Tuning of a nonlinear metamaterial band gap by an external magnetic field

M. Gorkunov* and M. Lapine
Department of Physics, University of Osnabrück, 49069 Osnabrück, Germany
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We consider tuning of permeability of a nonlinear metamaterial with resonant conductive elements by an external varying magnetic field. We propose two tuning regimes: slow tuning, when the permeability follows adiabatically the field variations, and high-frequency tuning, when changes of the pump microwave amplitude and/or frequency affect the permeability. We demonstrate how the metamaterial band gap can be tuned and describe the resulting metamaterial switching between transmitting, reflecting, and absorbing states. The details appear to depend drastically on the type of nonlinear components inserted into the resonant conductive elements. We perform practical estimates for typical diodes available for the microwave frequency range. We predict that the transmittance of a metamaterial slab can be modulated by several orders of magnitude already using a slab with thickness equal to one microwave wavelength in vacuum.

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I. INTRODUCTION

Metamaterial being a regular three-dimensional array of identical elements represents a direct analog of an optical crystal designed for the radio and microwave frequency range. Choosing appropriate structural elements and their arrangement one can adjust the macroscopic characteristics. Thus, in contrast to crystals, metamaterials can be tailored to provide desired electromagnetic response. Arising macroscopic properties can be even beyond the limits established in optics, as the research on negative refraction has shown.[1,2]

During the recent years, various resonant conductive elements (RCEs) have been widely used as structure units of metamaterials. As long as wavelengths of electromagnetic waves are much larger than RCE size, the elements can be well described by linear resonant circuits. Accordingly, the metamaterial acquires a notable magnetic resonance. In a certain frequency range above the resonance, the permeability of metamaterial made of RCEs is negative. As suggested recently,[3] this allows using the metamaterial as a compact photonic band gap crystal.

Inserting nonlinear components into the structure elements provides the metamaterial with nonlinear properties. A good example of this basic idea is the metamaterial made of RCEs with diode[4,5] or nonlinear dielectric[6] insertions proposed recently. Nonlinearity of the insertions current-voltage and/or capacitance-voltage characteristic leads to the nonlinear dependence of the metamaterial magnetization on magnetic field. In the limit of weak fields, the arising effects are universally described by the optical approach involving nonlinear magnetic susceptibility.[4]

Increasing the magnetic field allows enhancing the nonlinearity and achieving extraordinarily strong microwave coupling. However, this case, when the essentially nonlinear part of the nonlinear insertion characteristic is employed, cannot be treated similarly to optics. It is necessary to consider various nonlinear processes separately.

Here we study tuning of the metamaterial permeability by an external varying magnetic field. This field induces voltages in RCEs, which bias the nonlinear insertions and vary the effective capacitance and/or resistance of RCEs. This results in changes of the RCEs″ resonant frequency and quality factor, which in turn affect the resonant frequency and resonance width of the metamaterial permeability. The latter determines the band gap and controls the propagation of weak signal waves through the medium. Then, the metamaterial can be switched between transmitting, reflecting, and absorbing states.

Particular tuning capabilities depend strongly on the type of the nonlinear insertion. For instance, insertions with variable resistance (e.g., backward diodes[7]) allow affecting the effective RCE resistance and provide a tuning of the material transparency, i.e., switching between transmitting and absorbing states. Varactor diodes[8] allow varying the effective RCE capacitance, which defines the position of the metamaterial resonance. Shifting the resonance one can switch the medium between all the three states with respect to a signal wave at a given frequency. Thin ferroelectric films [9] operate similarly to varactor diodes. Although the ferroelectric devices demand higher biasing, they are more advantageous at higher frequencies, beyond the varactor frequency cutoff.

We propose two tuning regimes: tuning by a strong slow varying field and tuning by a high-frequency field. With the slow tuning, the refractive index of the metamaterial is changed adiabatically following time derivatives of the external field. With the high-frequency tuning, a strong pump microwave propagating through the medium causes a homogeneous variation of the refractive index, which can be controlled by adjusting the wave amplitude and/or frequency.

To estimate arising application possibilities we study microwave transmission through a slab of the nonlinear metamaterial. We show that even a relatively thin slab, having the thickness equal to one microwave wavelength in vacuum, can be used for a modulation of the transmitted signal microwave amplitude by several orders of magnitude.

II. TUNING THE METAMATERIAL

A. RCE with a nonlinear insertion

The electromagnetic response of a single RCE can be described in the approximation of linear resonant contour under the assumption that the wavelengths of electromagnetic waves are much larger than RCE dimensions.[10,11] Ohmic losses in the RCE material determine the contour resistance R_c; the geometric shape controls its self-inductance L_c and capacitance C_c. Note, that one can adjust the resonant frequency of the RCE, $(L_c C_c)^{-1/2}$, by choosing appropriate RCE dimensions.

Inserting in series a component with nonlinear electric properties (voltage dependent resistance and/or capacitance) makes the RCE response nonlinear. We assume the insertion resistance and capacitance to be known functions of the voltage drop on the insertion, $R_{\text{ins}}(U_{\text{ins}})$ and $C_{\text{ins}}(U_{\text{ins}})$, respectively. Then the current $I(t)$ in the RCE, induced by an external electromotive force $\mathcal{E}(t)$, obeys the equation

$$\mathcal{E}(t) = R_c I(t) + \frac{1}{C_c}\int_{-\infty}^{t} I(t')dt' + L_c \dot{I}(t) + U_{\text{ins}}(t), \tag{1}$$

where the voltage drop on the insertion can be generally written as

$$U_{\text{ins}}(t) = I(t)R_{\text{ins}}(U_{\text{ins}}(t)) + \frac{1}{C_{\text{ins}}(U_{\text{ins}}(t))}\int_{-\infty}^{t} I(t')dt'. \tag{2}$$

Obviously, Eq. (2) has no general solution, but for particular nonlinear processes certain approximations are applicable.

