PHYSICAL REVIEW B **70**, 241301(R) (2004)

Manifestation of the spin Hall effect through charge-transport in the mesoscopic regime

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(Received 13 September 2004; published 1 December 2004)

We study theoretically the manifestation of the spin Hall effect in a two-dimensional electronic system with Rashba spin-orbit coupling via dc-transport measurements in realistic mesoscopic H-shape structures. The Landauer-Buttiker formalism is used to model samples with mobilities and Rashba coupling strengths of current experiments and to demonstrate the appearance of a measurable Rashba-coupling dependent voltage. This type of measurement requires only metal contacts, i.e., no magnetic elements are present. We also confirm the robustness of the intrinsic spin Hall effect against disorder in the mesoscopic metallic regime in agreement with results of exact diagonalization studies in the bulk.

DOI: 10.1103/PhysRevB.70.241301 PACS number(s): 73.43.−f, 72.25.Dc, 72.25.Hg, 85.75.−d

INTRODUCTION

The ability to manipulate electronically spins and to generate spin currents in semiconductors is the *sine qua non* for the full development of semiconductor based spintronics.[1] The control of spin and spin-currents without applying external magnetic fields can be achieved through the spin-orbit (SO) coupling, which acts as an effective momentum-dependent Zeeman field. Within this context, the recently proposed intrinsic spin Hall effect (SHE) in *p*-doped semiconductors by Murakami *et al.*[2] and in a two-dimensional electron system (2DES) by Sinova *et al.*[3] offers possibilities for spin current manipulation and generation in high mobility paramagnetic semiconductor systems. The intrinsic spin Hall effect represents a spin-current response generated perpendicular to the driving electric field. The spins are tilted out of the plane due to the torque imparted by the SO coupling-induced effective Zeeman field. In the Rashba SO-coupled 2DESs the bulk intrinsic spin Hall conductivity was found to have a value of $e/8\pi$ in the clean limit for the case of both spin-split subbands being occupied and decreases linearly with the electron density for single spin-split subband occupation.[3]

The SHE has generated a tremendous interest in the research community.[4–15] Similarly to the long-standing debate on the origin of the anomalous Hall effect (AHE),[16] the robustness of the bulk intrinsic SHE against disorder and how it is related to the scattering mediated extrinsic spin Hall effect,[17–19] has been the focus of an intense theoretical debate.[4,8–13] While it was understood originally[3] that, unlike the quantum Hall effect, the universal value of the intrinsic SHE in the Rashba SO-coupled 2DESs will be reduced whenever the disorder broadening is larger or similar to the SO-coupling splitting, as was verified within a standard Born-approximation treatment,[4] taking into account the ladder vertex corrections through various methods suggests that the bulk spin Hall conductivity vanishes in the weak disorder dc limit.[9,11] However, these results have been challenged by other analytical calculations which also consider ladder vertex corrections.[12] Other recent studies,[13] using arguments that echo the long-standing debate between skew and side-jump scattering in the AHE, have argued that the intrinsic SHE vanishes in all regimes.

Given the ferraginous collection of analytical results, unbiased numerical calculations are needed to shed light on this controversy. An exact diagonalization treatment of disorder[10] has shown that the bulk intrinsic SHE is robust against weak disorder. In addition, several numerical studies utilizing the Landauer-Buttiker (LB) formalism in a tight-binding model representation of the Rashba SO-coupling Hamiltonian in the presence of disorder show similar conclusions in the limit that corresponds to the continuum effective mass model (see below).[14,15,20]

