Crossover to non-Fermi-liquid spin dynamics in cuprates

J. Bonča,[1,2] P. Prelovšek,[1,2] and I. Sega[1]
[1]*J. Stefan Institute, SI-1000 Ljubljana, Slovenia*
[2]*Faculty of Mathematics and Physics, University of Ljubljana, SI-1000 Ljubljana, Slovenia*
(Received 1 September 2004; revised manuscript received 16 September 2004; published 6 December 2004)

The antiferromagnetic spin correlation function $S_{\mathbf{Q}}$, the staggered spin susceptibility $\chi_{\mathbf{Q}}$, and the energy scale $\omega_{FL}=S_{\mathbf{Q}}/\chi_{\mathbf{Q}}$ are studied numerically within the *t-J* model and the Hubbard model, as relevant to cuprates. It is shown that ω_{FL}, related to the onset of the non-Fermi-liquid spin response at $T>\omega_{FL}$, is very low in the regime below the "optimum" hole doping $c_h<c_h^*\sim 0.16$, while it shows a steep increase in the overdoped regime. A quantitative analysis of the NMR spin-spin relaxation rate $1/T_{2G}$ for various cuprates reveals a similar behavior, indicating a pronounced crossover between a Fermi-liquid and a non-Fermi-liquid behavior as a function of doping.

DOI: 10.1103/PhysRevB.70.224505 PACS number(s): 74.72.−h, 75.20.−g, 71.27.+a

I. INTRODUCTION

The understanding of the phase diagram of cuprates continues to exemplify one of the major theoretical and experimental challenges.[1] Besides superconductivity (SC) and antiferromagnetic (AFM) ordering, several regimes with distinct electronic properties have been identified within the normal metallic phase. The behavior of the spin degrees of freedom, which are the subject of this paper, has been intensively studied using the inelastic neutron scattering[2,3] (INR) and NMR relaxation[4] experiments. They clearly reveal that in underdoped cuprates magnetic properties are not following the usual Fermi-liquid (FL) scenario within the metallic state above the SC transition $T>T_c$.

In a normal FL one expects a T-independent dynamical spin susceptibility $\chi''_{\mathbf{q}}(\omega)$ at low T,ω. However, INS results show that $\mathbf{q}$-integrated spin susceptibility exhibits in a broad range of ω and T an anomalous, but universal behavior $\chi''_L(\omega)\propto f(\omega/T)$, first established in $La_{2-x}Sr_xCuO_4$ (LSCO) at low doping.[3,5] This behavior can be even followed to lowest T in $YBaCu_3O_{6+x}$ (YBCO), where T_c has been suppressed by Zn doping.[6] At the same time, low-energy INS reveals at low T the saturation of the inverse AFM correlation length $\kappa=1/\xi$, at least in YBCO (Ref. 2) and in LSCO (Refs. 3 and 5) at low doping. The anomalous T dependence of the ^{63}Cu NMR spin-lattice relaxation rate $1/T_1$ and of the spin-spin relaxation rate $1/T_{2G}$ in underdoped cuprates is in general compatible with INS,[4] in particular $1/(T_1T)\propto \chi''_L(\omega,T)/\omega|_{\omega\to 0}\propto 1/T$.

On the other hand, cuprates at optimum doping and, moreover, in the overdoped regime show a strong reduction of the spin response at low energies ω. This is evident from the loss of INS intensity in the normal state and low NMR relaxation rates $1/T_1$, $1/T_{2G}$. At the same time, NMR confirms the approach to the normal FL behavior, $1/(T_1T)\sim$ const and $1/T_{2G}\sim$ const.[4] An anlogous message arises from the analysis of cuprates doped with nonmagnetic Li and Zn, where the impurity-induced spin susceptibility varies as $\propto 1/(T+T_K)$—i.e., with a Kondo-like behavior with a characteristic temperature $T_K(c_h)$ (Ref. 7)—where $T_K\sim 0$ in the underdoped regime, whereas it shows a strong increase and more FL-like behavior in the overdoped regime. There are also indications from transport properties and angle-resolved photoemission spectroscopy (ARPES) that the normal FL behavior is approached in the overdoped regime. Recent ARPES data for the bilayer splitting in BiSrCaCuO (BSCCO) are interpreted as an evidence for coherent (FL-like) electronic excitations for $T<T_X$ (Ref. 8) in the overdoped regime, where $T_X(c_h)$ shows a steep increase with doping. However, alternative ARPES measurements reveal splitting even in underdoped BSCCO.[9]

