

Collective pinning of the vortex lattice by columnar defects in layered superconductors

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The mixed phase of layered superconductors with no magnetic screening is studied through a partial duality analysis of the corresponding frustrated XY model in the presence of random columnar pins. A small fraction of pinned vortex lines is assumed. Thermally induced plastic creep of the vortex lattice within isolated layers results in an intermediate Bose glass phase that exhibits weak superconductivity across layers in the limit of weak Josephson coupling. The correlation volume of the vortex lattice is estimated in the strongly coupled Bose glass regime at lower temperature. In the absence of additional point pins, no peak effect in the critical current density is predicted to occur on this basis as a function of the Josephson coupling. Also, the phase transition observed recently inside of the vortex-liquid phase of high-temperature superconductors pierced by sparse columnar defects is argued to be a sign of dimensional crossover.

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I. INTRODUCTION

Random material defects that are correlated along the lines of magnetic induction are perhaps the most effective mechanism known to pin down the vortex lattice in high-temperature superconductors.¹ A Bose glass phase with a divergent tilt-modulus is predicted to exist at relatively low fields such that each and every flux line can be trapped by a correlated material defect.² Experimental studies of the opposite limit, where the number of flux lines exceeds the number of correlated pinning sites, have been made recently.³⁻⁵ Less is known theoretically about this regime in comparison to the former relatively low-field regime. The vortex lattice is pinned collectively here by the sparse columnar defects in the low-temperature limit.² A Bose glass phase that displays an infinite tilt modulus is therefore still expected. The collective pinning effect is naturally degraded by thermal fluctuations, however. In stark contrast to the case of point pinning, two classes of vortex lines exist in the case of sparse columnar defects:^{3,5} (i) those lines of vortices that are pinned down at a columnar defect, and (ii) those lines that are not. An *interstitial* phase is therefore predicted to exist theoretically at intermediate temperatures between the Bose glass and the vortex liquid phases, where only a fraction of the flux lines remain pinned down while the remaining lines are free to wander and thereby degrade the superconductivity.⁶

High-temperature superconductors are also layered materials.^{7,8} Thermal fluctuations of the vortex lines in layered superconductors are larger than in those of isotropic materials. This makes them ideal candidates to be a host for interstitial vortex matter. Monte Carlo simulations of the frustrated XY model have been performed recently in order to study extremely type-II layered superconductors in perpendicular magnetic field with sparse columnar pins located at random.⁹ These simulations find evidence for the existence of an interstitial liquid/glass phase as described above. We study this possibility here theoretically through a duality analysis of the same layered XY model.¹⁰ An intermediate Bose glass phase that shows weak superconductivity across layers¹¹ exists at temperatures that lie between the vortex-liquid phase and the Bose glass phase in the limit of weak

Josephson coupling between layers. We argue that the transition between the strongly coupled Bose glass that exists in the zero-temperature limit and the latter weakly coupled Bose glass is a crossover by comparison with the duality transformation of the layered XY model without frustration.¹² Numerical simulations find evidence for a sharper transition, however, which we suggest is an artifact of the relatively coarse model grid that was used.⁹ We also argue for the absence of a peak effect in the critical current density of the strongly-coupled Bose glass phase as a function of the Josephson coupling if no additional point pinning is present (see Ref. 13). This is based on an estimate for the Larkin correlation volume of the vortex lattice.^{7,14} Last, a vortex liquid-vortex liquid transition has been observed very recently in high-temperature superconductors with sparse columnar defects.⁵ Both the material defects and the external magnetic field were oriented perpendicular to the copper-oxygen planes. We shall interpret this phenomenon as a dimensional crossover transition that exists inside of the vortex-liquid phase of the defective superconductor,¹⁵ but that is absent in the pristine superconductor.¹⁶

