Magnetically induced Bragg scattering of electrons in quantum-dot crystals

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We demonstrate a novel manifestation of dynamic Bragg reflection in artificial quantum-dot crystals that is driven by the application of a magnetic field. This backscattering is coherently cascaded as the number of dots in the structure is increased, causing a superlinear damping of the electron wave function and the appearance of a series of gaps in the miniband spectrum. The evolution of the dynamic miniband structure as the magnetic field is varied gives rise to behavior analogous to a metal-insulator transition, which is manifest as a dramatic resonance in the magneto-resistance of the structures.

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Manipulation of matter on the nanoscale, to realize systems with artificially engineered bandstructures, is currently an active area of research, with the potential to impact on the development of a variety of electronic and optoelectronic technologies. In photonic crystals, for example, nanoscale patterning of conventional semiconductors is used to open forbidden gaps in the spectrum for electromagnetic-wave propagation.[1,2] At the other extreme, scanning tunneling microscopy can be used to construct linear chains of small numbers of atoms, and to study the emergence of one-dimensional energy bands associated with the ordered atomic arrangement.[3,4] Another versatile system, with the potential for use as the building block of periodically modulated nanostructures, is provided by semiconductor *quantum dots*.[5] These may be viewed as artificial atoms, with a discrete set of energy levels (*Darwin-Fock* states) that hybridize when several dots are coupled together to form an artificial molecule or crystal.[6,7]

A common feature of the systems considered above is the formation of a superlattice bandstructure, whose gaps and minibands are quite distinct from those of the bulk material.[8] A well known process that influences the bandstructure of real crystals is *Bragg reflection* of electrons, which occurs when their wavelength is commensurate with the interatomic spacing. When this occurs, constructive interference of electron waves, scattered from successive crystal planes, opens a forbidden gap in the electronic energy spectrum, usually at the Brillouin-zone boundary. At energies within this gap, electron propagation is described by a complex wavevector, whose wave function decays evanescently as the electron attempts to tunnel into the crystal. In the various nanostructures described above, Bragg reflections are also expected to arise at the new Brillouin zone boundary associated with their artificial periodicity.

In this paper, we report on a novel *dynamic* manifestation of Bragg reflection in artificial quantum-dot crystals. The scattering is driven by the application of a magnetic field, which opens a set of gaps in the miniband spectrum and gives rise to a dramatic resonance in the magneto-resistance. The dynamic nature of this effect (as the magnetic field is varied) is found to be due to the commensurability, at the resonant magnetic field, of the electron cyclotron motion with the internal dimensions of the dot (a condition which depends upon both the magnetic field and the Fermi velocity). The commensurability leads to an enhancement of backscattering that is progressively cascaded as the number of dots in the structure is increased, giving rise to a coherent increase of the magneto-resistance peak, beyond that expected merely from the number of dots. Numerical calculations reproduce the observed resonance, and show that the effect of the magnetic field is to induce a strong decay of the electron wavefunction in the array, which is accompanied by the appearance of forbidden gaps in its miniband spectrum. Our observations are therefore analogous to a metal-insulator transition, which is induced as the magnetic field is used to dynamically open gaps in the artificially engineered electronic band structure.

Our results were obtained in a comparative study of a single quantum dot, and of linear arrays comprised of three and seven such dots (Fig. 1). For a consistent comparison, the devices were fabricated on the same GaAs/AlGaAs heterojunction wafer, using the split-gate technique. The two-dimensional electron gas was located 92 nm below the top surface, and its density and mobility were 2.5×10^{11} cm^{-2} and 1.2×10^{6} cm^2/Vs (at 0.03-K), respectively. The sample

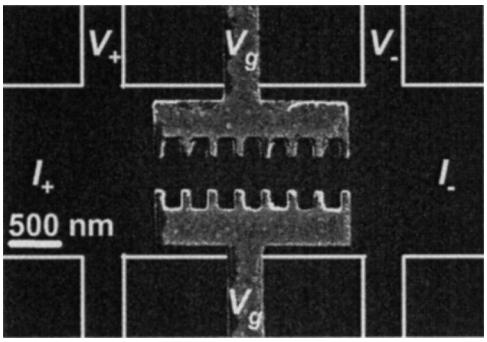

FIG. 1. A micrograph of a seven-dot array realized by the split-gate technique. Lighter regions are the metal gates and the white lines show schematically the Hall-bar structure on top of which the gates are deposited.

