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Deconfined quantum criticality and Néel order via dimer disorder

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Recent results on the nature of the quantum critical point between Néel and valence bond solid (VBS) ordered phases of two-dimensional quantum magnets are examined by an attack from the VBS side. This approach leads to an appealingly simple physical description, and further insight into the properties of the transition.

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Recent theoretical work[1,2] on quantum phase transitions in two dimensional spin-$1/2$ quantum antiferromagnets has unearthed some interesting phenomena dubbed "deconfined quantum criticality." The theory of such deconfined quantum critical points is described in terms of excitations that carry fractionalized quantum numbers which interact through an emergent gauge field. A precise characterization of the deconfinement is provided by the emergence of an extra global topological $U(1)$ symmetry not present at a microscopic level. This symmetry leads to an extra conservation law at the critical fixed point that is conveniently interpreted as the conservation of a gauge flux.

The most prominent example of such a deconfined quantum critical point arises at the transition between Néel-and valence bond solid (VBS) ordered phases of spin-$1/2$ magnets on a square lattice. A direct second-order transition is possible between these two phases despite their very different broken symmetries, and in contrast to naive expectations based on the Landau paradigm for phase transitions. Previous results[1,2] on this transition have been based primarily on an attack starting from the Néel ordered side. Here we will take the alternate approach of attacking from the VBS side. This approach provides for an appealingly simple physical description of the transition.

The Néel ordered state is described by an $O(3)$ vector order parameter. The VBS state, on the other hand, is described by a Z_4 clock order parameter. The four degenerate ground states associated with the Z_4 order parameter are illustrated in Fig. 1 for a specific VBS state in which the valence bonds have lined up in columns.[3] A naive approach to the transition from the Néel side would associate the critical fixed point with the usual $O(3)$ fixed point in $D=2+1$ dimensions. This expectation is incorrect. Similarly a naive approach to the transition from the VBS side would lead one to expect a critical fixed point in the Z_4 universality class in $D=2+1$. This expectation is again incorrect. As is well known, the *classical* Z_4 transition in three dimensions is actually in the $D=3$ *XY* universality class as the fourfold clock anisotropy is irrelevant at the latter fixed point (for instance, see Ref. 4). The critical theory discussed in Refs. 1 and 2 is emphatically not in the three-dimensional (3D) *XY* universality class.

Why do these naive expectations fail? The answer is rooted in the observation that the topological defects in either order parameter carry nontrivial quantum numbers. When the defects in one order parameter, say the Néel vector, proliferate and condense they kill long-range Néel order. At the same time, the quantum numbers they carry induce a different broken symmetry. The nontrivial structure of the defects is inherently quantum mechanical and is not captured in naive macroscopic treatments of the broken symmetry state. For the Néel ordered states, the structure of the defects (known as hedgehogs),[5] and their role in producing the VBS ordered paramagnet[6] was elaborated many years ago. This provided the basis for the theory of the transition developed in Refs. 1 and 2. Here we will expose this physics starting from the VBS side.

As the VBS order is described by a discrete Z_4 clock order parameter, the natural topological defects are domain walls.[7] Various kinds of walls between the four different broken symmetry states are possible. It is convenient to consider an "elementary" domain wall across which the clock angle shifts by $\pi/2$, and to assign an orientation to such a wall. An example is shown in Fig. 2. All other walls, where the clock angle shifts by higher multiples of $\pi/2$, may be built up from the elementary wall.

A key point is that four such elementary walls can come

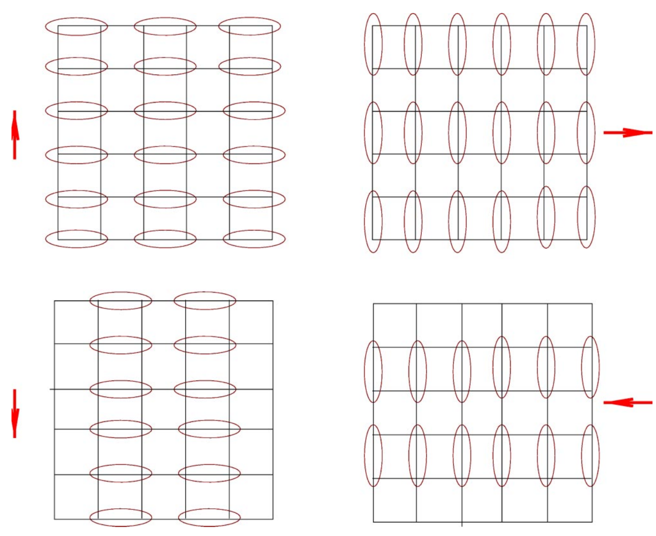

FIG. 1. (Color online) Schematic picture of the four degenerate ground states associated with the columnar VBS state. The encircled lines represent the bonds across which the spins are paired into a valence bond. The four ground states are associated with four different orientations of a Z_4 clock order parameter.

