 75005 Paris, France*

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We study the quantum Hall states that appear in the dilute limit of rotating ultracold fermionic gases when a single hyperfine species is present. We show that the  $p$ -wave scattering translates into a pure hard-core interaction in the lowest Landau level. The Laughlin wave function is then the exact ground state at filling fraction  $\nu=1/3$ . We give estimates of some of the gaps of the incompressible liquids for  $\nu=p/(2p\pm 1)$ . We estimate the mass of the composite fermions at  $\nu=1/2$ . The width of the quantum Hall plateaus is discussed by considering the equation of state of the system.

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The preparation and manipulation of ultracold atomic gases has led to many interesting developments in the study of quantum fluids undergoing fast rotation.<sup>1,2</sup> The Bose-Einstein condensates can be set in various rotation regimes with the characteristic response of a superfluid. The condensate does not acquire angular momentum below some velocity threshold. Then there is nucleation of one vortex and with increasing velocity more and more vortices are created. They have been observed forming the Abrikosov triangular lattice.<sup>3,4</sup> When the rotation frequency reaches the trapping frequency in the radial plane, it has been predicted that quantum Hall fractional states should become ground states of the system<sup>5-8</sup> if the gas enters a two-dimensional regime. Trapped Fermi gases may also exhibit superfluidity if they undergo BCS pairing condensation. If the pairing strength is varied, it may be possible to observe the crossover from molecular condensation at strong coupling to BCS phase transition at weak coupling. A possible signature of the superfluid-paired phase of fermions is the peculiar response to stirring, leading again to vortex formation. In the fast rotation limit, it is thus natural to ask if there is formation of fractional quantum Hall states as in the Bose case and what are their properties.

In this Rapid Communication we investigate the fractional quantum Hall effect (FQHE) appearing in atomic vapor made of a single hyperfine species of fermions. We show that the  $p$ -wave scattering between fermions can lead to the formation of the Jain principal sequence of FQHE fractions  $\nu=p/(2p\pm 1)$ , in addition to the celebrated Laughlin wave function at  $\nu=1/3$ , as well as a Fermi sea of composite fermions for half filling of the lowest Landau level (LLL). We give estimates of the gaps for the incompressible fluids governed by the  $p$ -wave scattering length and of the mass of the composite fermions. The equation of state of the system seen as the angular momentum of the ground state as a function of the rotation frequency displays plateaus corresponding to the FQHE fluids. Their widths can be estimated by taking into account the nucleation of quasiparticles.

We consider a gas of fermionic atoms and suppose that they are set in rotation, for example, by a stirring external potential<sup>9</sup> that can be applied for some time to transfer angular momentum to the gas and then is removed. We are then left with a rotating cloud and we assume that it attains ther-

mal equilibrium in the rotating frame. If  $\mathcal{H}$  stands for the Hamiltonian in the laboratory frame then it becomes  $\mathcal{H}_R = \mathcal{H} - \omega L_z$  in the rotating frame where  $\omega$  is the rotation frequency and  $L_z$  the angular momentum along the rotation axis. The Hamiltonian describing  $N$  particles of mass  $m$  in this frame can be written as

$$\mathcal{H}_R = \sum_{i=1}^N \frac{1}{2m} (\mathbf{p}_i - m\omega\hat{\mathbf{z}} \times \mathbf{r}_i)^2 + \frac{1}{2} m\omega_z^2 z_i^2 + \frac{1}{2} m(\omega_0^2 - \omega^2) \times (x_i^2 + y_i^2) + \sum_{i<j}^N V(\mathbf{r}_i - \mathbf{r}_j), \quad (1)$$

where the  $xy$  trap frequency is  $\omega_0$ , the axial frequency is  $\omega_z$  and the angular velocity vector is  $\omega\hat{\mathbf{z}}$ . For  $\omega$  close to  $\omega_0$ , the physics is that of charge- $e$  particles in a magnetic field  $\mathbf{B} = (2m\omega/e)\hat{\mathbf{z}}$ , corresponding to a magnetic length  $\ell = \sqrt{\hbar/(2m\omega)}$ . We assume the existence of a two-dimensional (2D) regime in which the wave function along the  $z$  axis is the ground state of the  $z$ -axis harmonic potential.