While considering the tuning phenomena, we focus on the effect of strong pump field on the propagation of weak harmonic signal wave, which probes the changes of linear metamaterial response at frequency ω. Correspondingly, we separate the current and voltage into pump (index p) and signal (index ω) components:

$$I(t) = I_p(t) + I_\omega e^{-i\omega t} + I_\omega^* e^{i\omega t}, \tag{3}$$

$$U_{\text{ins}}(t) = U_p(t) + U_\omega e^{-i\omega t} + U_\omega^* e^{i\omega t}. \tag{4}$$

For the signal terms much smaller than the pumping ones, it is possible to perform an expansion in (2). In the zero order approximation, with the signal contribution being neglected, Eq. (2) allows to determine the pump voltage for a given pump current. Next, in the linear approximation with respect to the signal, the signal voltage is proportional to the signal current, $U_\omega \propto I_\omega$. It is convenient to separate the real and imaginary parts of the proportionality factor and write:

$$U_\omega = \left(R_{\text{ins}}(t) + \frac{i}{\omega C_{\text{ins}}(t)}\right) I_\omega. \tag{5}$$

Here the effective insertion capacitance and resistance vary in time due to the pumping. The particular form of Eq. (5) is determined by the type of nonlinear insertion and the regime of tuning, as discussed in detail in Secs. III and IV.

Considering the harmonic signal, we obtain from Eq. (1)

$$\mathcal{E}_\omega = Z_c(\omega) I_\omega + U_\omega, \tag{6}$$

where $Z_c(\omega) = R_c - i\omega L_c + i(\omega C_c)^{-1}$ is the linear RCE impedance and $\mathcal{E}_\omega$ is the electromotive force at the signal wave frequency. Substituting the relation (5) and (6) yields that a RCE with nonlinear insertion acquires an effective impedance

$$Z(t) = R(t) - i\omega L_c + i(\omega C(t))^{-1}, \tag{7}$$

which is affected by the pumping through the overall resistance $R(t) = R_c + R_{\text{ins}}(t)$ and capacitance $C(t) = (C_c^{-1} + C_{\text{ins}}^{-1}(t))^{-1}$. Therefore, the main RCE characteristics, its resonant frequency, $\omega_0 = (L_c C)^{-1/2}$, and quality factor, $Q = R^{-1}\sqrt{L_c/C}$, are tuned by pumping.

B. Macroscopic permeability

We consider a planar metamaterial arrangement, when RCEs lie in parallel planes normal to the z-axis and form a regular lattice. The lattice constants are normally of the order of RCE size, i.e., also much smaller than microwave wavelengths. Then, the macroscopic description of the metamaterial electromagnetic response is valid, and corresponding permeability and permittivity tensors can be found using standard averaging procedure established in optics.[13,14]

The metamaterial made of RCEs has rather trivial electric properties: its permittivity is positive and shows no notable frequency dispersion. The detailed consideration[12] of the magnetic properties of such structure shows that it possesses a diagonal tensor of resonant effective permeability with components:

$$\mu_{xx} = 1, \quad \mu_{yy} = 1, \quad \mu_{zz}(\omega) = 1 - \frac{A\omega^2}{\omega^2 - \omega_r^2 + i\Gamma\omega}. \tag{8}$$

Mutual interaction of structure elements shifts the resonant frequency to

$$\omega_r = \omega_0 \left(\frac{L_\Sigma}{L} + \frac{1}{3}\frac{\mu_0 n S^2}{L}\right)^{-1/2}, \tag{9}$$

where n is the volume concentration of RCEs, S is the effective area of RCE contour, L_Σ is the combined mutual and self-inductance. The resonance height and width are $A = \mu_0 S^2 n L_c^{-1} \omega_r^2 \omega_0^{-2}$ and $\Gamma = \omega_r^2 \omega_0^{-1} Q^{-1}$, respectively.

As follows from Eq. (8), propagation conditions for electromagnetic waves with magnetic field directed along z-axis, substantially depend on the wave frequency. Namely, the medium is:

(1) transparent at frequencies lower than ω_r, since $\text{Re}[\mu_{zz}] > 0$ and $\text{Im}[\mu_{zz}] \ll 1$ there;

(2) absorptive at frequencies close to the resonance, $|\omega - \omega_r| \leqslant \Gamma$, where $\text{Re}[\mu_{zz}] \sim \text{Im}[\mu_{zz}]$ and incident waves dissipate in the bulk;

(3) reflecting in a certain band gap above ω_r, where $\text{Re}[\mu_{zz}] < 0$ and $\text{Im}[\mu_{zz}] \ll 1$, and incident waves are reflected from the surface;

(4) transparent above the band gap, where again $\text{Re}[\mu_{zz}] > 0$ and $\text{Im}[\mu_{zz}] \ll 1$.

Consequently, to achieve maximal tuning effect it is desirable to adjust the metamaterial resonance close to the frequency of the incident signal wave. Then by varying the parameters ω_r (by shifting ω_0) and/or Γ (by changing Q) it is possible to affect drastically the propagation of the signal in the metamaterial. Note that large values of the real part of the permeability at such frequencies lead to high refractive index. This reduces wavelengths and electromagnetic penetration depths inside the metamaterial. Therefore, already thin slab can be used for significant modulation of the signal.

III. TUNING BY SLOWLY VARYING EXTERNAL MAGNETIC FIELD

Adiabatic tuning of the metamaterial can be accomplished by applying a slowly varying external magnetic field. All the three tensor components of the permeability (8) are equal or close to unity at small frequencies. Therefore, pump field inside the medium, $H_p(t)$, equals to the externally applied one. We assume the latter to be homogeneous and perpendicular to RCE planes (parallel to z-axis). The electromotive force induced in RCE by the external field,

$$\mathcal{E}_p(t) = -\mu_0 S\dot{H}_p(t), \tag{10}$$

drives pump current, which, in turn, gives rise to pump voltages biasing the nonlinear insertion. The resulting tuning effect depends on the type of the insertion. Below we consider particular cases of variable resistance and variable capacitance insertions.