II. ISOLATED LAYER

Consider first a stack of isolated superconducting layers in a perpendicular external magnetic field. The XY model over the square lattice with uniform frustration then provides a qualitatively correct description of the mixed phase for each layer in the absence of Josephson coupling, as well as of magnetic screening. The neglect of the latter is valid at perpendicular fields that are far enough above the lower critical field such that

$$a_{vx} \ll \lambda_L, \quad (1)$$

where a_{vx} is the square root of the area per vortex in each layer, and where λ_L denotes the London penetration depth associated with supercurrents that flow within each layer. A weak Josephson coupling will be turned on in the next section. The corresponding Boltzmann distribution is set by the sum of energy functionals

$$E_{XY}^{(2)} = - \sum_{\mu=x,y} \sum_{\vec{r}} J_{\mu} \cos[\Delta_{\mu}\phi - A_{\mu}] \quad (2)$$

for the superfluid kinetic energy of each layer l written in terms of the superconducting phase $\phi(\vec{r}, l)$. Here $\Delta_{\mu}\phi(\vec{r}) = \phi(\vec{r} + a\hat{\mu}) - \phi(\vec{r})$ and $\vec{A} = (0, 2\pi f x/a)$ make up the local supercurrent, where f denotes the concentration of vortices over the square lattice with lattice constant a . The local phase rigidity $J_{\mu}(\vec{r})$ is assumed to be constant over most of the nearest-neighbor links $(\vec{r}, \vec{r} + a\hat{\mu})$ in layer l , with the exception of those links in the vicinity of the pinning sites that are located at random.

Macroscopic phase coherence is monitored by the two-dimensional (2D) phase rigidity given by one over the dielectric constant of the 2D Coulomb gas ensemble¹⁷ that corresponds to the frustrated XY model (2). Vortex/anti-vortex excitations *not* associated with displacements of the zero-temperature vortex lattice are suppressed exponentially at temperatures far below the Kosterlitz-Thouless transition. The total number of vortices is conserved in such case. This then ultimately leads to the result

$$\rho_s^{(2D)} = J_0 [1 - (\eta'_{vx}/\eta_{sw})] \quad (3)$$

for the 2D phase rigidity, where $\eta_{sw} = k_B T / 2\pi J_0$ is the spin-wave component of the phase correlation exponent, and where¹⁸

$$\eta'_{vx} = \pi \left\langle \left[\sum_{\vec{R}} \delta \vec{u} \right]^2 \right\rangle / N_{vx} a_{vx}^2, \quad (4)$$

monitors fluctuations of the center of mass of the vortex lattice.¹⁹ Above, $\delta \vec{u}$ is the displacement field of each vortex with respect to its location at zero temperature. Also above, N_{vx} denotes the total number of vortices, while $a_{vx} = a/f^{1/2}$. Finally, J_0 denotes the Gaussian phase rigidity.¹⁷ Generalized phase autocorrelation functions $C_l[q] = \langle \exp[i \sum_{\vec{r}} q(\vec{r}) \cdot \phi(\vec{r}, l)] \rangle_0$ within an isolated layer l can also be computed using the Villain approximation in the low-temperature limit.²⁰ This yields the form¹⁹

$$C_l[q] = |C_l[q]| \cdot \exp \left[i \sum_{\vec{r}} q(\vec{r}) \phi_0(\vec{r}, l) \right] \quad (5)$$

for such autocorrelation functions, where $\phi_0(\vec{r}, l)$ represents the zero-temperature configuration of an isolated layer. In the ordered phase, where $\rho_s^{(2D)}(T) > 0$, phase correlations are found to decay algebraically as

$$|C_l[q]| = g_0^{n_+} \cdot \exp \left[\eta_{2D} \sum_{(1,2)} q(1) \ln(r_{12}/r_0) q(2) \right] \quad (6)$$

at the asymptotic limit, $r_{12} \rightarrow \infty$, with a net correlation exponent $\eta_{2D} = k_B T / 2\pi \rho_s^{(2D)}$. Above, g_0 is equal to the ratio of the 2D phase rigidity with its value at zero temperature, while n_+ is equal to half the number of probes in $q(\vec{r})$. Last, r_0 is the natural ultraviolet scale. Phase correlations are short range in the disordered phase, on the other hand, where $\rho_s^{(2D)}(T) = 0$. In particular, the two-point phase autocorrelation function probed at $q(\vec{r}) = \delta_{\vec{r}, \vec{r}_1} - \delta_{\vec{r}, \vec{r}_2}$ retains the form (5), but its magnitude decays exponentially as

$$|C_l(1,2)| = g_0 e^{-r_{12}/\xi_{2D}} \quad (7)$$

at asymptotically large separations $r_{12} \rightarrow \infty$. Here ξ_{2D} denotes the 2D phase correlation length.