From the point of theoretical understanding, an approach to a FL behavior in the overdoped regime far from a metal-insulator transition seems plausible; nevertheless, solid theoretical evidence is still missing. A crossover from a strange metal to a coherent metal phase is, e.g., predicted within the slave-boson approach.[10] A frequently invoked interpretation is given in terms of the quantum critical point (QCP) at optimum doping c_h^* (masked, however, at low T by the SC phase), dividing the FL phase at $c_h>c_h^*$ and a (singular) non-Fermi-liquid (NFL) metal at $c_h<c_h^*$. While such a concept is well established in spin systems,[11] its application to metallic cuprates is controversial due to the absence of a critical length scale—e.g., $\xi(T\to 0)\to\infty$. Low-energy spin dynamics as emerges from INS and in particular from NMR experiments has been extensively analyzed within phenomenological theory,[12] describing a FL close to an AFM instability, where the spin-fluctuation energy is related to the characteristic FL scale, as discussed further on. Evidence for a NFL to FL crossover in the self-energy has also been found in the Hubbard model within the FLEX approximation.[13]

The present authors recently showed that an anomalous ω/T scaling, as observed at low doping, emerges from a general approach to $\chi_{\mathbf{q}}(\omega)$ under a few basic requirements:[14] (a) damping of the collective AFM mode in the normal state is large, and (b) equal-time correlations and the corresponding inverse correlation length $\tilde{\kappa}$ are finite and saturate at low T. It has been shown that ω/T scaling appears for $T>\omega_p$, where ω_p represents characteristic spin-fluctuation or related FL scale and can be very small in the low-doping regime.[14] In this paper we define an analogous FL scale ω_{FL} suitable for numerical studies and analyze its behavior within models relevant to cuprates—i.e., the planar *t-J* model and the Hubbard model. It can be also extracted directly from NMR-relaxation and INS experiments on cuprates, whereby the results obtained for ω_{FL} indicate a quite sharp but continuous crossover behavior between the NFL and FL regime as a function of doping.

In Sec. II we define and discuss the FL scale $\omega_{FL}(T)$. Section II is divided into three subsections where we present results for this quantity obtained numerically for the planar *t-J* model and the Hubbard model, respectively, and discuss the analysis of experimental NMR-relaxation and INS data on cuprates. Conclusions and the relation to other experiments and scenarios are discussed in Sec. III.

II. FERMI-LIQUID SCALE

Let us assume that the dominant collective magnetic mode is the AFM one at $\mathbf{Q}=(\pi,\pi)$. Considering the dynamical susceptibility $\chi_{\mathbf{q}}(\omega)$ in the normal state we note that in general the damping γ of the collective $\mathbf{Q}=(\pi,\pi)$ mode in the normal state is large, leading to an overdamped mode. At the same time, equal-time correlations $S_{\mathbf{Q}}=\langle S^z_{-\mathbf{Q}}S^z_{\mathbf{Q}}\rangle$ and the corresponding inverse correlation length $\tilde{\kappa}$ seem to saturate at low T. A nontrivial dependence of the static $\chi_{\mathbf{Q}}(T)$ then follows[14] from the fluctuation-dissipation relation

$$\frac{1}{\pi}\int_0^\infty d\omega\, \mathrm{cth}\frac{\omega}{2T}\chi''_{\mathbf{q}}(\omega)=S_{\mathbf{q}}. \tag{1}$$

Note that we use $\hbar=1$ and define $\chi_{\mathbf{q}}(\omega)$ in units of $g^2\mu_B^2$. Relation (1) leads to a ω/T scaling for $T>\omega_p$ where

$$\omega_p \sim \gamma e^{-2\zeta}, \quad \zeta \propto \gamma/\tilde{\kappa}^2. \tag{2}$$