A. Variable resistance insertions

For variable resistance devices, the capacitive contribution in Eq. (2) is not relevant, and their properties are usually described by current-voltage characteristic. Here we consider the reciprocal, voltage-current, characteristic $\mathcal{U}(I)$. Then, instead of using $R_{\text{ins}}(U_{\text{ins}})$ in Eq. (2), we can immediately write

$$U_{\text{ins}}(t) = \mathcal{U}(I(t)). \tag{11}$$

The pump current induced by slow varying e.m.f. (10), can be easily found as the solution of Eq. (1) with small signal terms neglected. The inductive and resistive terms are negligible at low frequencies and we get for such a capacitance-dominated RCE

$$\mathcal{E}_p(t) = \frac{1}{C_c}\int_{-\infty}^{t} I_p(t')dt', \tag{12}$$

which yields using Eq. (10):

$$I_p(t) = -C_c\mu_0 S\ddot{H}_p(t). \tag{13}$$

For a high frequency signal wave, all terms in Eq. (1) are important. Upon substituting the total current (3) in Eq. (11), and expanding with respect to weak signal contribution we arrive at the particular form of Eq. (5):

$$U_\omega = R_{\text{ins}}(t)I_\omega, \tag{14}$$

where

$$R_{\text{ins}}(t) \equiv \left.\frac{d\mathcal{U}(I)}{dI}\right|_{I=I_p(t)} \tag{15}$$

stands for the effective insertion resistance, which follows changes of the pump current. The contour resistance R_c is negligible in comparison with that of the insertion. Then, Eq. (15) represents the total resistive part of the effective impedance (7) and determines the RCE quality factor. Accordingly, the dissipation in the metamaterial depends on pumping:

$$\Gamma = \frac{\omega_r^2}{\omega_0^2}\frac{1}{L_c}R_{\text{ins}}(t). \tag{16}$$

For an illustrative estimate of tuning range and required pump field values, we consider a circular RCE with radius $r_0 = 5$ mm and wire diameter 1 mm. Then the contour self-inductance equals 12 nH and an implemented capacitance of 0.23 pF ensures the resonant frequency $\omega_0 = 6\pi \cdot 10^9$ rad/s ($\nu_0 = 3$ GHz) to be in the microwave frequency range. For the tetragonal arrangement of RCEs with lattice constants $b = 0.5r_0$ along the z-axis and $a = 2.1r_0$ in plane, the metamaterial resonance occurs[12] at frequency $\omega_r \approx 3.6\pi \cdot 10^9$ rad/s ($\nu_r \approx 1.8$ GHz). The above metamaterial parameters are used for estimates throughout this paper.

Among variable resistance devices, backward diodes are prominent for relatively high sensitivity and reliable operation without additional bias. Taking, for example, InGaAs diodes reported in Ref. 7 with cross section 10^3 μm^2, we conclude that $\mathcal{U}(I)$ changes sufficiently at current values about 1 mA. To produce such a pump current, in accordance with Eq. (13), the values $\ddot{H} \sim 10^{19}$ A/ms^2 are required, which demands, for instance, fields of about 10^5 A/m with a characteristic varying time about 0.1 μs.

The assumed diode cross section provides in the absence of pumping the resistance $R_0 = d\mathcal{U}/dI|_{I=0} = 23\Omega$; corresponding resonance width is $\Gamma_0 = 0.06\omega_r$ (RCE quality factor $Q_0 \approx 10$). Such medium has low absorption at frequencies $|\omega - \omega_r| \geqslant \Gamma_0$. Biasing the diode with appropriate $\ddot{H}$ one can achieve points on the current-voltage characteristic with higher $d\mathcal{U}/dI$. Clearly, already a 10 times increase yields $Q = 1$ and $\Gamma \sim \omega_r$, i.e., the absorption becomes substantial in a broad frequency band. Therefore, switching of the metamaterial between transparent and absorbing states in a wide frequency range is possible.

Remarkably, there exist points on the current-voltage characteristics of certain backward diodes, where $d\mathcal{U}/dI \to \infty$. Tuning the diode to such point would provide $Q \to \infty$, $\Gamma \to \infty$, which means that the medium becomes totally absorptive for signals at any frequency.

B. Variable capacitance insertions

Varactors possess a capacitance dependent on the voltage drop on the element. We consider this dependence $\mathcal{C}(U)$ to be a known function. Generally, varactor resistance is also voltage dependent. However, the arising resistive contribution to the nonlinearity is much smaller than the capacitive one. Moreover, there are also varactors reported to have constant resistance.[8] Accordingly, we neglect the dependence for simplicity and consider Eq. (2) with constant R_{ins}:

$$U_{\text{ins}}(t) = I(t)R_{\text{ins}} + \frac{1}{\mathcal{C}(U_{\text{ins}}(t))}\int_{-\infty}^{t} I(t')dt'. \tag{17}$$

To maximize the resulting RCE nonlinearity, it is favorable to adjust the varactor capacitance $\mathcal{C}(0)$ so that no additional capacitance C_c is necessary to achieve the desired resonant frequency. Then Eq. (1) takes the form:

$$\mathcal{E}(t) = L_c\dot{I}(t) + U_{\rm ins}(t), \tag{18}$$

where we omit negligible resistance R_c of RCE conductor.

While determining slowly varying pump current and biasing voltage, we can disregard the inductive term in Eq. (18) as well as the resistive term in Eq. (17). Then, using Eq. (10), we get

$$U_p(t) = -\mu_0 S\dot{H}_p(t), \quad I_p(t) = \frac{d}{dt}[U_p(t)\mathcal{C}(U_p(t))]. \tag{19}$$

Next, we expand Eq. (17) with respect to small current and voltage at the signal frequency ω. Substituting in Eq. (18) and accounting for linear in U_ω, I_ω terms yields:

$$\mathcal{E}_\omega = \left[-i\omega L_c + \left(R_{\rm ins} + \frac{i}{\omega C(t)}\right)\left(1 + U_p(t)\frac{C'(t)}{C(t)}\right)^{-1}\right]I_\omega, \tag{20}$$

where

$$C(t) \equiv \mathcal{C}(U)\big|_{U=U_p(t)}, \quad C'(t) \equiv \left.\frac{d\mathcal{C}(U)}{dU}\right|_{U=U_p(t)} \tag{21}$$

are the pump-controlled functions of time.