We shall assume now that the array of random columnar pins quenches-in unbound dislocations into the triangular vortex lattice of each layer in isolation at zero temperature.²¹ Direct Monte Carlo simulations of the weakly disordered 2D XY model (2) in the Coulomb gas representation indicate that this is indeed the case for sufficiently low frustration²² f , in which case substrate pinning of the 2D vortex lattice by the model grid is sufficiently weak. Direct Monte Carlo simulations of the corresponding layered XY model with sparse columnar pins confirm the above.⁹ Also, both simulations show that the dislocations in the vortex lattice appear either unbound or bound up into neutral pairs. In particular, dislocations do not line up to form low-angle grain boundaries.^{9,22} This is consistent with the incompressible nature of the vortex lattice in the extreme type-II limit. The motion of the most common type of grain boundary requires a combination of glide and climb by the two orientations of edge dislocations of which it is composed.²³ The total number of vortices is *not* conserved when a dislocation climbs, however. This is energetically costly in the incompressible limit. Grain boundaries cannot therefore move in or move out from the surface of the 2D vortex lattice at the extreme type-II limit. Last, a direct evaluation of the 2D phase rigidity, Eqs. (3) and (4), in the zero-temperature limit shows that macroscopic phase coherence persists in the limit of a dilute concentration of unbound dislocations.¹⁸ In particular, the thermal fluctuation of quenched-in dislocations about their home sites results in a vortex contribution to the phase correlation exponent η_{2D} that is small compared to the spin-wave contribution η_{sw} in the limit of a small number N_{df} of such dislocations compared to the number of pinned vortices N_{pin} :¹⁸ $\eta_{vx} \lesssim (N_{df}/N_{pin}) \eta_{sw}$. The phase-coherent vortex lattice state¹⁹ thus survives in the presence of a dilute concentration of dislocations as a hexatic vortex glass state.²⁴ It should melt into a phase-incoherent liquid state at a transition temperature $T_g^{(2D)}$ that is close to the 2D melting temperature of the pristine vortex lattice, $k_B T_m^{(2D)} \cong J/14$, in the present dilute limit $N_{df} \ll N_{pin}$. The existence of such a hexatic vortex glass state is confirmed by direct Monte Carlo simulation of the 2D XY model (2) in the Coulomb gas representation.²² In addition, current-voltage measurements of 2D arrays of Josephson junctions in external magnetic field indicate that the 2D superconducting/normal transition at $T = T_g^{(2D)}$ is second order.²⁵

III. JOSEPHSON COUPLING

Let us now add a weak Josephson coupling energy $-J_z \cos(\Delta_z \phi - A_z)$ to all of the vertical links in between adjacent layers of the three-dimensional (3D) XY model. Here, $J_z = J/\gamma'^2$ is the perpendicular coupling constant that we write in terms of the 2D phase rigidity at zero temperature J and of the model anisotropy parameter $\gamma' > 1$. Also, $A_z = -(2\pi d/\Phi_0) B_{\parallel} x$ is the vector potential that describes the parallel component of the magnetic induction B_{\parallel} , which we take

to be oriented along the y axis. The spacing between adjacent layers is denoted here by d . Study of the field equation that was derived by Bulaevskii and Clem in Ref. 26 for the difference of the superconducting phase across adjacent layers indicates that the effect of magnetic screening on the Josephson coupling can be neglected for Josephson penetration depths $\Lambda_0 = \gamma'a$ that are small compared to the London penetration depth associated with (Josephson) supercurrents that flow across layers, $\lambda_L^{(\perp)} = \gamma\lambda_L$. Here $\gamma = \Lambda_0/d$ is the physical anisotropy parameter. The previous condition is then equivalent to the inequality

$$d \ll \lambda_L, \quad (8)$$

which is notably independent of the anisotropy parameter. We shall further assume that the optimum phase configuration of an isolated layer is *unique*, despite the defective nature of the ordered state. The columnar pins are perfectly correlated across layers, however. This obviously implies that the zero-temperature configurations for each layer in isolation are the same:

$$\phi_0(\vec{r}, l) = \phi_{\Delta'}(\vec{r}). \quad (9)$$

Notice then that a small fraction of the vortex lines are pinned down by the columnar tracks in the case that the pinning per layer is sparse. The layered XY model can then be effectively analyzed in the selective high-temperature limit $k_B T \gg J_z$ through a *partial* duality transformation. It leads to a dilute Coulomb gas (CG) ensemble that describes the nature of the Josephson coupling in terms of dual charges that live on the vertical links in between adjacent layers.¹⁰ Below we review the results of this analysis.

Phase correlations across layers can be computed from the quotient

$$\left\langle \exp \left[i \sum_r p(r) \phi(r) \right] \right\rangle = Z_{\text{CG}}[p] / Z_{\text{CG}}[0] \quad (10)$$

of partition functions for a layered CG ensemble¹⁰

$$Z_{\text{CG}}[p] = \sum_{\{n_z(r)\}} y_0^{N[n_z]} \prod_l C_l[q_l] \cdot e^{-i \sum_r n_z A_z}. \quad (11)$$

Here the dual charge $n_z(\vec{r}, l)$ is an integer field that lives on links between adjacent layers l and $l+1$ located at 2D points \vec{r} . The ensemble is weighted by a product of phase autocorrelation functions for isolated layers l probed at the dual charge that accumulates onto that layer:

$$q_l(\vec{r}) = p(\vec{r}, l) + n_z(\vec{r}, l-1) - n_z(\vec{r}, l). \quad (12)$$

It is also weighted by a bare fugacity y_0 that is raised to the power $N[n_z]$ equal to the total number of dual charges $n_z = \pm 1$. The fugacity approaches $y_0 = J_z / 2k_B T$ in the selective high-temperature regime $J_z \ll k_B T$ reached at large model anisotropy $\gamma' \rightarrow \infty$. Observe now that the phase factors of the correlation functions (5) cancel out in the CG ensemble (11) for probes p that go directly across layers due to the perfect registry across layers of the zero-temperature phase configurations (9). These generalized autocorrelation functions can then be replaced by their magnitude $|C_l[q_l]|$ within the CG ensemble (11).

Expression (10) for phase correlations across layers can be evaluated perturbatively in the decoupled vortex-liquid phase at high temperature $T > T_g^{(2D)}$. Consider, in particular, the gauge-invariant phase difference between any two layers l and l' :

$$\phi_{l,l'}(\vec{r}) = \phi(\vec{r}, l') - \phi(\vec{r}, l) - (l' - l) \cdot A_z(\vec{r}). \quad (13)$$

Because of the cancellation of the zero-temperature phase (9) mentioned above, the lowest-order result for the corresponding phase auto-correlation function in powers of the fugacity y_0 of the dual CG ensemble (11) reads¹⁰

$$\langle e^{i\phi_{l,l+n}} \rangle = (y_0/a^2)^n \prod_{l'=l}^{l+n-1} \left[\int d^2 r_{l'} |C_{l'}(\vec{r}_{l'})| \right] |C_{l+n}(-\sum_{l'=l}^{l+n-1} \vec{r}_{l'})| \quad (14)$$

at zero parallel field, $A_z = 0$. Here $|C(\vec{r}_{12})|$ is the magnitude (7) of the phase autocorrelation function for an isolated layer. At $n=1$, the above expression reduces to Koshelev's formula for the interlayer cosine in the vortex liquid phase:²⁷ $\langle \cos \phi_{l,l+1} \rangle = y_0 \int d^2 r |C_l(\vec{r})| |C_{l+1}^*(\vec{r})| / a^2$. Only short-range phase coherence exists in the disordered phase of isolated layers at $T > T_g^{(2D)}$ over a scale equal to the phase correlation length ξ_{2D} . Koshelev's formula hence yields a result of order $g_0^2 y_0 (\xi_{2D}/a)^2$ for the interlayer cosine, or equivalently

$$\langle \cos \phi_{l,l+1} \rangle \sim g_0^2 (J/k_B T) (\xi_{2D}/\Lambda_0)^2. \quad (15)$$