In this paper we address a key question that has yet to be addressed directly: How to measure the intrinsic spin Hall effect through transport measurements? All previous numerical studies have focused on the controversy regarding the robustness of the effect against disorder and how disorder, Rashba coupling strength, etc., change the continuum effective mass model value of $e/8\pi$. Recently, a new theoretical question has arisen—whether the dissipationless currents or spin background currents can lead to spin accumulation or to a steady signal which can manifest such an effect.[6,7] This question can be addressed unambiguously in the mesoscopic regime where the effect of the leads and disorder can be taken into account through the explicit treatment of voltage and current probes within the LB formalism.[14,15,20] Here we consider an H-shape structure shown in Fig. 1 to demonstrate the appearance of the spin Hall effect through dc-transport measurements without any magnetic elements. A current is driven from lead 2 to 1 in the lower leg shown in Fig. 1(a) and the rest of the contacts are voltage probes, i.e., leads with total zero current. Then, for typical current utilized in experiments and typical parameters we find a voltage difference $\Delta V_{34} \equiv V_3 - V_4$ dependent on the Rashba coupling and coupled to the spin Hall conductance defined at, e.g., probe 6. The variation of this voltage, relative to the residual voltage obtained at zero Rashba coupling, is of the order μV for the maximum system sizes that we can model ($\sim 0.1\ \mu$m). In the mesoscopic regime, several of the controversies that have

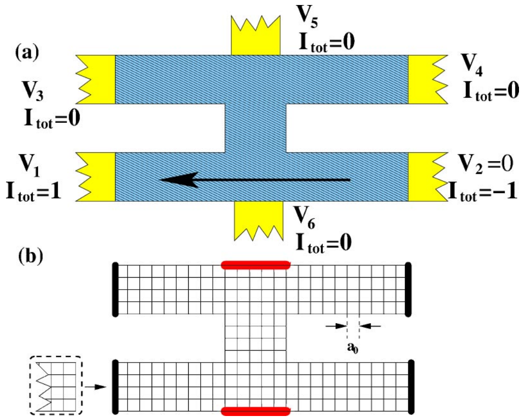

FIG. 1. (Color online) (a) Mesoscopic H-bar probe with metallic leads. (b) The continuum model is converted into a tight-binding model and the effects of the leads are treated through a self-energy.

arisen from the study of the bulk spin-transport coefficients can be addressed. Within this regime the only assumption made in describing the transport through the sample via the current and voltage probes, other than the applicability of the tight-binding approximation for the SO-coupled electronic structure, is that such contacts are perfectly metallic, i.e., an exact analytical expression is known for their Green's functions.

Several numerical studies have addressed the robustness of the intrinsic spin Hall effect within the mesoscopic regime utilizing the LB formalism.[14,15,20] Perhaps the most compendious of these tight-binding model numerical studies are Refs. 14 and 15, where the expected symmetry of the Hall conductance with respect to E_F and the metal-insulator transition as a function of SO coupling and disorder strength[21] were shown.

MODEL HAMILTONIAN AND LB TREATMENT OF THE SPIN HALL EFFECT

The experimental detection of the SHE through electrical means is conceptually challenging. Given the controversy surrounding the nature of the spin currents generated by electric fields, a measurement of the voltage between two metallic contacts[7] appears to be the most promising dc-transport approach to unambiguously determine the presence of the SHE signal. We focus our attention on the proposed H-shape device shown in Fig. 1 and demonstrate that within the mesoscopic metallic regime the intrinsic SHE is exhibited through the change in a voltage difference between two contacts as the Rashba interaction is varied.

The continuum effective mass model described by the 2DES Hamiltonian with the Rashba SO interactions is given by $\hat{H}=(\hat{p}^2/2m^*)+\lambda(\hat{\sigma}_x p_y-\hat{\sigma}_y p_x)+H_{dis}$, where the second term is the Rashba SO coupling[22,23] due to the asymmetry of the confining potential and H_{dis} is the disorder potential. To model the complex geometry of our disordered conductor within the LB formalism we use the tight-binding (or finite differences) approximation.[20] Within this approximation the continuum effective mass envelope function Hamiltonian becomes