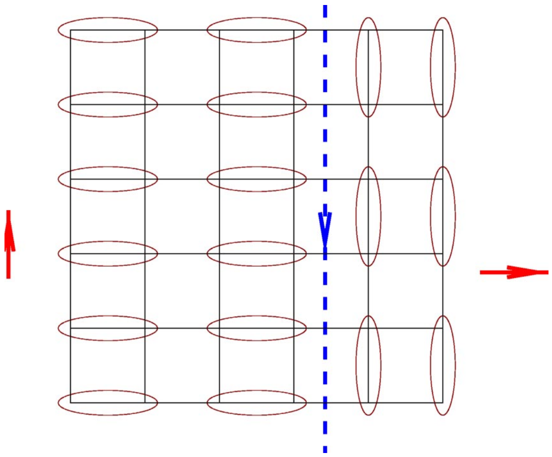

FIG. 2. (Color online) An example of an elementary domain wall in the VBS state across which the clock angle shifts by $\pi/2$.

together and terminate at a point. It is clear that such termination points may be associated with Z_4 vortices—the clock angle winds by 2π upon encircling such a termination point (see Fig. 3). Z_4 antivortices may be similarly defined.

What do such Z_4 vortices correspond to in terms of the underlying VBS configurations? An example is illustrated in Fig. 4. A remarkable property of this cartoon is that at the core of such a vortex there is a site with an unpaired spin—i.e., a spin that is not part of any valence bond. It is easy to see that this is a general property of any such vortex pattern of the VBS order parameter. Furthermore, translating the entire valence bond pattern by one lattice spacing reverses the direction of the winding—thus the Z_4 vortices are associated with one sublattice, say the A sublattice, and the Z_4 antivortices with the B sublattice.

Thus in this particular quantum problem, the Z_4 vortices (and antivortices) carry an uncompensated spin-1/2 moment. They may therefore be identified with "spinons." In the VBS-ordered phase, the energy required to separate a vortex from an antivortex increases linearly with distance, since such a pair is necessarily accompanied by four domain walls connecting the two defects (see Fig. 3). This means that the spinons are *confined* and do not exist as free excitations in this phase.

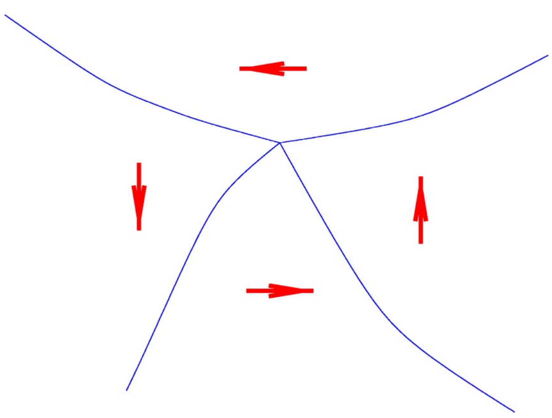

FIG. 3. (Color online) Macroscopic picture of a Z_4 vortex as a point where four oriented elementary domain walls meet and end.

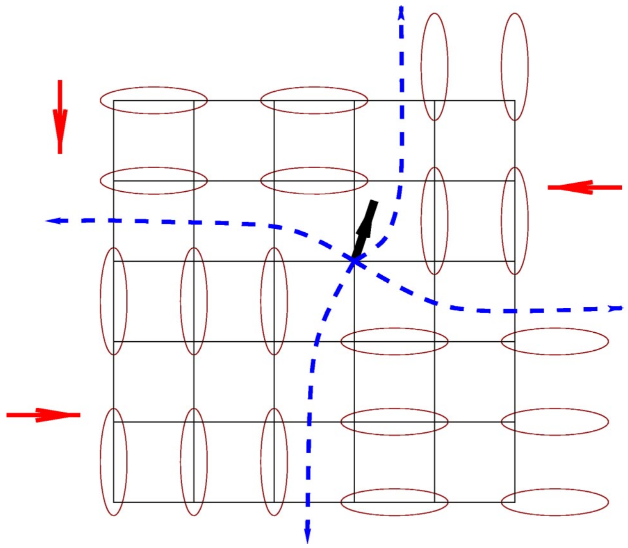

FIG. 4. (Color online) The Z_4 vortex in the columnar VBS state. The blue lines represent the four elementary domain walls. At the core of the vortex there is an unpaired site with a free spin-1/2 moment.