Here $\Lambda_0 = \gamma'a$ is the Josephson penetration length. The last factor in expression (14) constrains the $2n$ -dimensional integral at larger separations between layers $n \geq 2$. Its effect can be neglected in the asymptotic large- n limit, however. This yields the principal dependence¹⁵

$$\langle e^{i\phi_{l,l+n}} \rangle \propto \left(y_0 \int d^2 r |C(\vec{r})| / a^2 \right)^n \quad (16)$$

for the phase autocorrelation function at large separations between layers $n \rightarrow \infty$. The argument that is raised to the power n on the right-hand side of Eq. (16) above is hence of order $g_0 y_0 (\xi_{2D}/a)^2$. The prefactor that is not shown above on the right-hand side decays only polynomially with the layer separation n (see Refs. 10 and 15). Observe now that the phase correlation length across layers ξ_{\perp} is equal to the interlayer spacing d when the former argument is equal to $1/e$. This occurs at a dimensional crossover^{28,29} field

$$f\gamma'^2 \sim g_0 (J/k_B T) (\xi_{2D}/a_{\text{vx}})^2, \quad (17)$$

given in units of the naive decoupling scale Φ_0/Λ_0^2 , that separates 2D from 3D vortex-liquid behavior.^{10,15} This crossover field is traced out in Fig. 1. At a fixed field $f\gamma'^2$, we generally conclude that phase coherence across a few to many layers is absent in the decoupled vortex-liquid phase that lies at high temperature $T > T_{\times}$. Last, observe that the perturbative result (16) for the phase correlation across a macroscopic number of layers n diverges with the 2D phase correlation length ξ_{2D} at the 2D hexatic vortex glass transition. This implies that a transition to a Bose glass occurs at a critical temperature T_{BG} that lies inside of the window $[T_g^{(2D)}, T_{\times}]$, below which strict long-range phase coherence

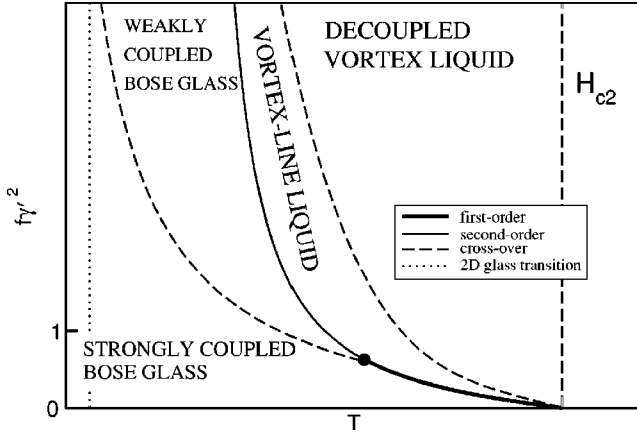


FIG. 1. Shown is the proposed phase diagram under the assumptions that the 2D vortex lattice contains a dilute concentration of unbound dislocations and that it melts through a continuous phase transition (see Ref. 25). The dashed line inside of the Bose glass phase corresponds to the decoupling crossover (22), while the dashed line inside of the vortex-liquid phase corresponds to the 2D-3D crossover, (17). The concentration of in-plane vortices f is held fixed, and a mean-field temperature dependence $J \propto T_{c0} - T$ is assumed. Also, the effect of substrate pinning by the 2D model grid is neglected.

exists across a macroscopic number of layers: $\xi_{\perp} \rightarrow \infty$. Indeed, the above perturbative result for the phase autocorrelation function across layers indicates that the Bose-glass melting transition occurs at a field approximately e times smaller than the 2D-3D crossover field (17), in which case the argument raised to the power n on the right-hand side of Eq. (16) is set to unity instead.