If we consider a single hyperfine species of fermions, then the  $s$ -wave scattering is forbidden by the Pauli principle. The next allowed partial wave, the  $p$  wave, leads to much weaker interactions<sup>10</sup> and this leads to difficulties when cooling fermionic vapors. They can be evaded, for example, by sympathetic cooling<sup>11</sup> with a different atom. However it is also feasible to use a scattering resonance, such as a Feshbach resonance, to dramatically enhance  $p$ -wave scattering. This has been demonstrated with <sup>40</sup>K atoms.<sup>12</sup> The scattering even reaches values comparable to  $s$ -wave scattering. We will see that this means that FQHE gapped states will have characteristic energies in the same range as for similar bosonic states. At small wave vector, i.e., in the low-energy limit, the  $p$ -wave phase shift of the two-body scattering problem behaves as

$$\delta_1(k) \sim \frac{1}{3} k^3 a_1^3, \quad (2)$$

where  $a_1$  defines the  $p$ -wave scattering length. As a consequence, the scattering amplitude is no longer isotropic,

$$f_1(\theta) \sim a_1^3 k^2 \cos \theta, \quad (3)$$

where  $\theta$  is the angle between ingoing and outgoing wave vectors. We now use an effective potential which mimics the behavior in Eq. (3) when treated in the Born approximation. It is given by

$$\hat{U}_p = \frac{12\pi a_1^3}{m} \hat{\mathbf{p}} \delta^{(3)}(\mathbf{r}) \hat{\mathbf{p}}, \quad (4)$$

where the quantities  $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$  and  $\mathbf{p} = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2)$  pertain to the relative particle. The use of such a potential is enough for our purpose since we will study the dynamics of the LLL only; the massive degeneracy is lifted at first order in the potential (higher orders involve Landau level mixing).

We now turn to the quantum Hall regime for fermions when  $\omega = \omega_0$  in Eq. (1). Assuming a 2D regime with  $\ell_z = \sqrt{\hbar/m\omega_z}$  the confinement length along  $z$ , the interaction potential can be written as  $g_f \ell^4 \hat{\mathbf{p}} \delta^{(2)}(\mathbf{r}) \hat{\mathbf{p}}$  where the vectors  $\mathbf{r}$  and  $\mathbf{p}$  are now 2D with

$$g_f = \sqrt{\frac{2\hbar^2}{\pi m}} \frac{a_1^3}{\ell_z \ell^4}. \quad (5)$$

The coupling constant  $g_f$  sets the scale of the FQHE phenomenon.

The interaction Hamiltonian in the LLL can be written as

$$\mathcal{H}_{LLL} = \sum_m \sum_{i,j} V_m \hat{P}_m(i,j), \quad (6)$$

where  $m$  is the relative angular momentum (RAM), hence the sum runs over all positive odd integers for fermions, and  $\hat{P}_m(i,j)$  is the projection operator for particles  $i$  and  $j$  onto RAM  $m$ . The coefficients  $V_m$  fully characterize the interaction problem in the LLL. They are called pseudopotentials after Haldane.<sup>13</sup> With the pure  $p$ -wave interaction Eq. (4), only  $m=1$  scattering is allowed and the interaction Hamiltonian reduces immediately to the pure hard-core model for which  $V_1 \neq 0$  and all the other pseudopotentials are zero. The hard-core model<sup>13-15</sup> is known to constitute an excellent blueprint of the FQHE. Notably the celebrated Laughlin wave function,<sup>16</sup>

