Controlling Fano and Dicke effects via a magnetic flux in a two-site Anderson model

P. A. Orellana,[1] M. L. Ladrón de Guevara,[1] and F. Claro[2]
[1]*Departamento de Física, Universidad Católica del Norte, Casilla 1280, Antofagasta, Chile*
[2]*Departamento de Física, Pontificia Universidad de Chile, Casilla 306 Santiago 22, Chile*
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The electronic transport through two coupled quantum dots in a parallel configuration is studied under a magnetic flux. We model the system by means of a non interacting two-site Anderson Hamiltonian. We find that the conductance shows Fano and Dicke effects that can be controlled by the magnetic flux.

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Resonant tunneling through two parallel quantum dots has attracted much interest recently. For instance, Holleitner *et al.*[1] studied how the molecular states of semiconductor quantum dots connected in parallel to the leads can be coherently probed and manipulated in transport experiments, while Kubala and König[2] reported a level attraction in an Aharonov-Bohm interferometer with two quantum dots in its arms. Moreover, Kang and Cho[3] and Boese *et al.*[4] studied the double quantum dot in the parallel geometry in the presence of a magnetic flux.

In Ref. 5 we reported on the transition from a series to a parallel arrangement of two coupled quantum dots attached to leads. The existence of two different pathways for the electron transport produces conductance spectra composed by a Breit-Wigner resonance and a Fano-like resonance at the bonding and antibonding frequencies, respectively. We found that close to the symmetrical parallel configuration the conductance shows an ultranarrow peak that eventually disappears completely in the symmetrical one, a feature we called the ghost Fano effect. The general features of the conductance spectrum taking place in the series to parallel transition of Ref. 5 are given in the parallel coupled double quantum dot embedded in an Aharonov-Bohm flux, as discussed by Kang and Cho[3] and Bai *et al.*[6] The two conductance peaks (Breit-Wigner and Fano-like line shapes) depend sensitively on the external magnetic field and exhibit Aharonov-Bohm-type oscillations.

In this Brief Report we consider the electron transport through parallel coupled double quantum dots embedded in an Aharonov-Bohm interferometer connected asymmetrically to leads. We show that with a period of a quantum of flux ($\Phi_0 = h/e$) the magnetic field allows interchanging the roles of the bonding and antibonding states in the transmission spectrum. For intermediate values of the flux (namely, semi-integer multiples of a quantum of flux) the parallel coupled double quantum dot behaves as if it were connected in series. We also find that whenever the flux is close to integer multiples of Φ_0, the density of states shows an ultranarrow and a broad peak at the energies of the molecular states, associated with Fano and Breit-Wigner line shapes in the conductance. When the flux has exactly the above values, the conductance experiences the suppression of the Fano line shape, indicating a localization of the corresponding molecular state, similarly to what takes place for the symmetrical case in the absence of magnetic field.[5] We find that these results hold even under a strong left-right asymmetry. This phenomenon resembles the Dicke effect in optics, which takes place in the spontaneous emission of a pair of atoms radiating a photon with a wavelength much larger than the separation between them.[7] The luminescence spectrum is characterized by a narrow and a broad peak, associated with long- and short-lived states, respectively. The former state, coupled weakly to the electromagnetic field, is called *subradiant*, and the latter, strongly coupled, *superradiant* state.

The Dicke effect was predicted and experimentally verified long ago in atomic systems;[9–11] however, only recently predictions were made for it to occur in transport through mesoscopic systems.[8] The appearance of the Dicke effect in resonant tunneling was predicted to appear in the conductance by Shahbazyan and Raikh in a work on a tunneling junction with two impurities.[12] Later, Shahbazyan and Ulloa[13] studied this effect in a system of localized states in a strong magnetic field. More recently, Vorrath and Brandes[14] studied the stationary current through a double quantum dot interacting via a common phonon environment, and Wunsch and Chudnovskiy investigated the Dicke effect in a ring coupled to a reservoir.[15]

We consider two coupled single-level quantum dots, attached asymmetrically to leads. The system is modeled by a noninteracting two-impurity Hamiltonian, and the conductance and densities of states are obtained by the equation of motion approach for the Green's function.