Consider now temperatures below the 2D hexatic vortex-glass transition $T < T_g^{(2D)}$, where the 2D phase autocorrelation functions decay algebraically following Eq. (6). A Hubbard-Stratonovich transformation of the dual CG ensemble (11) reveals³⁰ that it is equivalent to a renormalized Lawrence-Doniach (LD) model that shows no explicit dependence on the component of the magnetic field perpendicular to the layers:

$$E_{LD} = \rho_s^{(2D)} \int d^2r \sum_l \left[\frac{1}{2} (\vec{\nabla} \theta_l)^2 - \Lambda_0^{-2} \cos \theta_{l,l+1} \right], \quad (18)$$

where $\theta_{l,l+1}(\vec{r}) = \theta_{l+1}(\vec{r}) - \theta_l(\vec{r}) - A_z(\vec{r})$. The local phase rigidity of each layer in the LD model is equal to the macroscopic one $\rho_s^{(2D)} = k_B T / 2\pi \eta_{2D}$, while the Josephson coupling in the LD model is set by the Josephson penetration length $\Lambda_0 = \gamma' a$. Also, the LD model inherits the ultraviolet cutoff r_0 from the autocorrelation functions (6) of isolated layers in the ordered phase. A standard thermodynamic analysis¹⁵ then yields that the strength of the local Josephson coupling is given by

$$\langle \cos \phi_{l,l+1} \rangle = y_0 + g_0 \langle \cos \theta_{l,l+1} \rangle. \quad (19)$$

Likewise, the system shows phase rigidity across a macroscopic number of layers equal to¹⁵

TABLE I. Listed are the ‘‘cosine,’’ the phase rigidity, and the phase correlation length across layers for the various regimes inside of the mixed phase of an extremely type-II superconductor at weak Josephson coupling between layers, and with sparse columnar pinning.

| regime/phase | $\langle \cos \phi_{l,l+1} \rangle$ | ρ_s^{\perp} / J_z | ξ_{\perp} / d |
|-----------------------------|-------------------------------------|------------------------|-------------------|
| strongly coupled Bose glass | unity | unity | ∞ |
| weakly coupled Bose glass | fraction | fraction | ∞ |
| vortex-line liquid | fraction | 0 | unity, or greater |
| decoupled vortex liquid | fraction | 0 | fraction |

$$\rho_s^{\perp} / J_z = g_0 \langle \cos \theta_{l,l+1} \rangle. \quad (20)$$

A Gaussian approximation of the LD model (18) yields the result

$$\langle \cos \theta_{l,l+1} \rangle = (r_0 / \Lambda_J)^{\eta_{2D}} \quad (21)$$

for the LD ‘‘cosine,’’ where Λ_J is of order Λ_0 . Notice how the dependence of the LD model with the perpendicular field enters implicitly through the natural ultraviolet cutoff r_0 , which lies somewhere in the range between the scale of the model grid, a , and the intervortex scale a_{vx} . Because we have $g_0(T) = \rho_s^{(2D)}(T) / \rho_s^{(2D)}(0)$, inspection of Eq. (20) implies that 3D scaling of the phase rigidities breaks down at small LD cosines, $\langle \cos \theta_{l,l+1} \rangle \ll 1$. In particular, since $g_0 \leq 1$, Eq. (20) implies that only weak superconductivity can exist across a macroscopic number of layers in such a case: $\rho_s^{\perp} \ll J_z$. By Eq. (21), this requires weak enough Josephson coupling such that the perpendicular field/anisotropy be larger than

$$f\gamma_D'^2 \sim (r_0 / a_{vx})^2 e^{1/\eta_{2D}}. \quad (22)$$

The above decoupling scale is astronomically large, however, at low temperature $\eta_{2D} \ll 1$.

The phase diagram displayed by Fig. 1 in conjunction with the physical properties that are listed by phase in Table I summarize the above predictions. At zero temperature, the correlated nature of the pinning implies that optimum superconductivity exists across layers: $\langle \cos \phi_{l,l+1} \rangle = 1 = \rho_s^{\perp} / J_z$. All of the vortex lines are either pinned or caged in by the columnar tracks in such a case. This phase is therefore a strongly coupled Bose glass. Quasi-2D plastic creep of the vortex lattice that is driven by thermally fluctuating edge dislocations²³ sets in at fields/anisotropies above the decoupling scale $f\gamma_D'^2$ and this degrades the superconductivity across layers in comparison to the result expected from scaling the 2D superconductivity inside of each isolated layer.¹⁵ We shall refer to this phase as a weakly coupled Bose glass. Suppose now that the Bose-glass melting transition is continuous (to be argued for below), and compare the result for the interlayer cosine at low temperatures $T < T_g^{(2D)}$, Eqs. (19) and (21), with the result for the same quantity deep inside the decoupled vortex-liquid phase, Eq. (15) at $\xi_{2D} \sim a_{vx}$. It suggests that the 2D correlation exponent in Eq. (21) should be replaced by an effective exponent of order unity in the vicinity of the Bose-glass melting transition due to the proximity of the vortex-liquid result (15). This indicates that the decou-