$$H=\sum_{j,\sigma}\epsilon_j c^\dagger_{j,\sigma}c_{j,\sigma} - t\sum_{j,\vec{\delta},\sigma} c^\dagger_{j+\vec{\delta},\sigma}c_{j,\sigma} + t_{SO}\Big[\sum_j -i(c^\dagger_{j,\uparrow}c_{j+a_y,\downarrow} + c^\dagger_{j,\downarrow}c_{j+a_y,\uparrow}) + \sum_j (c^\dagger_{j,\uparrow}c_{j+a_x,\downarrow} - c^\dagger_{j,\downarrow}c_{j+a_x,\uparrow}) + \text{H.c.}\Big], \quad (1)$$

where $t=\hbar^2/2m^*a_0^2$ and $t_{SO}=\lambda/2a_0$, a_0 is the mesh lattice spacing, and $\vec{\delta}=\pm a_0\hat{x},\pm a_0\hat{y}$. The first term represents a quenched disorder potential and disorder is introduced by randomly selecting the on-site energy ϵ_j in the range $[-W/2,W/2]$. Within the leads the SO coupling is zero and therefore each lead should be considered as having two independent spin channels. These leads constitute reservoirs of electrons at chemical potential $\mu_1,\ldots,\mu_N$ where N is the number of leads which we consider to be either four (leads 1–4 in Fig. 1) or six (leads 1–6 in Fig. 1).

In the low temperature limit $k_BT \ll E_F$ and for low bias voltage, the particle current going through a particular channel is given within the LB formalism by[20] $I_{p,\sigma}=(e/h)\Sigma_{q\sigma'}T_{p,\sigma;q,\sigma'}[V_p-V_q]$, where p labels the lead and $T_{p,\sigma;q,\sigma'}$ is the transmission coefficient at the Fermi energy E_F between the (p,σ) channel and the (q,σ') channel. This transmission coefficient is obtained by $T_{p,\sigma;q,\sigma'}=\mathrm{Tr}[\Gamma_{p,\sigma}G^R\Gamma_{q,\sigma'}G^A]$ where $\Gamma_{p,\sigma}$ is given by $\Gamma_{p,\sigma}(i,j)=i[\Sigma^R_{p,\sigma}(i,j)-\Sigma^A_{p,\sigma}(i,j)]$. The retarded and advance Green's function of the sample $G^{R/A}$ with the leads taken into account through the self energy $\Sigma^{R/A}_{p,\sigma}(i,j)$ has a form $G^{R/A}(i,j)=[E\delta_{i,j}-H_{i,j}-\Sigma_{p,\sigma}\Sigma^{R/A}_{p,\sigma}(i,j)]^{-1}$. Here the position representation of the matrices $\Gamma_{p,\sigma}$, G^R, $H_{i,j}$, and Σ^R are in the subspace of the sample. Since the SO-coupled Hamiltonian preserves time reversal symmetry, the transmission coefficients obey the relation $T_{p,\sigma;q,\sigma'}=T_{q,-\sigma';p,-\sigma}$.[24] Within the above formalism the spin current through each channel is given by $I^s_{p,\sigma}=(e/4\pi)\Sigma_{q\sigma'}T_{p,\sigma;q,\sigma'}[V_p-V_q]$ and the spin Hall conductance is defined as

$$G_{SH}=\frac{(I^s_{6\uparrow}-I^s_{6\downarrow})}{V_1-V_2}, \quad (2)$$

as indicated in Fig. 1. All the voltages are obtained by imposing the boundary conditions $I_{i,\uparrow}+I_{i\downarrow}=0$ for $i=3-6$, $I_{1,\uparrow}+I_{1\downarrow}=1$, and $I_{2,\uparrow}+I_{2\downarrow}=-1$. The arbitrary zero of voltage is fixed by setting $V_2=0$. These are later translated to a realistic voltages by setting the current to a typical value of 10 nA.