$$\Psi = \prod_{i<j} (z_i - z_j)^3 e^{-\sum_i |z_i|^2/4\ell^2}, \quad (7)$$

is an exact zero-energy ground state of the hard-core model. In fact this is the most spatially compact zero-energy state for the hard-core model. It has a well-defined angular momentum  $L_z = 3N(N-1)/2$ . It corresponds to filling  $1/3$  of the LLL and thus is the exact ground state for ultracold fermions. This is analogous to the Bose problem<sup>7</sup> where  $s$ -wave scattering leads to the Laughlin ground state (with exponent 2) at filling  $1/2$ . It is also known that the hard-core model exhibits the prominent Jain sequence<sup>17</sup> of incompressible fluids for fillings of the form  $\nu = p/(2p \pm 1)$ . We have estimated the gaps at some of these fractions by performing exact diagonalizations in the spherical geometry.<sup>18,19</sup> A sphere of radius  $R$  is threaded by a flux  $4\pi R^2 B$ , which is an integral multiple ( $2S$ ) of the flux quantum by Dirac quantization condition.

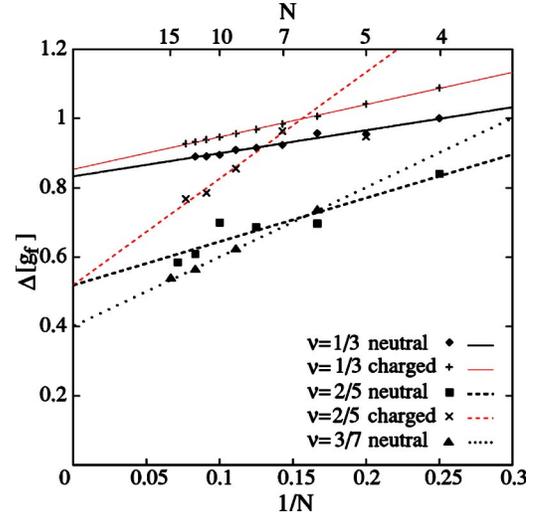


FIG. 1. Energy gaps for neutral and charged excitations at  $\nu = 1/3$ ,  $\nu = 2/5$ , and neutral gaps for  $\nu = 3/7$ . The lines are our best fits and energies are in units of  $g_f$ .

Incompressible fluids appear for special matching of the number of particle vs  $2S$ . We obtain the low-lying energy levels for a small number of particles and then perform finite-size scaling to obtain estimations of the thermodynamic limit.

For the most stable fluid at  $\nu = 1/3$  the low-lying neutral excited states are dominated by a well-defined collective density mode and we have obtained the corresponding gap by studying up to  $N=13$  fermions. The gap can also be evaluated from the charged excitations, i.e., the Laughlin quasiparticles. For the  $p$ -wave problem, there is the peculiarity that the quasiholes are gapless at  $\nu = 1/3$ . It is thus enough to study the quasielectrons that can be nucleated by removing one flux quantum from the reference  $\nu = 1/3$  situation. We find that both estimates scale nicely to a common value  $\approx 0.8g_f$  [see Fig. 1]. For the fraction  $\nu = 2/5$  the gap is  $\approx 0.5g_f$  obtained by the study of systems for  $N=4 \dots 12$  fermions. In this case there are the two types of charged excitations, quasiholes, and quasielectrons, each having a non-zero gap and we find that the sum of these gaps converges to a value compatible with the neutral gap. There are fewer available values of the number of particles as we go down the hierarchy, and for the next fraction  $\nu = 3/7$ , we estimate the gap to be  $\approx 0.4g_f$ , and for  $4/9$  it is more difficult to give a reliable estimate; the gap is smaller and of the order of  $\approx 0.3g_f$ . For values of the parameters  $\ell_z$ ,  $\ell$ , and  $a_1$  typical of present experiments<sup>3,12</sup> the gaps may be of the order of the nanoKelvin.

For fillings less than  $1/3$ , the fermion system no longer displays incompressibility because there are proliferating zero-energy states when we increase the angular momentum. Some of these are edge excitations of the droplet and for larger angular momentum they are quasiholes. Due to particle-hole symmetry, these very same modes fill any gap in the region  $1 \geq \nu \geq 2/3$ . The Jain sequence has an appealing interpretation in terms of composite fermions (CF). These entities are naively fermions bound to an even number of flux quanta of a fictitious field. The total field acting upon