Within such an approach it is natural that the $(T=0)$ spin fluctuation scale ω_p remains finite within the whole paramagnetic regime. Assuming that it is the lowest energy scale in the normal state it plausibly plays the role of the relevant FL scale ω_{FL}. Namely, for $T<\omega_{FL}$ one expects the FL behavior with quite T-independent $\chi_{\mathbf{q}}(\omega)$, while for $T>\omega_{FL}$ the T dependence becomes essential. Due to the strong dependence on ζ, $\omega_p(c_h)$ can show quite a sharp crossover from very small values in the underdoped regime to a large increase in overdoped systems.

One possibility is to obtain ω_{FL} from the full T dependence of various magnetic quantities, in particular from static $\chi_{\mathbf{Q}}(T)$ and $S_{\mathbf{Q}}(T)$. It is evident that in the NFL regime $T>\omega_{FL}$ a relation follows from Eq. (1):

$$\frac{S_{\mathbf{Q}}}{T\chi_{\mathbf{Q}}}=\left[1-\frac{\Delta}{S_{\mathbf{Q}}}\right]^{-1}, \tag{3}$$

which evolves into the "classical" relation for $\Delta \ll S_{\mathbf{Q}}$. Note that $\Delta(T)$ arises from Eq. (1) as the integral over the large-ω tail $\chi''_{\mathbf{Q}}(\omega>T)$. We are interested in the low-T regime in the paramagnetic phase where $S_{\mathbf{Q}}(T)$ already saturates. The saturation is quite evident from the numerical analysis of various models.[1,15] Equation (3) indicates that even constant $S_{\mathbf{Q}}$ can be compatible with T-dependent $\chi_{\mathbf{Q}}(T)\propto 1/T$ which appears to be the essence of the NFL regime in cuprates. In contrast, one expects a finite $\chi_{\mathbf{Q}}(T\to 0)$ within the FL regime.

It follows that the characteristic energy scale of spin fluctuations can be defined as

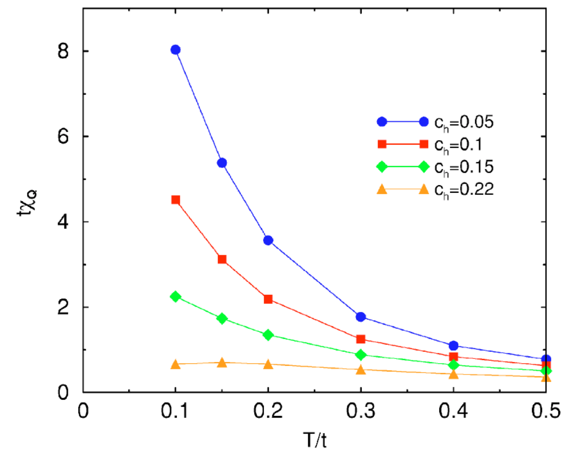

FIG. 1. (Color online) AFM susceptibility $\chi_{\mathbf{Q}}$ vs T for different doping values c_h within the *t-J* model.

$$\omega_{FL}(T)=\frac{S_{\mathbf{Q}}(T)}{\chi_{\mathbf{Q}}(T)}, \tag{4}$$

with the corresponding $T=0$ limit $\omega_{FL}(0)$. Note that $\omega_{FL}(0)$ is just the first frequency moment of the shape function $\chi''_{\mathbf{Q}}(\omega,T=0)/\omega$ for $\omega>0$,

$$\omega_{FL}(0)=\langle\omega\rangle=\frac{2}{\pi\chi_{\mathbf{Q}}}\int_0^\infty \chi''_{\mathbf{Q}}(\omega)d\omega. \tag{5}$$

On the other hand, one can extract ω_{FL} also from experiments, in particular from NMR $1/T_{2G}$ relaxation data, which give rather straightforward information on $\chi_{\mathbf{Q}}(T)$.