The factor in square brackets in (20) represents the particular form of the effective RCE impedance (7). Correspondingly, we obtain the time dependent parameters of the permeability resonance:

$$\omega_0(t) = (L_c[C(t) + U_p(t)C'(t)])^{-1/2}, \tag{22}$$

$$\Gamma = \frac{\omega_r^2}{\omega_0^2}\frac{R_{\rm ins}}{L_c}\left(1 + U_p(t)\frac{C'(t)}{C(t)}\right)^{-1}. \tag{23}$$

The metamaterial resonant frequency is tuned proportionally to ω_0, and the resonance height A is not affected. Small variations of the resonant width are not valuable compared to the tuning of the metamaterial resonance. The latter determines the position of the metamaterial band gap, affecting critically the signals having frequencies close to the band gap edges and enables the transmission-reflection switching.

For qualitative estimates, we take typical examples of varactor diodes[8] and ferroelectric varactors.[9] To reach a remarkable tuning effect, the biasing voltage U_p should provide considerable changes of $\mathcal{C}(U)$. For varactor diodes, corresponding values of the biasing voltage are of the order of 0.5 V. Using Eq. (19), we obtain that $\dot{H}_p \sim 5\cdot 10^9$ A/m s is required (for instance, an external field of about $5\cdot 10^2$ A/m with a varying time about 0.1 μs). For ferroelectric varactors, the biasing voltage should be one order of magnitude higher, about 5 V, which enlarges the necessary pump field accordingly.

Note, that for tuning with slow fields the varactor insertions demand field values of one or even two orders of magnitude lower than backward diodes do. There are two reasons for the robustness of backward diodes in the case considered. First, the requirement of the original metamaterial transparency demands the overall RCE reactance to be much larger than its resistance (the quality factor Q of at least 10). Therefore, the pump voltage drop on varactors is of the order of external electromotive force, while the voltage drop on backward diodes is much smaller. Second, the pump current (13) is proportional to the second time derivative of the external field, whereas the varactor pump current (19) involves the first derivative. With slow varying fields, this also decreases the resulting effectiveness of backward diodes.

IV. TUNING BY A HIGH-FREQUENCY EXTERNAL MAGNETIC FIELD

As follows from Eqs. (13) and (19), in order to achieve substantial tuning using much lower pump magnetic field, one should decrease the characteristic varying times. The most convenient way to provide a fast varying magnetic field is to let a pump microwave propagate through the metamaterial. We assume pump frequency Ω to lie in the transparency region below the metamaterial resonance, but still being comparable with the resonant frequency ω_r.

Note that the macroscopic pump field in the bulk of the metamaterial differs from the externally applied field by a factor of the order of $\mu_{zz}(\Omega)$. The details of field distribution depend, especially for small samples, on particular sample shape and dimensions. However, if the sample is large enough, it is possible to obtain the pump wave with constant amplitude throughout the sample bulk. Thus, we assume the pump wave inside the metamaterial to have the z-component

$$H_z^{(\Omega)}(\mathbf{r},t) = H_p\cos(\Omega t - \phi(\mathbf{r})), \tag{24}$$

with homogeneous amplitude H_p and arbitrary phase ϕ changing at distances much larger than RCE size. Clearly, a plane running wave is a particular case of Eq. (24).

The amplitude of the pump current can be easily found from the macroscopic magnetization $\mathbf{M}$ of the medium on the pump frequency, which has only one nonzero component $M_z = nSI$. Using the relation $M_z = (\mu_{zz}(\Omega) - 1)H_z^{(\Omega)}$ between the magnetization and macroscopic magnetic field, we arrive at

$$I_p(t) = I_0\cos(\Omega t - \phi(\mathbf{r}_{\rm RCE})), \tag{25}$$

where the current amplitude is

$$I_0 = (\mu_{zz}(\Omega) - 1)\frac{H_p}{nS}, \tag{26}$$

and the phase is taken at the position $\mathbf{r}_{\rm RCE}$ of the RCE under consideration. Remarkably, it appears that the tuning effect is determined by the amplitude of I_p oscillation and is insensitive to its phase (see below). Therefore, the pump wave (24) produces homogeneous changes of the permeability.

As seen from Eq. (26), choosing Ω closer to the resonance one can take an advantage of the resonant magnetization enhancement, which provides higher pump currents. However, there are at least two reasons to use pump frequencies outside the resonance region. First, substantial absorption of the pump wave would otherwise alter the wave amplitude inside the metamaterial and cause inhomogeneity of the tuning effect. Second, the medium occurs to be extremely nonlinear with respect to strong waves with frequencies close to resonance. Clearly, harmonic oscillating plane waves are not necessarily the eigenmodes of such highly nonlinear medium. Accordingly, we suggest that the pump frequency lies in the transparency region and use $(\mu_{zz}(\Omega)-1)\simeq 1$ for our estimates. At the level of RCE response, we concentrate on the case of weak RCE nonlinearity, although with the nonlinear insertions operating in highly nonlinear regime. Therefore, we have to ensure that the nonlinear contribution to U_{ins} in Eq. (2) is small compared to linear in I terms in Eqs. (1) and (2) at the pump frequency.

For the frequency Ω being comparable with the signal wave frequency ω, tuning can be accompanied by excitation of harmonics $\omega \pm l\Omega$ with any integer l. However, this is not relevant unless the phase matching conditions are specially met. Here we neglect the harmonics generation and concentrate on tuning.