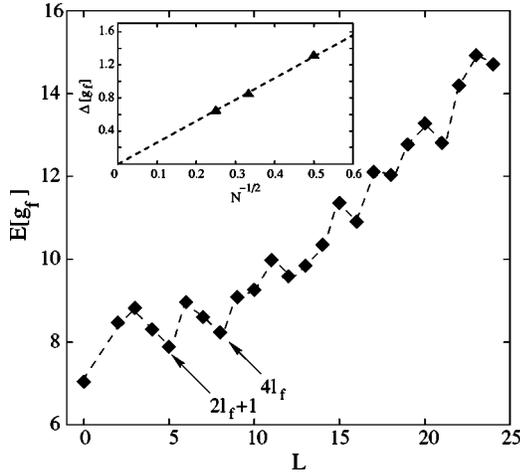


FIG. 2. Energy spectrum for  $N=9$  fermions in the spherical geometry at filling  $\nu=1/2$ . The CFs feel zero flux and can be interpreted as forming a closed shell. The inset shows the scaling of the gap of the closed shell states  $N=4, 9, 16$ . Energies are in units of  $g$  and the horizontal axis is total angular momentum.

the CFs is then the sum of the external field and the fictitious field. When treated in a mean-field manner this explains the FQHE of electrons as the integer quantum Hall effect of CFs. The Jain sequence has an accumulation point at  $\nu=1/2$ ; at this filling the CF experience zero net flux and form a Fermi-liquid-like ground state.<sup>20,21</sup> These CFs have remaining interactions and also an effective mass  $m^*$  which is entirely due to interactions. To estimate this mass, we use a special matching of the flux  $2S=2(N-1)$  giving zero net field on the sphere<sup>22,23</sup> for the CFs. In the Coulomb case, it has been shown that many features of the spectrum can be successfully interpreted by reasoning with free CFs, eventually supplemented by second Hund's rule. There are closed-shell configurations when the number of fermions is a square;  $N=(\ell+1)^2$  and  $\ell$  is thus the total angular momentum of the highest occupied orbital. It is the equivalent of the Fermi momentum on the sphere and thus we call it  $\ell_F$ . We expect that the closed shell sequence  $N=(\ell_F+1)^2=4, 9, 16, \dots$  display ground states with zero total angular momentum and should have good scaling properties towards the thermodynamic limit, as is the case for Coulomb interactions.<sup>23,24</sup> From a closed-shell configuration one can form particle-hole (ph) excitations; the lowest-lying excitations are obtained by promoting a fermion from the shell with momentum  $\ell_F$  to the empty shell at momentum  $\ell_F+1$ , leading to a branch extending from  $L=1$  up to  $2\ell_F+1$ . Above this branch we should find two-particle-two-hole states extending up to  $4\ell_F$  and so on. This is exactly what we find for the hard-core model. The low-lying levels of  $N=9$  fermions are displayed in Fig. 2. Above the singlet ground state we clearly identify the two branches predicted by the free CF model.

We can obtain an estimate of the CF mass at  $\nu=1/2$  by using the free CF model.<sup>23,24</sup> A free CF on the sphere has an energy given by  $E=l(l+1)/(2m^*R^2)$  where  $l$  is the angular momentum. As a consequence, the gap of the one ph branch is  $\Delta=(\ell_F+1)/(m^*R^2)$ . If we fix the density  $\rho$ , the scaling law of the gap becomes  $\Delta_N=4\pi\rho\sqrt{N}/[m^*(N-1)]$ . In the

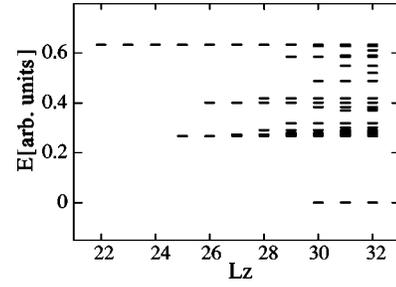


FIG. 3. Energy spectrum for  $N=5$  fermions in the disk geometry as a function of the angular momentum. The Laughlin state is the unique zero-energy state at  $L_z=30$ . The lowest-energy state at  $L_z=25$  is the quasielectron with a finite gap.