pling field/anisotropy (22) is *not* exponentially big there, unlike the low-temperature limit. Last, the dual CG ensemble (11) that was used to obtain the above results is formally identical to the one derived from the layered XY model with no frustration,¹² provided that the 2D transition at $T=T_g^{(2D)}$ is second order. The latter is consistent with current-voltage measurements of 2D arrays of Josephson junctions in external magnetic field.²⁵ The standard 3D XY model exhibits a unique order-disorder transition, however, despite the presence of extreme anisotropy.²⁸ The former equivalence then indicates that the transition between the weakly coupled and strongly coupled Bose glass phases must be a decoupling crossover, and not a true phase transition. The unique 3D XY transition is identified instead with the Bose-glass melting transition, at which point long-range phase coherence across layers vanishes. Last, study of the CG ensemble (11) indicates that a first-order decoupling transition³¹ occurs at temperatures outside of the 2D critical regime, $\xi_{2D} \sim a_{vx}$, due to the absence of a diverging length scale. [See Fig. 1, and see Ref. 10, Eq. (62)].

IV. DISCUSSION AND CONCLUSIONS

The following physical picture emerges from the previous analysis. A dilute concentration of straight lines of edge dislocations that are quenched in by the sparse columnar pins thread the vortex lattice from top to bottom in the zero-temperature limit. The pinning of the vortex lattice is therefore collective.² It has a transverse scale for positional correlations R_c that is set by the separation between unbound dislocations,^{18,32} and a longitudinal scale for positional correlations L_c given by the length of the columnar pins. The Larkin correlation volume^{7,14} $L_c R_c^2$ is then notably independent of the strength of the Josephson coupling. No peak effect in the critical current density is therefore expected as a function of field/anisotropy $B_\perp \Lambda_0^2 / \Phi_0$, in the zero-temperature limit. It is important to observe that a macroscopic scale L_c for correlations of the vortex lattice along the (perpendicular) field direction is not possible in the case of point pinning for weakly coupled layers (see Refs. 18 and 33). Double-kink excitations that lie within the glide planes of the lines of dislocations,²³ as well as bound pairs of dislocations, are excited thermally within isolated layers at elevated temperatures $T \sim T_g^{(2D)}$. In the presence of weak Josephson coupling, however, such excitations are possible only at temperatures above the decoupling crossover scale T_D , where they act to degrade phase coherence across layers. The integrity of the thermally fluctuating lines of dislocations is then finally lost at the Bose-glass melting transition temperature T_{BG} that lies above T_D , at which point macroscopic phase coherence across layers also vanishes. Last, the nature of phase correlations in isolated layers, Eqs. (5) and (9), indicates that the dilute concentration of vortex lines that are pinned to columnar defects at zero temperature remain pinned up to the temperature that marks the end of the 2D critical regime, $\xi_{2D} \sim a_{vx}$ (see Refs. 3 and 5). Both the weakly coupled Bose glass regime and the 3D vortex-line-liquid regime that straddle Bose-glass melting at $T=T_{BG}$ lie below this cutoff in temperature at field/anisotropy in the

quasi-2D regime $f\gamma^2 > 1$. They can therefore both be properly identified as *interstitial* phases,^{6,9} where the remaining lines of unpinned vortices show considerable thermal fluctuations in the form of plastic creep.