As we show below, the high frequency regime allows decreasing the required pump field values by several orders of magnitude. Clearly, the adiabatic approximation of Sec. III is no more valid and we develop a separate approach to describe this tuning regime. Particular tuning effect of the pump current (25) is determined by the nonlinear insertion type and characteristics. Below we consider the cases of variable resistance and variable capacitance.

A. Variable resistance insertions

As in previous, we consider the relation (11) for the voltage drop on the insertion. The current in RCE is given by Eq. (3) with pumping term Eq. (25). Substitution in Eq. (1) and expansion with respect to small I_ω yields

$$\mathcal{E}_\omega = \left[-i\omega L_c + \frac{i}{\omega C_c} + R(I_0) \right] I_\omega, \tag{27}$$

where we can neglect ohmic losses in the RCE conductor, and rewrite the overall RCE resistance as

$$R(I_0) = \left\langle \left. \frac{d\mathcal{U}(I)}{dI} \right|_{I=I_0 \cos(\Omega t-\phi)} \right\rangle. \tag{28}$$

The involved zero Fourier component of the oscillating instantaneous resistance is equivalent to the value, averaged over period $2\pi/\Omega$; it is denoted here by $\langle \ldots \rangle$-brackets. The averaged value depends neither on Ω nor on ϕ, which ensures the homogeneous tuning by a running wave.

As seen from Eq. (27), the pumping affects the effective resistance of RCE. This finally results in tuning of the metamaterial resonance width:

$$\Gamma(H_p,\Omega) = \frac{\omega_r^2}{\omega_0^2} \frac{1}{L_c} R(I_0), \tag{29}$$

which appears to be dependent via I_0 on the pump field amplitude and frequency [see Eq. (26)].

To estimate the arising tuning capabilities with backward diodes[7] we model the typical current-voltage characteristics [Fig. 1(a)] assuming feasible diode cross section $10^3\ \mu\text{m}^2$. Corresponding relative resistance changes $R(I_0)/R(0)$ are shown in Fig. 1(b). For some of the modelled characteristics, the effective resistance demonstrates a remarkable peak. It is easy to explain the origin of this sharp peak: it occurs when the pump current amplitude matches the local maximum of the current-voltage characteristic, where $d\mathcal{U}/dI$ tends to infinity. Corresponding contribution to the averaged value (28) increases drastically then, since the diode spends considerable time interval in the vicinity of this critical point.

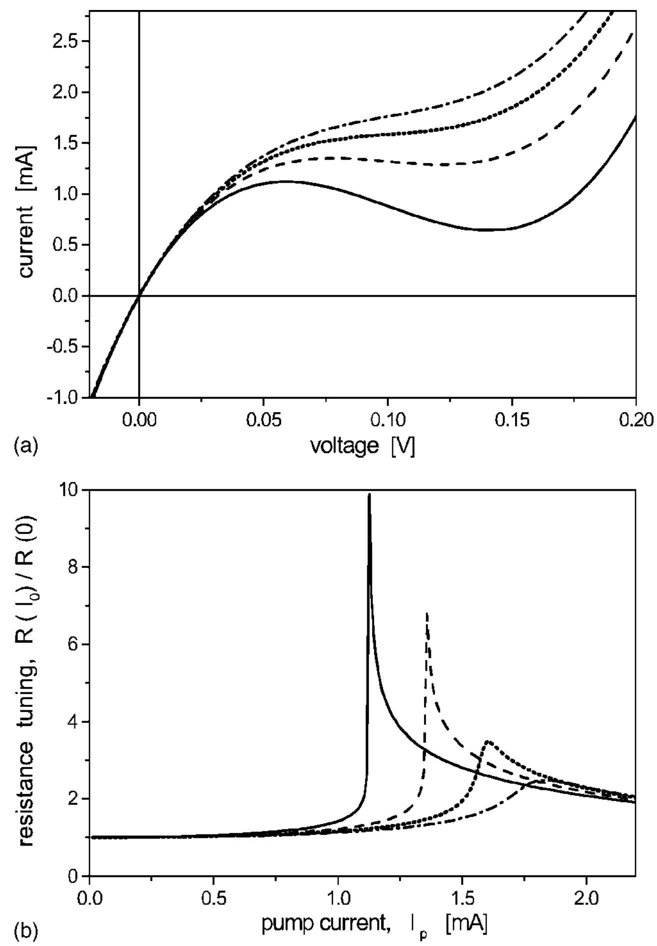

FIG. 1. Resistance tuning with backward diodes. (a) Modelled current-voltage characteristics of backward diodes. (b) Corresponding dependencies of the relative resistance change on pump current.

We conclude that for the modelled diodes, the effective RCE resistance can be increased by up to one order of magnitude. This requires the amplitude of the pump current about 1 mA. Using Eq. (26) with the chosen metamaterial parameters (see Sec. III A) we estimate the pump field amplitude of the order of 0.3 A/m to be sufficient. Comparing with the estimates in Sec. III A shows that the high-frequency tuning indeed demands much lower field amplitudes. On the other hand, achievable Γ modulation is smaller.

B. Variable capacitance insertions

We consider, like in Sec. III B, a RCE with the capacitance being provided by a varactor insertion. The pump voltage drop on the varactor can be obtained from Eq. (17) for the known pump current (25). At this step, we can neglect ‘the weak signal contribution as well as the small resistance in (17). Then the pump voltage obeys the transcendental equation

$$U_p(t)\mathcal{C}(U_p(t)) = \frac{1}{\Omega} I_p(t). \tag{30}$$

Obviously, $U_p(t)$ is a $2\pi/\Omega$ periodic function; it can be obtained from Eq. (30) for a particular varactor with known $\mathcal{C}(U)$.