spherical geometry we have to take into account the fact that there is a nontrivial shift between the flux  $2S$  and the thermodynamic limit value  $N/\nu$ . For finite numbers of particles the density is not exactly equal to the thermodynamic limit value. Better scaling properties<sup>19</sup> are obtained by rescaling the magnetic length by a factor  $\sqrt{N/(2S\nu)}$  (going to unity for  $N\rightarrow\infty$ ). We thus find  $\Delta_N=4\pi\rho/(m^*\sqrt{N})$ . This scaling is obeyed for the sizes  $N=4, 9, 16$  [see inset of Fig. 2] and this leads to an estimate of the effective mass

$$m^* \approx 0.5m \frac{\ell_z^2}{a_1^3}. \quad (8)$$

When the number of fermions lie between closed shell values we have checked that the ground state angular momentum is given by second Hund's rule (maximum  $L$ ), as is the case for Coulomb interactions.<sup>22</sup>

Finally we discuss the width of the Hall plateaus in the case of trapped atomic vapors. Contrary to the 2D electron systems in semiconductor devices, there is no source of disorder to pin the quasiparticles that are nucleated when we deviate from the finetuning of a quantum Hall fraction. The role of disorder is thus played by the finite number of particles of the system and the quantum Hall plateaus are expected to be of vanishingly small width in the thermodynamic limit. We can give precise estimates by considering the equation of state of the rotating system, i.e., the value of the ground-state angular momentum as a function of the rotation frequency  $\langle L_z \rangle(\omega)$ . At the critical frequency, the Hamiltonian is rewritten as a magnetic field problem [see Eq. (1)]. If the frequency is slightly less,  $\omega_0 - \delta\omega$ , then we have the small field  $-\delta\omega L_z$  acting upon the purely magnetic problem. It leads to a trivial shift of the energies of the FQHE problem that will change the ground state when increased. This is seen from the typical spectrum displayed in Fig. 3 in the condition of the critical rotation. The Laughlin state at  $L_z^{\text{Laughlin}}=3N(N-1)/2$  is the ground state and the first excited state is the quasielectron at  $L_z^{\text{Laughlin}}-N$  with a nonzero gap  $\Delta_{qe}$ . When adding a  $-\delta\omega L_z$  shift the Laughlin will remain the ground state till the quasielectron energy becomes lower for a critical value equal to

$$\delta\omega_c = \frac{\Delta_{qe}}{N\hbar} \approx \frac{10^2 \text{Hz}}{N} \left( \frac{\Delta_{qe}}{1 \text{mK}} \right). \quad (9)$$

If we increase  $\delta\omega$  beyond this value, quasielectrons are nucleated forming a fluid that will condense into a new

FQHE fluid. We expect the result above in Eq. (9) to be generic. This picture is essentially dual to the nucleation of vortices at small rotation frequency.<sup>3</sup>

To experimentally characterize FQHE states for Fermi gases, one possibility is to generate quasiholes by laser manipulation and detect the effect of fractional statistics by Ramsey interferometry.<sup>25</sup> Another signature is the peculiar surface modes of the cloud, the so-called edge modes that rule that transport properties of electron gases. Their detection would also give evidence for FQHE formation.<sup>26</sup> The collective magnetoroton mode, which is a gapped density excitation, is also a possible signature of incompressibility.

We have shown the appearance of the Jain principal sequence of quantum Hall fractions in ultracold rotating fermionic vapors. The composite fermion picture gives a successful account of the observed fractions as well as their collective mode excitations. The gaps we estimate from exact diagonalizations are of the order of  $\hbar^2 a_1^3 / m \ell_z \ell^4$ . At half filling of the lowest-Landau level, there is a Fermi-liquid-like state of composite fermions and their effective mass  $m^*/m$  is controlled by  $\ell^2 \ell_z / (a_1^3)$ .