A. *t-J* model

Let us first consider the *t-J* model

$$H=-t\sum_{\langle ij\rangle s}\tilde{c}^\dagger_{js}\tilde{c}_{is}+J\sum_{\langle ij\rangle}\left(\mathbf{S}_i\cdot\mathbf{S}_j-\frac{1}{4}n_in_j\right), \tag{6}$$

with the nearest-neighbor hopping on a square lattice, which we analyze for $J/t=0.3$, as relevant for cuprates (for comparison with cuprates we use $t\sim 400$ meV). Results for $S_{\mathbf{Q}}(T)$ and $\chi_{\mathbf{Q}}(T)$ are evaluated using the finite-T Lanczos method[16] (FTLM). In this way we analyze systems with $N=18$ sites for arbitrary hole doping $c_h=N_h/N$ and with $c_h\leqslant 3/20$ for $N=20$. It should be also noted that in general FTLM results are rather insensitive to finite-size effects for $T>T_{fs}$, whereby for systems considered $T_{fs}\geqslant 0.1t$ (Ref. 16).

In Fig. 1 we present results for the T dependence of $\chi_{\mathbf{Q}}$ for several dopings c_h ranging from low to high doping. We notice that $\chi_{\mathbf{Q}}$ is small and essentially T independent for $c_h>0.2$. On the other hand, in the low-doping regime $\chi_{\mathbf{Q}}$ is strongly T dependent down to lowest reliable $T\sim T_{fs}$ where it also reaches large values. An analogous or even more transparent message follows from Fig. 2 where we present $\tilde{\chi}=4T\chi_{\mathbf{Q}}$ as a function of c_h for various $T>T_{fs}$. Note that the limiting value within the *t-J* model is $\tilde{\chi}(T\to\infty)=1-c_h$. Two distinct regimes become immediately evident from Fig. 2. The crossing of curves $\tilde{\chi}(c_h)$ with different T can be used as the definition of the "optimum" doping $c_h^*\sim 0.16$, whereby it

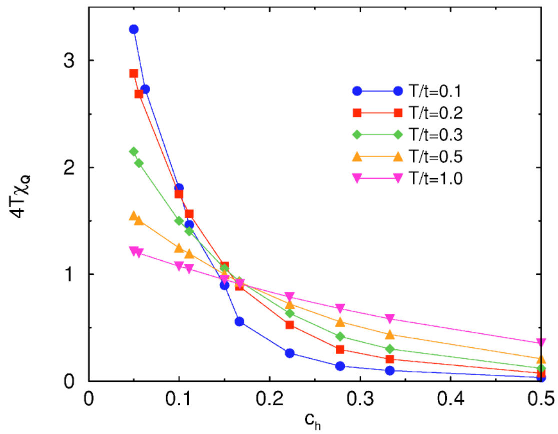

FIG. 2. (Color online) $4T\chi_{\mathbf{Q}}$ vs doping c_h for various T/t.

is indicative that the same value is obtained analyzing cuprates with highest T_c (Ref. 17). In the underdoped regime $\tilde{\chi}$ increases by lowering T (down to reachable $T \sim T_{fs}$) and appears to saturate to the NFL behavior, Eq. (3), consistent with the anomalous ω/T scaling.[14] On the other hand, at $c_h > c_h^*$ the tendency of $\tilde{\chi}(T)$ is the opposite; i.e., $\chi_{\mathbf{Q}}(T)$ saturates for $T<J$, indicating a "normal" FL behavior. If $\tilde{\chi}(c_h)$ curves would, even for lowest T, indeed cross at $c_h=c_h^*$, we would have been dealing with a singularity resembling a QCP with diverging $\chi_{\mathbf{Q}}(T\to 0)\propto 1/T$. Moreover, $\chi_{\mathbf{Q}}(T\to 0)$ would have been divergent in the whole regime $c_h<c_h^*$. Although the present results cannot exclude this possibility, the deviation (shift to the left) visible at lowest $T=0.1t$ is more in accordance with a crossover between FL and NFL regimes.