Next, we substitute the voltage drop (3) into Eqs. (17) and (18), and expand them with respect to the signal voltage U_ω, obtaining:

$$\mathcal{E}_\omega = \left[- i\omega L_c + \left(R_{\rm ins} + \frac{i}{\omega}\left\langle \frac{1}{C(t)} \right\rangle \right) \times \left(1 + \left\langle U_p(t) \frac{C'(t)}{C(t)} \right\rangle \right)^{-1} \right] I_\omega. \tag{31}$$

Note, that it has a form resembling Eq. (20) with an important distinction that the averaging over the period $2\pi/\Omega$ is performed. Taking Eq. (30) into account, it is easy to obtain that the averaged values are controlled by pumping via parameter $g = I_0/\Omega$. Accordingly, the RCE resonant frequency and the metamaterial resonance width are g-dependent:

$$\omega_0(g) = \left\langle \frac{1}{C(t)} \right\rangle^{1/2} \left(L_c \left(1 + \left\langle U_p(t) \frac{C'(t)}{C(t)} \right\rangle \right) \right)^{-1/2}, \tag{32}$$

$$\Gamma = \frac{\omega_r^2}{\omega_0^2} \frac{R_{\rm ins}}{L_c} \left(1 + \left\langle U_p(t) \frac{C'(t)}{C(t)} \right\rangle \right)^{-1}. \tag{33}$$

The shift of the metamaterial resonant frequency ω_r is proportional to the shift (32) of ω_0. The resonance height A remains constant.

We remind that in order to deal with harmonic pump wave, we have to ensure that it is only slightly distorted by nonlinear insertion influence. Therefore, the pump-induced modulation of the effective RCE capacitance should not be strong. This means, in particular, that the parameters (32) and (33) should not be altered significantly by pumping, i.e., $|\omega_0(g) - \omega_0(0)| \ll \omega_0(0)$ and $|\Gamma(g) - \Gamma(0)| \ll \Gamma(0)$. Such changes of the resonance width are hardly observable, whereas the resonance shift affects crucially propagation of signal waves with frequencies close to ω_r.

For illustrative estimates, we model characteristic dependencies $\mathcal{C}(U)$ for varactor diodes[8] and ferroelectric varactors[9] as shown in Figs. 2(a) and 3(a), respectively. We calculate the corresponding relative shift of the resonance $\omega_r(g)/\omega_r(0)$, taking RCE and metamaterial parameters as in Sec. III A. Figures 2(b) and 3(b) show that tuning of the resonant frequency by 10% is achievable with $g \sim 5 \cdot 10^{-14}$ A s for the modelled varactor diodes and $g \sim 2 \cdot 10^{-13}$ A s for the ferroelectric varactors. In accordance with Eq. (26), corresponding pump field amplitudes are $H_p \sim 0.1$ A/m and $H_p \sim 0.5$ A/m.

Comparing the estimates for the high frequency tuning, we see that the pump field amplitudes required for remarkable tuning are of the same order for backward diodes and varactors. As expected, these field amplitudes are several orders of magnitude lower than those for the slow-field tuning. This makes the high frequency tuning much more attractive for practice.

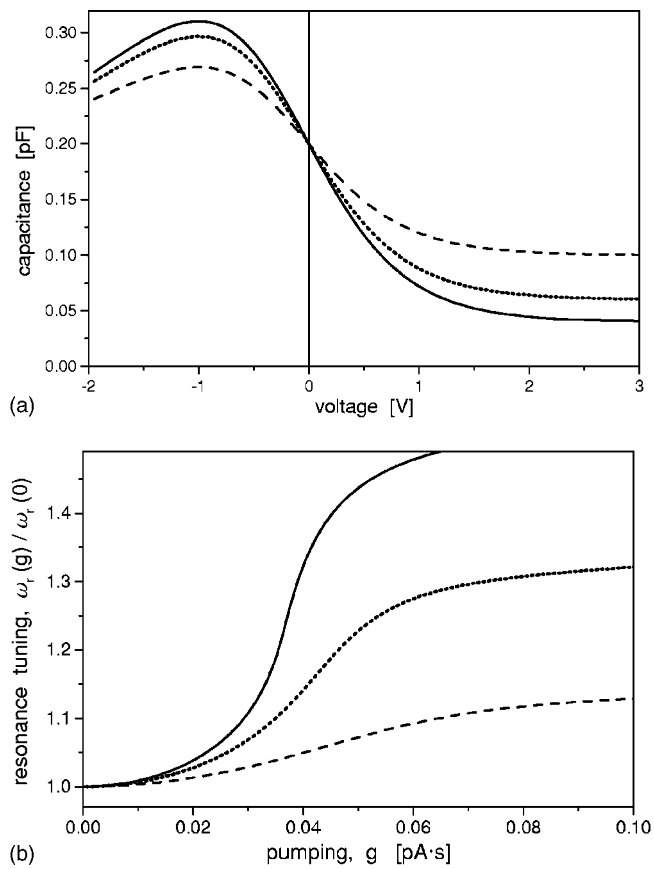

FIG. 2. Resonance tuning with varactor diodes. (a) Modelled capacitance-voltage characteristics of varactor diodes. (b) Corresponding dependencies of the relative resonance shift on pumping parameter g.

V. TUNED TRANSMISSION OF A METAMATERIAL SLAB

To get a better insight into the application capabilities of the metamaterial tuning, we turn to a simple example and study a transmission-reflection problem for a metamaterial slab with tunable permeability. Schematic of the problem is presented in Fig. 4. Since the metamaterial interacts only with the z-component of magnetic field, we choose the corresponding polarization of the incident (I) signal wave. For simplicity, we disregard the interaction of RCEs with the electric field (which is weak relative to the resonant magnetic interaction) and assume the permittivity to be given by a unit tensor: $\varepsilon_{ij} = \delta_{ij}$. The chosen coordinate axes are the principal axes of both permeability and permittivity tensors and therefore transmitted (T) and reflected (R) waves are polarized in the same way as the incident wave.