From the diagonal elements of the Green's function we can get the spectral densities $A_{\pm} = -(1/\pi)\mathrm{Im}\, G^{r}_{\pm\pm}$. Summing over the $\pm$ states we obtain the density of states of the two coupled quantum dots,

$$\rho(\varepsilon) = \sum_{\sigma=-,+} A_{\sigma}, \tag{1}$$

where

$$A_{-} = \frac{1}{\pi\Lambda}\cos^2(\phi/4)\tilde{\Gamma}[(\varepsilon - t_c)^2 + 4\Gamma_L\Gamma_R \sin^4(\phi/4)], \tag{2}$$

$$A_{+} = \frac{1}{\pi\Lambda}\sin^2(\phi/4)\tilde{\Gamma}[(\varepsilon + t_c)^2 + 4\Gamma_L\Gamma_R \cos^4(\phi/4)], \tag{3}$$

with

$$\Lambda = \tilde{\Gamma}^2[\varepsilon - t_c \cos(\phi/2)]^2 + [(t_c + \varepsilon)(\varepsilon - t_c) + \Gamma_L \Gamma_R \sin^2(\phi/2)]^2, \quad (4)$$

where Γ_L (Γ_R) is the left (right) level broadening, $\tilde{\Gamma} = \Gamma_L + \Gamma_R$, and ϕ the Aharonov-Bohm phase $\phi = 2\pi\Phi/\Phi_0$, with Φ the magnetic flux.

The transmission in turn results[16]

$$T(\varepsilon) = \frac{1}{\Lambda} 4\Gamma_L \Gamma_R [\varepsilon \cos(\phi/2) - t_c]^2. \quad (5)$$

The conductance is related to the transmission according to the Landauer formula at zero temperature,[2] $G(\varepsilon) = (2e^2/h)T(\varepsilon)$. We note in Eq. (5) that when $\phi = 0$ and $\Gamma_L = \Gamma_R = \Gamma_0$ (with Γ_0 the level broadening of a single quantum dot) the result without magnetic field is recovered.[5] Also, as follows from Eqs. (1)–(4), the density of states is the sum of a Lorentzian with width $\Gamma_- \to 2\Gamma_0$ and a δ function centered at the antibonding energy ($\Gamma_+ \to 0$). In other words, in this limit the antibonding state is decoupled from the continuum, while the bonding state has reduced its lifetime to a half. This is due to quantum interference in the transmission through the two different discrete states (the two quantum-dot levels) coupled to common leads. This result is similar to the Dicke effect in optics that takes place in the spontaneous emission of two closely lying atoms radiating a photon into the same environment.[7] The phenomenon is analogous to the formation of a bonding and an antibonding state by the coherent coupling of the two energy levels, with the difference that this effect is the splitting of the decay rates (level broadening) into a fast (superradiant) and a slow (subradiant) mode.[8]

The Dicke effect in the parallel two coupled quantum dots occurs also for arbitrary left-right asymmetry ($\Gamma_L \neq \Gamma_R$), and whenever the Aharonov-Bohm phase approaches an integer multiple of 2π. In fact, when $\phi = 2\pi n$, with n an integer, the density of states takes the form

$$\rho(\varepsilon) = \begin{cases} \frac{1}{\pi}\left[\frac{\tilde{\Gamma}}{(\varepsilon - \varepsilon_-)^2 + \tilde{\Gamma}^2}\right] + \delta(\varepsilon - \varepsilon_+), & n \text{ even} \\ \delta(\varepsilon - \varepsilon_-) + \frac{1}{\pi}\left[\frac{\tilde{\Gamma}}{(\varepsilon - \varepsilon_+)^2 + \tilde{\Gamma}^2}\right], & n \text{ odd}. \end{cases} \quad (6)$$

We observe that the positions of the long-lived and the short-lived states can be interchanged depending on the parity of the magnetic flux. Notice that when $t_c = 0$ the Dicke effect is still valid for $\phi = 2\pi n$, but when the system is degenerate ($\varepsilon_+ = \varepsilon_- = 0$) the narrow and the wide peaks in the density of states are superimposed, as in the original Dicke effect.

In order to evaluate the above expressions numerically, we set $\Gamma_L = (1 - \Delta_A)\Gamma_0$, $\Gamma_R = (1 + \Delta_A)\Gamma_0$, where Δ_A is the asymmetry parameter. Figure 1 shows the density of states for the cases discussed above for $\Delta_A = 0.5$. Figure 1(a) is for $t_c = \Gamma_0$ (solid line) and $t_c = 0$ (dashed line) at $\phi = 0.1\pi$. When $t_c = \Gamma_0$, we see that the narrow peak (corresponding to the "subradiant" state) develops around the antibonding state, and the broad peak (corresponding to the "superradiant" state) develops around the bonding state. In the case with $t_c = 0$, both the broad and narrow peaks are centered at $\varepsilon = 0$. For $\phi = 1.9\pi$ and $t_c = \Gamma_0$ the two peaks interchange roles, as displayed in Fig. 1(b), while when $t_c = 0$ they again are superimposed at $\varepsilon = 0$.