In Fig. 3 we show corresponding FTLM results for $\omega_{FL}(c_h)$ at $T \leqslant J$. Besides $T \geqslant T_{fs} \geqslant 0.1t$ we present also values extrapolated to $T\to 0$. Note that in this T window $S_{\mathbf{Q}}(T)$ is essentially T independent, with its extrapolated values being in close agreement with the $T=0$ results obtained via the usual Lanczos technique. Such a behavior is consistent with the fact that the AFM correlation length ξ saturates for $T\to 0$ and remains shorter than the system size $\xi<L$, at least for $c_h>0.05$. $S_{\mathbf{Q}}$ results are shown in the inset of Fig. 3 and overall follow surprisingly well the linear variation

$$1/S_{\mathbf{Q}} = K c_h, \quad K \sim 15. \tag{7}$$

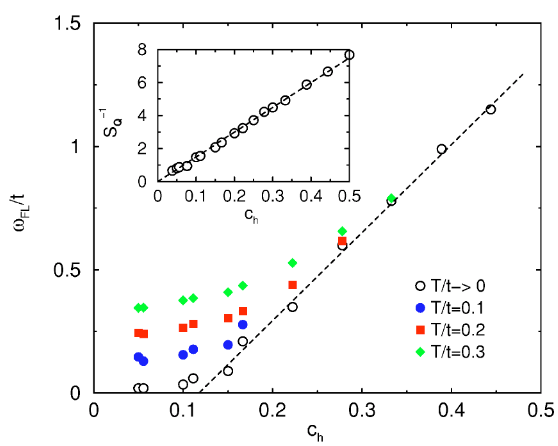

FIG. 3. (Color online) FL scale ω_{FL}/t vs c_h, obtained for the t-J model using the FTLM for $T>0$ and their $T\to 0$ extrapolated values. The inset shows $T=0$ results for $1/S_{\mathbf{Q}}$ vs c_h. Dashed lines are guide to the eye only.

In contrast, the FL scale ω_{FL} reveals a nonuniform variation with doping. Again, for $c_h>c_h^*$, ω_{FL} is already rather T independent for $T<J$. On the other hand, in the regime $c_h<c_h^*$ we find a strong T dependence of ω_{FL} even at $T\sim T_{fs}$ with an approximate behavior

$$\omega_{FL} \sim T + \omega_{FL}(0). \tag{8}$$

Note that at high enough $T>\omega_{FL}(0)$ the linear variation $\omega_{FL}(T)\sim T$ naturally follows from Eq. (3). We can summarize results in Fig. 3 as follows: (a) in the overdoped regime $\omega_{FL}(0)\sim\alpha(c_h-c_{h0})$ with $c_{h0}\sim 0.12$ and a large slope $\alpha \sim 3.5t \sim 1.4$ eV, and (b) in the underdoped regime our results indicate on a smooth crossover to very small $\omega_{FL}(0) \ll J$.

B. Hubbard model

The alternative relevant model is the Hubbard model on a square lattice,

$$H = -t\sum_{\langle ij\rangle s} (c_{is}^{\dagger}c_{js} + \text{H.c.}) + U\sum_i n_{i\uparrow}n_{i\downarrow}, \tag{9}$$

which in the case of strong Coulomb repulsion $U \gg t$ and close to half-filling maps onto the t-J model with $J=4t^2/U$. We calculate $S_{\mathbf{Q}}$ in the ground state as a function of hole doping c_h within the Hubbard model on a square lattice and at $U=8t$ using the constrained-path quantum Monte Carlo (CPMC) method.[18] In this method, the ground-state wave function is projected from a known initial wave function by a branching random walk in the overcomplete space of Slater determinants. Since the method is most efficient in closed-shell cases, we extend our calculations to various tilted square lattices where the number of sites, N, ranges between $N=34$ and $N=164$. The susceptibility $\chi_{\mathbf{Q}}=\partial m_{\mathbf{Q}}/\partial B_{\mathbf{Q}}$ is calculated by computing the sublattice magnetization $m_{\mathbf{Q}}$ induced by a small staggered magnetic field $B_{\mathbf{Q}}$.