For a normal incidence of the signal wave, the slab transmittance T and reflectance F take the form[13]

$$T = \frac{|4\Psi\sqrt{\mu_{zz}(\omega)}|^2}{|\Psi^2(1 - \sqrt{\mu_{zz}(\omega)})^2 - (1 + \sqrt{\mu_{zz}(\omega)})^2|^2}, \tag{34}$$

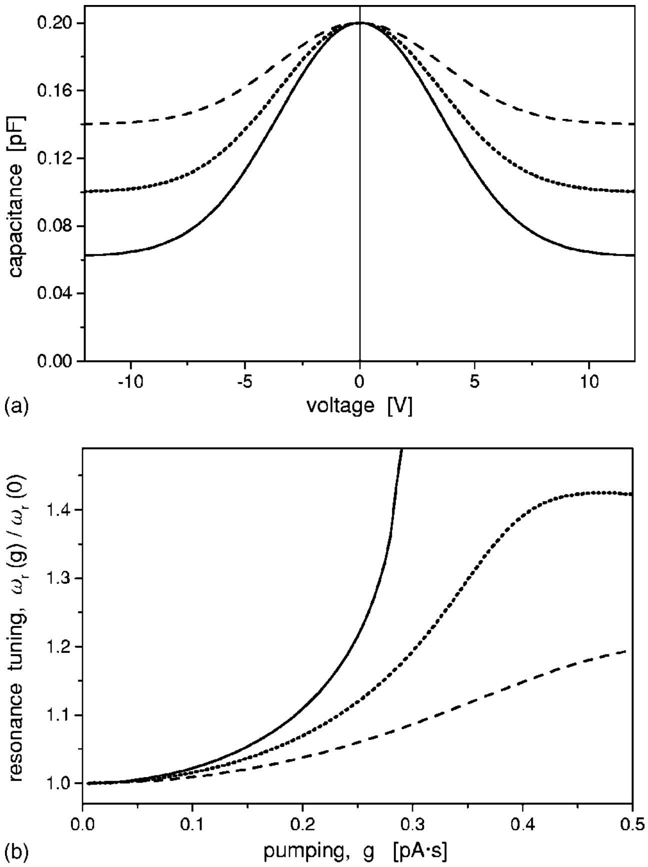

FIG. 3. Resonance tuning with ferroelectric varactors. (a) Modelled capacitance-voltage characteristics of ferroelectric varactors. (b) Corresponding dependencies of the relative resonance shift on pumping parameter g.

$$F = \frac{|(1-\Psi^2)(1-\mu_{zz}(\omega))|^2}{|\Psi^2(1-\sqrt{\mu_{zz}(\omega)})^2-(1+\sqrt{\mu_{zz}(\omega)})^2|^2}, \tag{35}$$

where $\Psi = \exp(id\sqrt{\mu_{zz}(\omega)}\omega c^{-1})$.

Note, that the transmittance and reflectance are controlled by only two parameters: the slab thickness d and permeability μ_{zz}. We distinguish between three qualitatively different states: transmitting slab, with $T\approx 1$, $F \ll 1$; reflecting slab, with $T \ll 1$, $F\approx 1$; and absorbing slab, with $T\ll 1$, $F\ll 1$. Adjusting the permeability by external magnetic field one can vary the coefficients (34) and (35) and switch the slab between these three states.

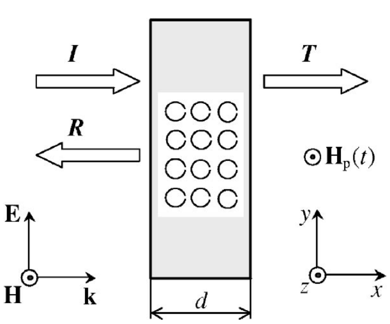

FIG. 4. Transmission-reflection setup for metamaterial slab. Orientation of RCEs is schematically shown.

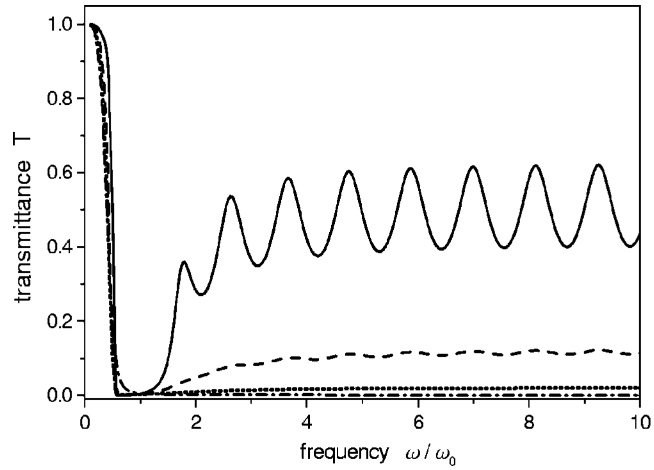

FIG. 5. Transmittance of metamaterial slab in a broad frequency range with the RCE quality factor $Q=10$ (solid line), 2 (dashed line), 1 (dotted line), and 0.5 (dashed-dotted line).

It appears that a slab thickness comparable with the microwave wavelength in vacuum, $\lambda_0 = 2\pi c/\omega_0$, is large enough for substantial transmittance modulation. Setting the thickness $d=\lambda_0$, we concentrate on the transmission coefficient T as a function of the signal frequency ω and study the changes caused by the pumping.

Variable resistance insertions allow controlling the RCE quality factor and, as a consequence, the metamaterial resonance width. We have demonstrated that for both considered types of tuning (slow and high frequency pumping) a 5–10-fold decrease of the quality factor is achievable. Resulting changes in the transmittance are presented in Fig. 5. Taking the RCE quality factor $Q=10$ in the absence of pumping ensures that the slab is initially transparent ($T\sim 0.5$) in a wide frequency range above the band gap. We characterize the tuning efficiency with the transmittance modulation depth (ratio of the untuned to tuned transmittance). Figure 6