Recent Monte Carlo simulations of the same XY model studied here for the mixed phase of layered superconductors with sparse columnar pins also find an intermediate regime between the Bose glass and the vortex liquid that exhibits relatively low phase rigidity across layers.⁹ The present calculation strongly suggests the identification of this intermediate phase with the weakly coupled Bose glass that is shown in Fig. 1 and that is listed in Table I. Measurements of the tilt modulus of the vortex lattice by these Monte Carlo simulations indicate that the boundary that separates the strongly coupled Bose glass from the intermediate phase represents a true phase transition, however, as opposed to a crossover. This may be an artifact of the XY model grid, which could be checked by performing simulations over finer model grids (or smaller f). Last, the anisotropy that was used in the simulations reported in Ref. 9 was only moderate: $f\gamma^2=1$. Equation (17) then predicts that a decoupled vortex liquid emerges outside of the 2D critical region, $\xi_{2D} \sim a_{vx}$, at high temperature $k_B T \gg J$.

The effects of sparse correlated pinning on the mixed phase of layered superconductors have also been studied recently in experiments on Bismuth-based high-temperature superconductors that were irradiated to produce columnar tracks. An intermediate “nanoliquid” phase is observed at temperatures and perpendicular magnetic fields that lie just above the melting line of the vortex lattice.⁵ This intermediate vortex liquid phase shows a resistivity ratio between the perpendicular field direction and the parallel layer direction that is at least an order of magnitude smaller than that shown by the more anisotropic vortex liquid phase that lies at higher temperature. It is very possible then that the boundary separating the two liquid phases observed experimentally is just the dimensional cross-over line (17) shown in Fig. 1, at which point the phase correlation length across layers becomes equal to the interlayer spacing. This identification requires that the vortex lattice of the unirradiated crystal *sublimate* into the decoupled vortex liquid phase, however, since the intermediate 3D liquid phase is absent in such case.⁵ A direct sublimation transition between a vortex solid and a decoupled vortex-liquid phase is in fact consistent with previous experimental studies of the unirradiated system.¹⁶ It is also predicted to occur theoretically in the vortex lattice state of pristine layered superconductors at sufficiently weak Josephson coupling, provided that the vortex lattice in isolated layers melts through a first-order transition.¹⁵ Last, the bismuth based high-temperature superconductor that was studied experimentally in Ref. 5 is highly anisotropic,⁸ with a zero temperature London penetration depth of about $\lambda_L(0) = 0.2 \mu\text{m}$, and with a layer spacing of $d=1.5 \text{ nm}$. The first condition (1) for the extreme type-II limit then yields a threshold field of 500 G at zero temperature, which is in the general vicinity of the observed nanoliquid phase. The second condition (8) for the extreme type-II limit, on the other hand, is easily met.

A polycrystalline vortex lattice phase is also observed below the melting line of the same bismuth-based high-

temperature superconductor.⁴ As discussed previously at the end of Sec. II, grain boundaries do not occur in the vortex lattice of the frustrated XY model (2) used here,^{9,22} since it describes incompressible vortex matter. More generally however, these measurements find that the vortex solid melts through a second-order phase transition at perpendicular magnetic fields above a certain critical point.³ This is consistent with the 3D-XY universality class for the Bose-glass melting transition that was argued for at the end of the previous section. The same set of experiments find that the melting transition of the vortex lattice becomes first-order at fields below the critical point.³ This phenomenon is then consistent with the first-order decoupling transition^{10,31} argued for at the end of the previous section at temperatures outside of the 2D critical regime, $\xi_{2D} \sim a_{vx}$ (see Fig. 1).

In conclusion, an intermediate Bose glass phase that shows weak superconductivity across layers, $\rho_s^+ \ll J_z$, exists

at weak coupling in extremely type-II layered superconductors in external magnetic field, with only a sparse arrangement of columnar pins oriented perpendicular to the layers. This phase is predicted to melt into a 3D vortex-line liquid that shows phase coherence across layers on length scales that are large compared to the spacing between adjacent layers. We believe that the phase transition observed recently inside of the vortex-liquid regime of high-temperature superconductors pierced by sparse columnar tracks⁵ reflects layer decoupling by such a 3D vortex-line liquid.¹⁵ This proposal is consistent with the absence of such a dimensional crossover transition in the vortex-liquid phase of the unirradiated (pristine) superconductor.¹⁶

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