RESULTS AND DISCUSSION

In order to address the key issue of how the spin Hall effect can manifest itself through a dc-transport measurement without ferromagnetic contacts we choose realistic parameters for our calculations that model currently available systems.[25] We consider an effective mass of $m^*=0.05m_e$ and a disorder strength of $W=0.09$ meV corresponding to the mobility of 250 000 $\mathrm{cm^2/V\,s}$, which is typical for a semiconductor such as (In,Ga)As. We take the Rashba parameter λ in the range from 0 to 80 meV nm that are easily obtained in experiments,[26,27] and we choose the electron concentration of approximately $n_{2D}=10^{12}\ \mathrm{cm^{-2}}$. The Fermi energy is

RAPID COMMUNICATIONS

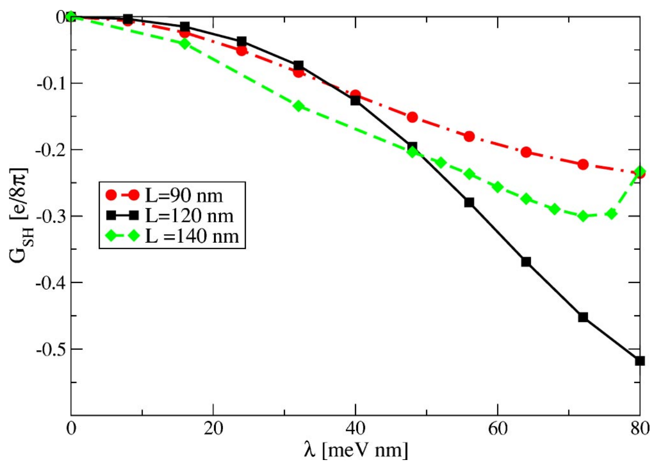

FIG. 2. (Color online) Spin Hall conductance defined at lead 6 (shown in Fig. 1) vs spin-orbit coupling strength for different size systems for $m_0=0.05m_e$, $\mu=250\,000\ \mathrm{cm}^2/\mathrm{V\,s}$, and flowing current of 10 nA in the bottom leg. Here L is divided in 42 points. Only a few disorder realizations are needed for convergence in samples with these mobilities.

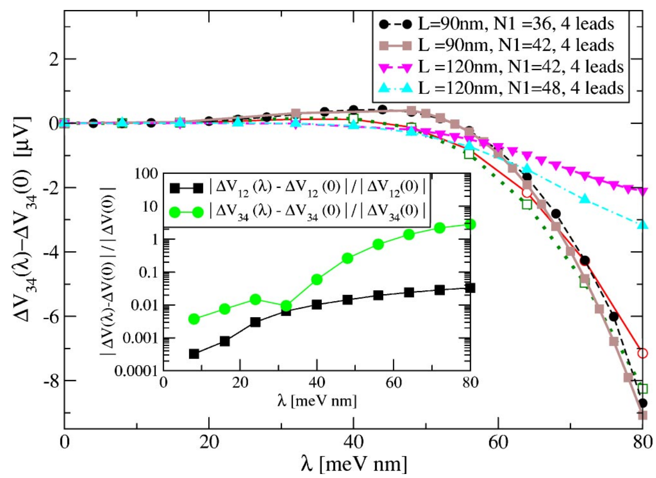

FIG. 3. (Color online) The voltage difference between leads 3 and 4 as a function of Rashba coupling λ for H probe for different size systems and meshes for $m_0=0.05m_e$, $\mu=250\,000\ \mathrm{cm}^2/\mathrm{V\,s}$, and flowing current of 10 nA in the bottom leg. The filled and open symbols correspond to H probe with 4 and 6 leads, respectively. In the inset the magnitude of the relative voltage changes between 1 and 2, and 3 and 4 leads are plotted as a function of λ for $L=120$ nm.

obtained from the chosen electron concentration assuming an infinite 2DES. This gives a small difference of a few percent when considering our finite tight-binding model but the leads themselves are the ones providing the reservoir of electrons and therefore such fluctuations are small as verified by numerical calculations (not shown). Parameters considered here correspond to $t_{so}\approx 0-0.2t$, $E_F/W\sim 500$ in other theoretical studies with small variations due to mesh scaling as physical system size and effective mass is kept constant.[14,15]