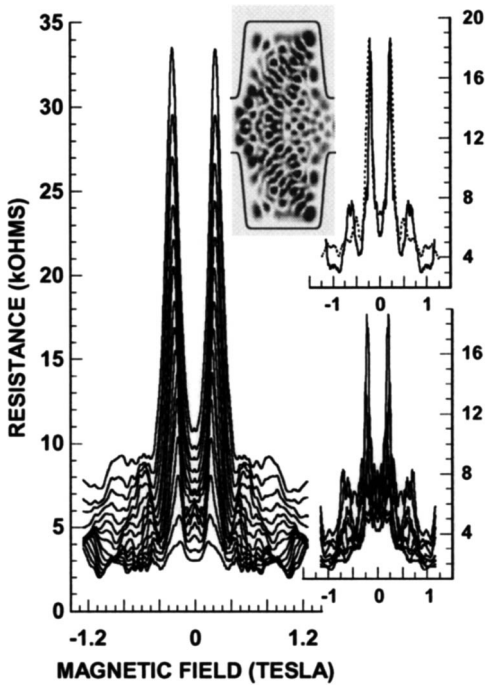

FIG. 2. Main panel: Magneto-resistance of the seven-dot array at several different gate voltages. Lower right inset: Simulated gate-voltage dependent magneto-resistance of the array. Upper right inset: A comparison of the experimental (dotted line) and computed (solid line) magneto-resistance. Also shown in the central inset is the probability density at 0.2 T in the first cell of the seven-dot array. Darker regions indicate higher probability density.

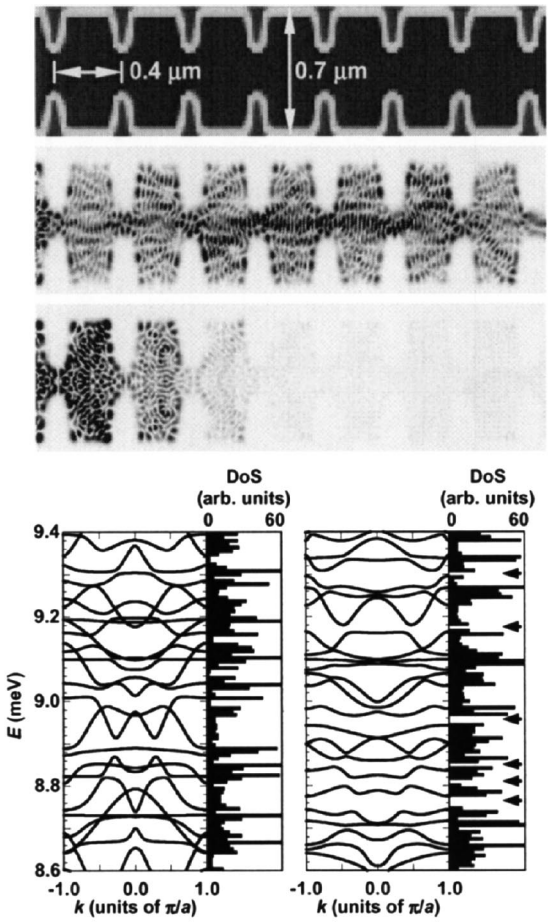

FIG. 3. The top panel shows an example of a potential profile that is used in calculations. The middle two panels show the calculated electron probability density in the array at 0 T (upper plot) and 8 T (lower plot). The bottom figures show the computed bandstructure and density of states (DoS) of an infinite chain of quantum dots at 0 T (left) and 0.2 T.

was clamped to the mixing chamber of a dilution refrigerator, and its magneto-resistance was measured at the base temperature of 10-mK (unless stated otherwise). Low-frequency ($\sim$11 Hz) constant currents ($\sim$5 nA) and lock-in detection were used for these measurements.