Our results for $S_{\mathbf{Q}}$ again reveal a linear variation $1/S_{\mathbf{Q}} \sim Kc_h$ with $K\sim 14$. Such a result is in qualitative agreement with previous QMC calculations for $U/t=4$ (Ref. 1), where in the latter case $K\sim 14.3$. In Fig. 4 we present the corresponding $\omega_{FL}(0)$. The qualitative behavior of ω_{FL} is similar to the result within the t-J model, Fig. 3. In the overdoped regime one can again approximate the variation of ω_{FL} as linear, with $c_{h0}\sim 0.1$ and $\alpha\sim 4.8t$, while for $c_h<c_{h0}$, ω_{FL} becomes very small. Altogether, obtained ω_{FL} does not differ much from that within the t-J model. Note that the rather surprising agreement in the thermodynamic properties of the t-J and Hubbard models at $T>0$ has been previously found using the FTLM in a large doping regime for strong enough correlations—i.e., $U/t \geqslant 8$ (Ref. 19). Still, the crossover within the Hubbard model seems to be less abrupt, which could be plausibly attributed to weaker correlations at $U=8t$. In Fig. 4 we display also the corresponding free fermion result. We notice that on approaching the empty band $c_e=1-c_h\to 0$ both curves converge. However, close to half-filling there is a huge qualitative difference.

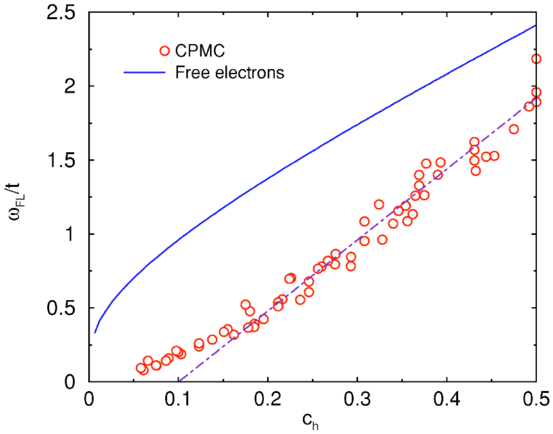

FIG. 4. (Color online) FL scale ω_{FL}/t vs doping c_h, as obtained via the CPMC method for the Hubbard model with $U/t=8$; the dashed line is a guide to the eye.

C. NMR relaxation analysis

Let us finally estimate $\chi_{\mathbf{Q}}(T)$ and consequently ω_{FL} directly from experiments on cuprates. Within the normal state we use the results for the NMR spin-spin relaxation time T_{2G}, obtained from the ^{63}Cu spin-echo decay, related to static $\chi_{\mathbf{q}}$ as[12]

$$\frac{1}{T_{2G}^2} = \frac{0.69}{8}\left[\frac{1}{N}\sum_{\mathbf{q}}[F(\mathbf{q})\chi_{\mathbf{q}}]^2 - \left(\frac{1}{N}\sum_{\mathbf{q}}F(\mathbf{q})\chi_{\mathbf{q}}\right)^2\right]. \quad (10)$$

Assuming that $\chi_{\mathbf{q}}$ is peaked at commensurate $\mathbf{q}=\mathbf{Q}$ and can be described by a Lorentzian form

$$\chi_{\mathbf{q}} = \frac{\chi_{\mathbf{Q}}\kappa^2}{(\mathbf{q}-\mathbf{Q})^2+\kappa^2} \quad (11)$$

(e.g., consistent with INS in YBCO) with $\kappa \ll \pi$, the second term in Eq. (10) can be neglected and the form factor replaced by $F(\mathbf{Q})$. This leads to the simplified relation