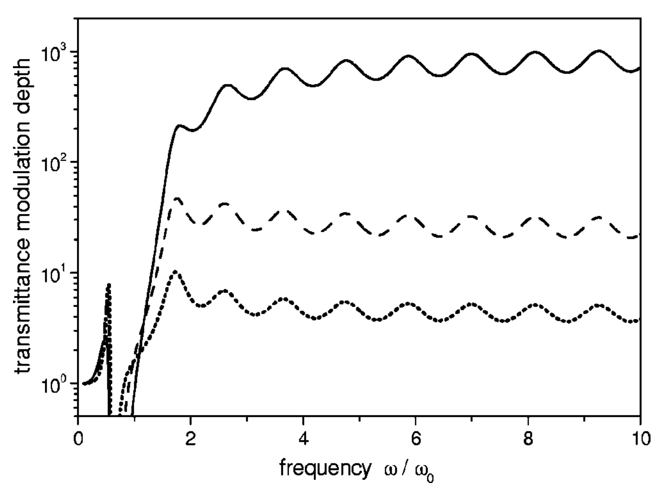

FIG. 6. Relative change of slab transmittance for the fivefold (dotted line), tenfold (dashed line), and 20-fold (solid line) decrease of the RCE quality factor.

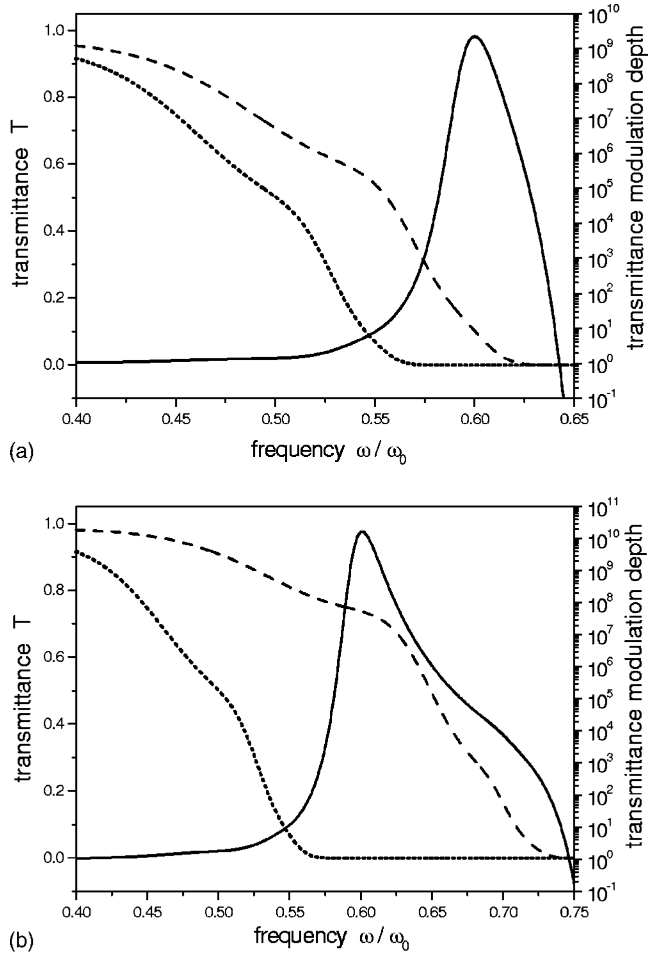

FIG. 7. Tuning of the metamaterial slab transmittance with 10% (a) and 30% (b) shift of the band gap. Transmittances of the nontuned slab (dotted line), of the slab with shifted resonance (dashed line), and resulting transmission modulation (solid line) are shown.

shows that a fivefold decrease of the quality factor leads to noticeable absorption of the signal wave resulting in a fivefold decrease of the transmittance. A tenfold decrease of the quality factor lowers the transmitted signal intensity by about 30 times. A 20-fold decrease of the quality factor practically blocks the transmission. Such an extreme modulation is apparently not feasible with the high frequency tuning, but can be realized with the slow tuning regime.

Variable capacitance insertions provide a possibility to tune the metamaterial resonance and, consequently, to shift the band gap. With both regimes of tuning, the resonant frequency can be altered by up to 30% and more. In Fig. 7 we show two examples of corresponding changes in the transmittance. Shifting the band gap by 10% allows varying the transmittance by up to nine orders of magnitude in a narrow frequency range between the original and shifted band gap edges [see Fig. 7(a)]. However, the transmittance at such frequencies remains below 0.2, i.e., the most part of the incident power is absorbed or reflected anyway. Switching to a state with higher transmittance becomes possible if the shift of the resonance exceeds the width of the transition region around the band gap edge. Thus, a 30% resonance shift makes possible to modulate the transmittance between 0.8 and practically zero.

VI. SUMMARY

We have analyzed the possibility to tune the linear electromagnetic properties of nonlinear metamaterial with external varying magnetic field. We have shown that the variable resistance insertions provide an opportunity to tune the width of the metamaterial resonance and, accordingly, to control the medium absorbance. Variable capacitance insertions allow shifting the metamaterial magnetic resonance, which determines the band gap.

Slow-field tuning regime allows broad variation of the metamaterial properties but requires extremely high values of external field. The high-frequency tuning by a pump microwave appears to demand much lower fields and we conclude it to be the most promising for applications.

We have performed estimates for available backward diodes, varactor diodes, and ferroelectric varactors employed as nonlinear insertions. We conclude that the high frequency tuning requires the amplitude of magnetic field in pump microwave to be of the order of 0.1 A/m, which is quite feasible.

We demonstrated that a relatively thin slab of metamaterial having the thickness comparable with microwave wavelength allows performing the broadband transmittance modulation by about 30 times, when variable resistance insertions are used. Using the variable capacitance insertions provides a narrowband modulation by several orders of magnitude.

We conclude that nonlinear metamaterial appears to be not only an interesting research object as an emerging media, but have clear application potential as well. The reported metamaterial tunability is analogous to electro-optic effect known for more than a century and widely exploited nowadays in optic technology. We expect nonlinear metamaterials to become key elements of innovative microwave devices for an opticlike manipulation with microwaves.

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*Email address: mgorkoun@uos.de; Permanent address: Institute of Crystallography RAS, Leninski prosp., 59, 117333, Moscow, Russia.

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