It is the nontrivial structure of the Z_4 vortex in this problem that distinguishes the VBS state from a more ordinary state with a Z_4 order parameter. Such an ordinary state obtains for instance in a simple lattice quantum $O(2)$ rotor model with a fourfold anisotropy. In this case the Z_4 vortices in the ordered state have featureless cores. The disordering transition in this simple model may be described by the usual three-dimensional classical Z_4 model and is hence in the 3D XY universality class (since the clock anisotropy is irrelevant). In contrast, disordering transitions out of the VBS phase must necessarily take into account the presence of the spin-1/2 moment in the cores of the Z_4 vortices. Any mapping to a classical 3D Z_4 model is then complicated by the need to incorporate this vortex structure.

Consider moving out of the VBS phase by proliferating and condensing the Z_4 vortices. Clearly once the vortices proliferate, long-ranged Z_4 order cannot be sustained. Furthermore, as these vortices carry spin, the resulting state will break spin symmetry, and as argued below may be identified as the Néel state.

These simple considerations, therefore, provide a mechanism for a direct second-order transition between the VBS and Néel phases. As for the usual Z_4 model, it is reasonable to expect that the clock anisotropy will be irrelevant at this transition as well. Indeed, as we will argue later, this is strongly supported by the evidence from Refs. 1 and 2. For the present, let us explore the consequences of the expected irrelevance of the clock anisotropy.

The critical theory will then be that of a (quantum) XY model in $D=2+1$ but with vortices that carry spin-1/2 (See Fig. 5). The spinon nature of these vortices will change the universality class from $D=3$ XY to something different. Clearly to expose this difference and to obtain a description of the resulting new universality class, it will be most convenient to go to a dual basis in terms of the vortices and their interactions (analogous to the familiar Coulomb gas description of classical 2D XY models).

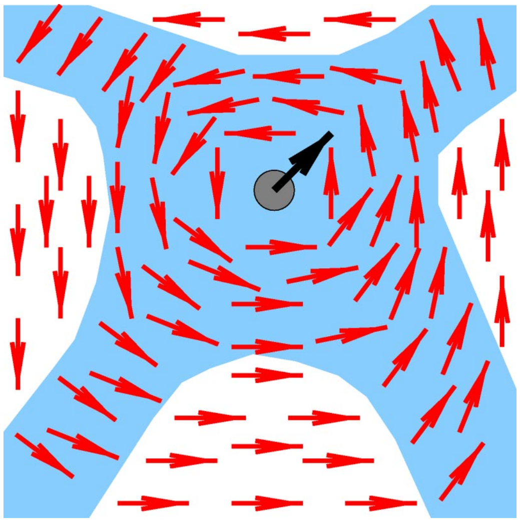

FIG. 5. (Color online) The structure of the Z_4 vortex close to the transition. As one approaches the critical point, the domain walls (depicted as a blue-shaded region) become thicker. At the same time, the vortex core where the spin-1/2 moment resides (depicted as a gray circular region) becomes larger. The domain wall thickness ξ_{VBS} and the vortex core size ξ both diverge at the transition but because the clock anistropy is irrelevant, the former diverges faster. Therefore, at intermediate length scales (larger than ξ but smaller than ξ_{VBS}) the clock angle winds smoothly as in a regular *XY* vortex.

The structure of such a dual vortex reformulation is well known. The basic idea is to regard the phase mode of the *XY* model as the photon associated with a fictitious noncompact $U(1)$ gauge field. The vortices then correspond to gauge charges that are minimally coupled to this photon field. At the critical point, the vortices are gapless: the critical theory may be constructed as a theory of gapless vortex fields minimally coupled to a fluctuating noncompact $U(1)$ gauge field. For the problem at hand, the spinon nature of the vortices is readily incorporated by introducing a two-component spinor field, z_a, to represent the vortices ($a=1,2$ is the spin index). The transition out of the VBS phase to the Néel phase will then be described by a theory of gapless spinon vortices coupled minimally to a fluctuating noncompact $U(1)$ gauge field (which is the dual of the *XY* phase mode).

The critical theory is readily written down. The most general theory consistent with the $U(1)$ gauge structure, $SU(2)$ symmetry, and vortex/antivortex exchange symmetry of the microscopic model (the latter required by the symmetry under sublattice exchange $A \leftrightarrow B$), is described by the action $S_z=\int d^2r d\tau \mathcal{L}_z$, with

$$\mathcal{L}_z = \sum_{a=1}^{2} |(\partial_\mu - i a_\mu) z_a|^2 + s|z|^2 + u(|z|^2)^2 + \kappa(\epsilon_{\mu\nu\kappa}\partial_\nu a_\kappa)^2. \tag{1}$$

The transition occurs as the parameter s is tuned. The a_μ represent the components of a fluctuating gauge field.