$$\frac{1}{T_{2G}} \sim 0.083\kappa F(\mathbf{Q})\chi_{\mathbf{Q}}. \quad (12)$$

$1/T_{2G}$ relaxation rates have been measured and summarized in Ref. 4—i.e., from underdoped to optimally doped YBCO with $0.63<x<1$, underdoped $YBa_2Cu_4O_8$, nearly optimum doped $Tl_2Ba_2Ca_2Cu_3O_{10}$ (Tl-2223), and overdoped $Tl_2Ba_2CuO_{6+\delta}$ (Tl-2201), whereby the normalization with corresponding $F(\mathbf{Q})$ has been already taken into account (see Fig. 8b in Ref. 4). Note that κ relevant to $\chi_{\mathbf{q}}$ is the one appropriate for low-ω spin dynamics, as measured directly by INS (plausibly $\kappa<\tilde{\kappa}$). For YBCO, $\kappa(x)$ has been summarized in Ref. 20. For the cuprates considered here appropriate hole concentrations c_h have been estimated in Ref. 17. Assuming a continuous variation of $\kappa(c_h)$ we determine also κ for $YBa_2Cu_4O_8$, Tl-2223, and Tl-2201 (for the latter we take $\kappa=1.2/a_0$), not available experimentally. In this way we evaluate $\chi_{\mathbf{Q}}(T)$, presented in Fig. 5. We notice qualitative and even quantitative correspondence with the model results in Fig. 1 (note that $t\sim400$ meV). In particular, for overdoped cuprates $\chi_{\mathbf{Q}}$ is FL like—i.e., small and essentially T independent—while for underdoped cuprates $\chi_{\mathbf{Q}}$ is strongly T dependent even at lowest $T>T_c$.

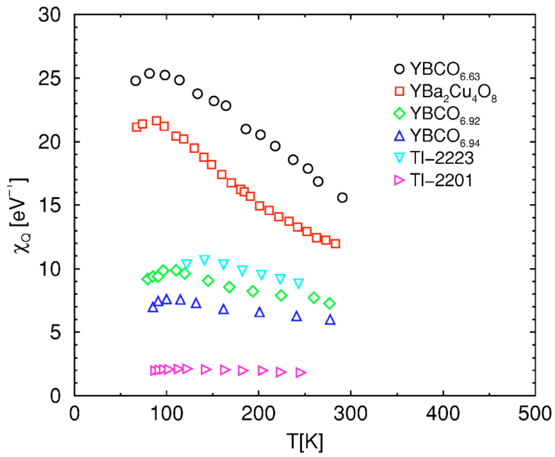

FIG. 5. (Color online) $\chi_{\mathbf{Q}}$ in units of $g^2\mu_B^2$ vs T, as evaluated from the NMR relaxation rate $1/T_{2G}$ (Ref. 4) and the INS width κ (Ref. 20) for various cuprates.

Equal-time correlations $S_{\mathbf{Q}}$ are so far not directly accessible by INS. As shown before they are nearly model independent, so we assume here the t-J model results to finally extract corresponding $\omega_{FL}(T)$ as presented in Fig. 6 for various cuprates. For T well above T_c, $\chi_{\mathbf{Q}}(T)$ extracted from $1/T_{2G}$ follows the Curie-Weiss behavior—i.e.,

$$\chi_{\mathbf{Q}}(T) \propto \frac{1}{T+\Theta}. \quad (13)$$

Such a behavior emerges also within our analytical approach in the scaling regime.[14] Hence, for $T>150$ K, we can well parametrize

$$\omega_{FL}(T) \sim \omega_{FL}(0)\left(1+\frac{T}{\Theta}\right) \quad (14)$$

and present in the inset of Fig. 6 the doping dependence of $\omega_{FL}(0)$ and Θ. It is evident that $\omega_{FL}(0)\sim\Theta$. This is consistent with Eq. (3) which requires that, for higher T, $\omega_{FL}\sim T$.

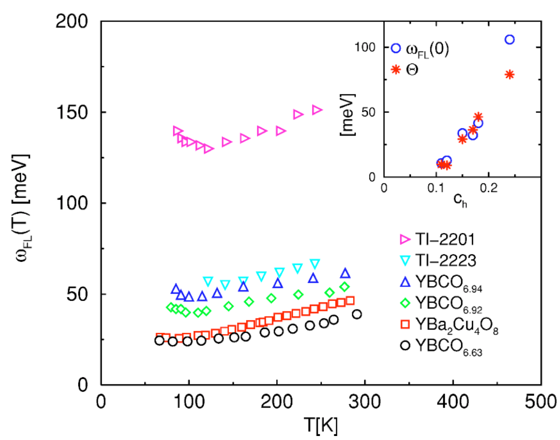

FIG. 6. (Color online) ω_{FL} vs T, for various cuprates. The inset shows the extrapolated scales $\omega_{FL}(0)$ and Θ vs doping c_h.