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- <sup>1</sup>M. R. Matthews, B. P. Anderson, P. C. Haljan, D. S. Hall, C. E. Wieman, and E. A. Cornell, *Phys. Rev. Lett.* **83**, 2498 (1999).  
<sup>2</sup>P. C. Haljan, I. Coddington, P. Engels, and E. A. Cornell, *Phys. Rev. Lett.* **87**, 210403 (2001).  
<sup>3</sup>K. W. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, *Phys. Rev. Lett.* **84**, 806 (2000); F. Chevy, K. Madison, and J. Dalibard, *Phys. Rev. Lett.* **85**, 2223 (2000).  
<sup>4</sup>J. R. Abo-Shaeer, C. Raman, J. M. Vogels, and W. Ketterle, *Science* **292**, 476 (2001).  
<sup>5</sup>N. K. Wilkin, J. M. F. Gunn, and R. A. Smith, *Phys. Rev. Lett.* **80**, 2265 (1998).  
<sup>6</sup>N. R. Cooper and N. K. Wilkin, *Phys. Rev. B* **60**, R16 279 (1999).  
<sup>7</sup>N. K. Wilkin and J. M. F. Gunn, *Phys. Rev. Lett.* **84**, 6 (2000).  
<sup>8</sup>N. R. Cooper, N. K. Wilkin, and J. M. F. Gunn, *Phys. Rev. Lett.* **87**, 120405 (2001).  
<sup>9</sup>P. Rosenbusch, D. S. Petrov, S. Sinha, F. Chevy, V. Bretin, Y. Castin, G. Shlyapnikov, and J. Dalibard, *Phys. Rev. Lett.* **88**, 250403 (2002).  
<sup>10</sup>B. DeMarco, J. L. Bohn, J. P. Burke Jr., M. Holland, and D. S. Jin, *Phys. Rev. Lett.* **82**, 4208 (1999).  
<sup>11</sup>C. J. Myatt, E. A. Burt, R. W. Ghrist, E. A. Cornell, and C. E. Wieman, *Phys. Rev. Lett.* **78**, 586 (1997).  
<sup>12</sup>C. A. Regal, C. Ticknor, J. L. Bohn, and D. S. Jin, *Phys. Rev. Lett.* **90**, 053201 (2003).  
<sup>13</sup>F. D. M. Haldane, *Phys. Rev. Lett.* **51**, 605 (1983).  
<sup>14</sup>S. A. Trugman and S. Kivelson, *Phys. Rev. B* **31**, 5280 (1985).  
<sup>15</sup>A. H. MacDonald, in *Mesoscopic Quantum Physics, Les Houches Proceedings*, edited by E. Akkermans, G. Montambaux, and J. L. Pichard (Elsevier Science, Amsterdam 1994).  
<sup>16</sup>R. B. Laughlin, *Phys. Rev. Lett.* **50**, 1395 (1983).  
<sup>17</sup>J. K. Jain, *Phys. Rev. Lett.* **63**, 199 (1989).  
<sup>18</sup>F. D. M. Haldane and E. H. Rezayi, *Phys. Rev. Lett.* **54**, 237 (1985).  
<sup>19</sup>R. H. Morf, N. d'Ambrumenil, and S. Das Sarma, *Phys. Rev. B* **66**, 075408 (2002); N. d'Ambrumenil and R. Morf, *ibid.* **40**, 6108 (1989).  
<sup>20</sup>A. Lopez and E. Fradkin, *Phys. Rev. B* **44**, 5246 (1991).  
<sup>21</sup>B. I. Halperin, P. A. Lee, and N. Read, *Phys. Rev. B* **47**, 7312 (1993).  
<sup>22</sup>E. Rezayi and N. Read, *Phys. Rev. Lett.* **72**, 900 (1994).  
<sup>23</sup>R. Morf and N. d'Ambrumenil, *Phys. Rev. Lett.* **74**, 5116 (1995).  
<sup>24</sup>M. Onoda, T. Mizusaki, T. Otsuka, and H. Aoki, *Phys. Rev. Lett.* **84**, 3942 (2000).  
<sup>25</sup>B. Paredes, P. Fedichev, J. I. Cirac, and P. Zoller, *Phys. Rev. Lett.* **87**, 010402 (2001).  
<sup>26</sup>M. A. Cazalilla, *Phys. Rev. A* **67**, 063613 (2003).