Remarkably this is exactly the same field theory as the one proposed in Refs. 1 and 2 for the Néel-VBS transition based on an approach that attacked from the Néel side. We have thus shown how to recover that field theory in an approach from the VBS side.

These considerations may be formalized as follows. First we note that z_a represents a Z_4 spinon vortex, and hence must transform as a spinor under physical SU(2) spin rotations. The antivortex must also transform as a spinor; we must therefore represent antivortices by $-i\sigma^y_{ab} z^*_b$, where σ^y is the usual Pauli matrix. As discussed pictorially above, elementary lattice translations take vortices to antivortices so that $z_a \to -i\sigma^y_{ab} z^*_b$. It follows that the vector $\vec{N}=z^*_a \vec{\sigma}_{ab} z_b$ changes sign under an elementary lattice translation. We may therefore identify it with the Néel order parameter. Thus for instance a uniform condensate of z_a corresponds to the Néel state.

We may formally justify the critical theory in Eq. (1) above as follows. The arguments developed above show that the critical theory is that of an *XY* model where the vortices are spinons. Consider the conserved current J_μ of this *XY* model. In the ordered phase this may be expressed in terms of the *XY* phase field χ through

$$J_\mu = K \partial_\mu \chi, \tag{2}$$

where K is the stiffness of the *XY* model. To access the *XY* disordered phase, it is necessary to include vortex configurations and account for the periodicity of the phase χ. The vortex current j_μ is given by

$$j_\mu = \frac{1}{2\pi} \epsilon_{\mu\nu\lambda} \partial_\nu \partial_\lambda \chi. \tag{3}$$

Note that j_μ must be invariant under physical spin rotations even though it is carried by spinons. The conservation condition on J_μ may be implemented by expressing it as

$$J_\mu = \frac{1}{2\pi} \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda. \tag{4}$$

This equation defines the field a_μ, which may be interpreted as a noncompact $U(1)$ gauge field. Clearly it is defined only up to a gauge transformation $a_\mu \to a_\mu + \partial_\mu \theta$. The vortex current may now be reexpressed in terms of a_μ,

$$j_\mu = \frac{1}{4\pi^2 K} \epsilon_{\mu\nu\lambda} \partial_\nu b_\lambda, \tag{5}$$

where $b_\lambda = \epsilon_{\lambda\alpha\beta} \partial_\alpha a_\beta$ is the gauge-invariant field strength associated with the a_μ field. This equation now takes the form of the familiar Ampere law. The duality is completed by requiring a continuum field theory of the spinon vortices z_a whose equations of motion reduce to this Ampere law equation. The action in Eq. (1) above has precisely this property as is readily checked.

Note that as usual the conserved density J_0 of the *XY* model is simply the magnetic flux density in the dual description. As the phase χ is the conjugate operator, the operator $e^{4i\chi}$ simply increases the total gauge flux by 8π. We may therefore identify it with a quadrupled monopole operator of the dual gauge theory. Thus the quartic anisotropy in the *XY* model corresponds precisely to the quadrupled monopole operator. Strong evidence for the irrelevancy of this operator at the critical fixed point of Eq. (1) was presented in Ref. 2.

Is it possible for quantum fluctuations to destroy the VBS order without inducing Néel order? Clearly the answer is yes. One possibility is a transition to a topologically ordered Z_2 spin-liquid state. To obtain a topologically ordered state from the VBS state, it is as usual necessary to condense *paired* vortices[8] in the VBS order parameter—but here these vortices are spinons. To get a spin singlet state it is necessary to form a singlet pair of these spinons.[9] As discussed above, vortices live on one sublattice and antivortices on another. Consequently we need to condense a spin-singlet pair of spinons living on the same sublattice to obtain the Z_2 spin liquid. All of this is completely consistent with existing gauge theoretic descriptions of Z_2 spin liquids.[10]

To conclude, we have examined the nature of the Néel-VBS transition by an attack from the VBS side. This approach leads to a simple physical description of the transition and is completely consistent with the alternate approach of attacking from the Néel side. All of the physics associated with the transitions out of the VBS phase may be fruitfully understood from the perspective of this paper.

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[9] Readers familiar with the gauge theoretic description of the Z_2 spin liquid will recognize that this is the same as the usual procedure of condensing a gauge charge-2 spinon pair to obtain the Z_2 phase.

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