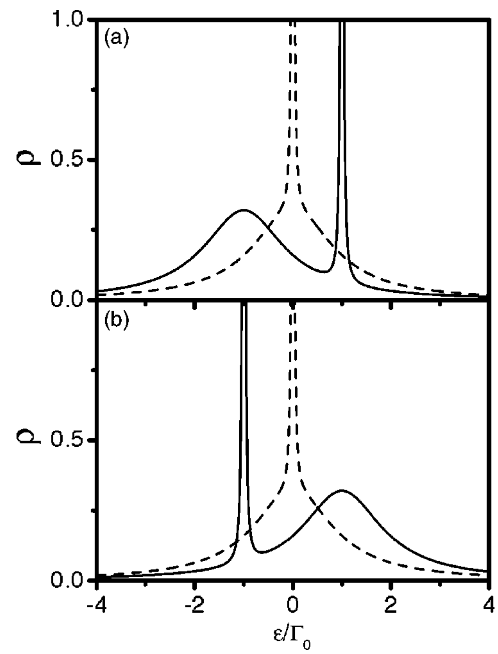

FIG. 1. Density of states ρ as a function of the Fermi energy for (a) $t_c = \Gamma_0$ (solid line) and $t_c = 0$ (dashed line) at $\phi = 0.1$ and (b) $t_c = \Gamma_0$ (solid line) and $t_c = 0$ (dashed line) at $\phi = 1.9\pi$.

Next we show how the Dicke effect is present in the conductance. We define the dimensionless conductance by $g = G/(2e^2/h)$. This quantity is given by

$$g(\varepsilon) = \frac{4\Gamma_L \Gamma_R}{\Lambda}[\varepsilon \cos(\phi/2) - t_c]^2. \quad (7)$$

In general the conductance spectrum is composed of Breit-Wigner and Fano line shapes, as shown previously by Kang and Cho[3] and us.[5] There is a correspondence between the narrower (wider) peak in the density of states and the Fano (Breit-Wigner) resonance in the conductance. And the widths of these lines are also controlled by the magnetic flux. In fact, when ϕ is around $2\pi n$, for n even (odd), the Fano line shape is associated with the antibonding (bonding) state. In the limit $\phi = 2\pi n$, the Fano resonance is suppressed and only the Breit-Wigner signal survives. The latter develops around the bonding or the antibonding energy depending on whether n is even or odd, respectively,

$$g_\pm = \frac{4\Gamma_L \Gamma_R}{[(\varepsilon - \varepsilon_\pm)^2 + \tilde{\Gamma}^2]}. \quad (8)$$

These features are displayed in Fig. 2 for $\Delta_A = 0.5$. The

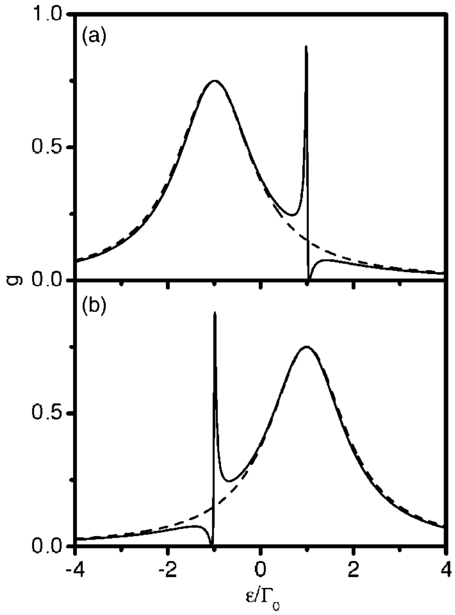

FIG. 2. Dimensionless conductance g as a function of the Fermi energy for $t_c=\Gamma_0$. (a) $\phi=0.1\pi$ (solid line) and $\phi=0$ (dashed line), and (b) $\phi=1.9\pi$ (solid line) and $\phi=2\pi$ (dashed line).

curves g versus Fermi energy for $\phi=0.1\pi$ (solid line) and $\phi=0$ (dashed line) are shown in Fig. 2(a). Figure 2(b) gives g versus Fermi energy for the magnetic flux $\phi=1.9\pi$ (solid line) and $\phi=2\pi$ (dashed line). As happens for a double coupled quantum dot attached symmetrically to leads in the absence of a magnetic field,[5] when the magnetic flux is an integer number flux quanta, the long-lived state is decoupled from the continuum and is suppressed from transmission. For a flux close to any of these points the system would be in a regime of Dicke effect (see Fig. 2).