At the same time, it is a nontrivial confirmation that assumed $S_{\mathbf{Q}}$ is consistent with experimental NMR and INS results. The derived $\omega_{FL}(0)$ is also well in agreement with the model results in Figs. 3 and 4, in particular regarding the large slope in the overdoped regime and a clear change of scale between the underdoped and overdoped cuprates.

III. CONCLUSIONS

To summarize, we have presented evidence, based both on numerical results within *t*-*J* and Hubbard models as well as on the analyses of NMR and INS experimental data on cuprates, that the FL scale ω_{FL} exhibits pronounced crossover between a steep increase in the overdoped regime and very low $\omega_{FL} \ll J$ in the underdoped regime for $c_h < c_h^*$. Note that in the latter regime within cuprates one can easily reach values $\omega_{FL}(0)$ smaller than T_c. This can explain why anomalous NFL scaling of the spin response is observed throughout the normal phase at $T > T_c$. Also note that the electron self-energy $\Sigma_{\mathbf{k}}(\omega)$, usually identifying the FL behavior, is expected to become NFL like for ω, $T > \omega_{FL}$ due to its relation to $\chi_{\mathbf{q}}(\omega)$ (Ref. 21).

Our results are at least in qualitative agreement with other experimental evidence for the existence of crossover to the FL behavior in cuprates—e.g., with the FL scale T_X and its doping dependence—as revealed by ARPES on BSCCO.[8] A similar doping-dependent scale T_K, analogous to a Kondo scale in metals, arises from the analysis of the local-moment susceptibilities in YBCO with in-plane nonmagnetic Li and Zn impurities.[7] Experiments show an abrupt and steep increase of T_K on approaching the optimum doping. It has been recently shown that the impurity-induced susceptibility is related to bulk $\chi_{\mathbf{q}}$ in a uniform system; hence, T_K should be related to Θ (Ref. 22).

Still, our numerical results cannot exclude the possibility of the existence of a QCP-like transition at $T=0$. From our analysis, the latter can be present at the point where $\omega_{FL}(0)$ vanishes on approaching from the overdoped side—i.e., in our model systems at $c_h \sim c_{h0} < c_h^*$. An analogous interpretation might follow also from experimental values in Fig. 6, as well as from results on the Kondo temperature $T_K(c_h)$ (Ref. 7). The main obstacle to an ordinary QCP scenario is that there is no evidence for an ordered AFM phase for $c_h < c_{h0}$, neither from calculated $S_{\mathbf{Q}}$ within the *t*-*J* and Hubbard models nor from experiments. As our analysis shows, $\omega_{FL}(0)$ remains finite throughout the normal phase at all dopings c_h down to the onset of the ordered AFM phase at $c_h^{AFM} < c_{h0}$. The experimental distinction between the sharp $T=0$ (QCP) transition and the present crossover scenario is that in principle $\omega_{FL}(0) > 0$ in the normal phase even in the heavily underdoped regime; hence, one should be able to detect this experimentally by suppressing the SC phase—e.g., as investigated with INS on YBCO system.[6] However, the theory[14] reveals that $\omega_{FL}(0) \propto \omega_p$ can be extremely small in the underdoped regime.

In our attempt to speculate on the origin of the crossover and on its location, we note that within the model c_h^* coincides with the "optimum" doping regime, characterized by the largest entropy[16,19] and degeneracy of states [in cuprates corresponding also to highest T_c (Ref. 17)]. The optimum doping c_h^* is furthermore roughly governed by the ratio $c_h^* \propto J/t$—i.e., by the interplay between the spin exchange and the itinerant electron character.

ACKNOWLEDGMENT

This work was sponsored by the Slovene Ministry of Education, Science and Sports under Grant No. P1-0044.

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