In Fig. 2 we show the spin Hall conductance G_{SH} as a function of the Rashba parameter for the H-shape structure when current flows through the bottom leg as shown in Fig. 1. Here we consider the system with six leads and with leg lengths L varying from 90 to 140 nm. The total size of the system is L in the x direction and $L/2$ in the y direction. The horizontal connection bar is $L/6$ by $L/6$. The width of the legs is $L/6$ with the attached leads of the same width. These ratios were chosen for typical fabricated samples (of larger system size) but any shape is feasible to do and a search for an optimal geometry is underway.[28] This particular H-shape structure allows for minimalization of the residual voltage drop due to charge flow. G_{SH} is calculated accordingly to Eq. (2). The calculations are conducted for a few different meshes $N_1=L/a_0$ to check the convergence of results for $a_0\to 0$. The magnitude of G_{SH} is around 0.2–0.6 in $e/8\pi$ units for Rashba coupling 70–80 meV nm and $L=90-140$ nm. We also note that within these parameters we are well within the metallic regime[15,21] and both spin-split subbands are occupied.

The spin Hall conductance cannot be measured by the paramagnetic leads and ferromagnetic leads can introduce spurious effects coming from the impedance mismatch preventing ballistic contacts.[24,29,30] However, we expect that the spin current which flows between leads 5 and 6 can generate a secondary effect, the induction of a voltage difference in the top leg between leads 3 and 4. This is in the same spirit as the initially proposed setup by Hirsch[18] but in a far simpler configuration without the need of considering a bridged conductor nor any unknown scattering mechanisms other than the effects of disorder which is actually small in this case.[14,15] We illustrate this in Fig. 3 where we show the nonzero voltage difference $\Delta V_{34}(\lambda)-\Delta V_{34}(\lambda=0)$ as a function of Rashba coupling for different size systems, different meshes, and with and without the additional leads 5 and 6. We find the increase of ΔV_{34} with the increase of Rashba coupling. The induced voltage variation is of the order of few μV for $\lambda=60-80$ meV nm and can be easily measured. We also note that the inclusion or omission of leads 5 and 6, which in reality cannot measure directly the spin Hall conductance, do not influence the voltage difference. They do, however, influence the total residual voltage background $\Delta V_{34}(\lambda=0)$. The convergence of the results as a function of mesh size is illustrated in Fig. 3. The inset to Fig. 3 shows the magnitude of the relative changes of voltage between leads 1 and 2, as well as 3 and 4 as a function of λ. The changes for leads 3 and 4 are almost two orders of magnitude larger than ones for terminals 1 and 2 for large λ. This excludes the possibility that V_{34} is generated by a change in the longitudinal charge conductivity due to the variation of λ.[31] We also note that for mobilities typical for experimental samples the results do not depend on the disorder strength if we, say, double it. Hence, we emphasize two important facts: (i) the induced voltage is likely originating from an intrinsic spin Hall effect, (ii) it is not related to the constant in-plane polarization induced by a current in a 2DES with Rashba coupling (the Levitov effect[32]), since then V_{34} would be proportional to the mobility.

SUMMARY

We have calculated, as a function of the Rashba SO-coupling strength λ, the voltage drop ΔV_{34} that occurs in an H-shape sample in a response to a driving dc current between leads 1 and 2. The voltage difference follows qualitatively the changes of the calculated SHE conductance, G_{SH}, with λ. Moreover, ΔV_{34} increases with the increase of the Rashba coupling for the constant disorder strength W indicating that the observed voltage signal is connected with the intrinsic SHE. Our work provides another confirmation of the robustness of the intrinsic SHE against weak disorder and shows the feasibility of detecting SHE signals through dc-transport measurements in structures with realistic experimental parameters.

We thank N. A. Sinitsyn, K. Nomura, F. Sols, A. H. MacDonald, Q. Niu, B. K. Nikolić, and E. I. Rashba for stimulating discussions. We also acknowledge financial support from DFG (SFB410), the Czech Republic GACR (202/02/0912), and the DARPA SpinS program.

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