Let us pay some attention to the special case when the quantum dots are disconnected from each other ($t_c=0$). Now, the Fano antiresonance is localized at $\varepsilon=0$ independent of the magnetic flux, except for $\phi=2\pi n$. Equation (7) evaluated at the center of the band gives

$$g=\begin{cases}0 \quad (\phi \neq 2\pi n, n \text{ integer})\\ \dfrac{4\Gamma_L\Gamma_R}{(\Gamma_L+\Gamma_R)^2}=1-\Delta_A^2 \quad (\phi=2\pi n, n \text{ integer}).\end{cases} \tag{9}$$

That is, the conductance is different from zero periodically in the magnetic field, with a period of one quantum of flux.

On the other hand, it is interesting to note that when $\phi \to 2\pi(n+1/2)$ (n integer), $\cos(\phi)\to 0$ and hence the conductance is reduced to a convolution of two Breit-Wigner line shapes with the same width centered in the bonding and antibonding energies, respectively,

$$g=\frac{\Gamma_L\Gamma_R t_c^2}{[(\varepsilon-i\Gamma_L)(\varepsilon-i\Gamma_R)-t_c^2][(\varepsilon+i\Gamma_L)(\varepsilon+i\Gamma_R)-t_c^2]}. \tag{10}$$

No Fano line shape develops. The double quantum dot in the parallel configuration behaves as a serial one for transmission. When the electron crosses the upper (lower) arm, it accumulates a phase difference $\pi/2$ $(-\pi/2)$. The contribution to the wave function of both paths interfere destructively and mutually cancel at the leads. Therefore, the paths that contribute to the conductance are only those that cross the system through both quantum dots sequentially, as in a serial configuration. Note that when $t_c=0$ the conductance vanishes independently of the energy and perfect reflection is reached. A similar result was obtained previously by Kubala and König for a parallel double quantum-dot system connected symmetrically to the leads.[2]

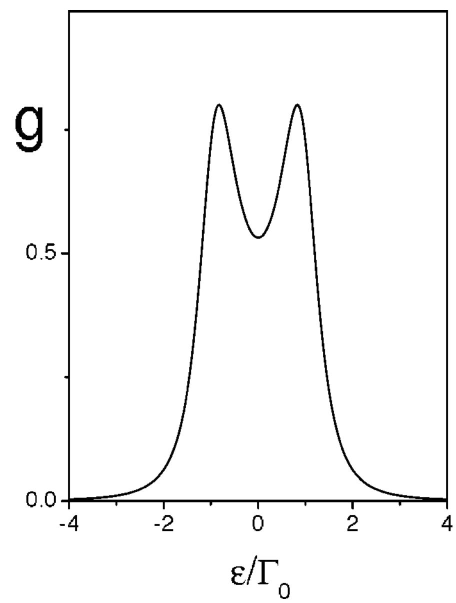

FIG. 3. Dimensionless conductance g as a function of the Fermi energy for $t_c=\Gamma_0$ and $\phi=\pi$.

In summary, we studied the conductance and the density of states at zero temperature of two coupled quantum dots connected asymmetrically to leads in a parallel configuration under a magnetic flux. We show that the magnetic flux can control the different regimes of conduction through the system. In particular, when the magnetic flux is near an integer number of flux quanta, the system is in the Dicke regimen. The conductance spectrum is composed of Breit-Wigner and Fano line shapes at the bonding and antibonding energies, or vice versa, depending on whether this number is even or odd, with their line broadenings controlled by the magnetic flux. The narrowing (broadening) of a line in the conductance can be interpreted as an increase (reduction) of the lifetime of the corresponding molecular state. From the densities of states it can be deduced that the antibonding (bonding) state becomes progressively localized as the magnetic flux tends to an integer number of flux quanta. When the magnetic flux is exactly an integer, tunneling through the antibonding (bonding) state is totally suppressed and the bonding (antibonding) state is the only participating state in the transmission. Moreover, when the magnetic flux is a half integer of flux quanta, the double quantum dot in the parallel configuration behaves as a serial one for the conductance. The control of the decoherence processes with the magnetic field exhibited by the present system may have applications in quantum computing.

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