The main panel of Fig. 2 shows the magneto-resistance of the seven-dot array at several gate voltages, and reveals the presence of two striking peaks near ± 0.2 T that rise to as much as 300% of the background resistance (which remains less than 10 kΩ over the entire range of experiment). To clarify the origin of the peaks, we simulate magneto-transport using the transfer-matrix method that we have previously applied to open quantum dots.[9,10] The simulations are performed by discretizing the Schrödinger equation onto a finite-difference mesh and using it in its discrete form to set up a numerically stabilized variant of the transfer matrix approach[11] to solve the transport problem. By imposing an electron flux from the left, and translating across the structure, one obtains the transmission coefficients that enter the Landauer-Büttiker formula to give the conductance. These coefficients also facilitate the reconstruction of the dot wave functions via backward substitution. The calculations use a device profile that mimics what one expects from self-consistent calculations.[9] A typical profile is shown in Fig. 3, and we note from this that the point contact providing the coupling between neighboring dots are *open* and so do not serve as tunnel barriers (consistent with the low background resistance found in experiment). In the lower inset of Fig. 2, we show the calculated magneto-resistance of the seven-dot array, and the agreement with experiment is excellent. This can also be seen in the upper inset of Fig. 2, which compares the computed magneto-resistance to an experimental trace that was chosen due to its similar background resistance. To obtain such quantitative agreement, it was necessary to superimpose a random background potential, of rms amplitude 2.8 mV, over the potential of the device. This fluctuation corresponds to less than 30% of the Fermi energy, and is consistent with the expected effects of the random dopant distribution in the heterojunction.[12]

Insight into the origin of the magneto-resistance peaks is also provided in Fig. 3, where we plot the electron wave function in the array at a magnetic field $B=0$ and 0.2 T. The calculations are performed for a Fermi energy equivalent to that in experiment (8.9 meV), and it is clear that the magnetic field very dramatically modifies the form of the wave function. At zero field, the electron probability density is distributed rather *uniformly* along the length of the array, with the resulting impression being one of an *extended* state. At 0.2 T, however, the probability density *decays* rapidly along length of the array, consistent with the striking

increase of the resistance that we observe at this magnetic field. (The apparently broken symmetry of the wave function in this figure is actually just a consequence of the transfer-matrix method,[11] in which the transmission is computed by assuming that a wave is incident on the structure from a particular direction. This is actually close to the situation in experiment, where a net electron flow is created by applying a small bias across the structure). The driving force for this effect, which is analogous to a *metal-insulator transition*, can be seen in the central inset to Fig. 2, which provides an expanded view of the wave function in the leftmost cell of the array. The role of magnetic focusing[13–17] is apparent in this figure, which shows the signature of two electron orbits (or possibly a single, connected, orbit) whose cyclotron curvature deflects them to the top and bottom corners of the dot, after which they are reflected back to its entrance. Indeed, the electron cyclotron radius at this magnetic field ($r_c=\hbar k_F/eB=375$ nm) is comparable to the period of the quantum-dot crystal (Fig. 3), and is consistent with the deflection of the electron orbits that can be discerned in the inset to Fig. 2. The evanescent decay of the wave function at 0.2 T, therefore, appears to result from a *cascading* of this magnetically driven dynamic backscattering, and the resonant nature of the associated magneto-resistance peaks can be attributed to the fact that this backscattering is effective, *only* near the magnetic field for which the cyclotron radius and the internal dimensions of the dot are commensurate with each other.

The decay of the wave function at 0.2 T is reminiscent of that which occurs in periodic structures, when Bragg reflection of electrons opens a set of forbidden gaps in the their energy spectrum. The magnetically driven dynamic backscattering mechanism that we discuss is analogous to such Bragg reflection. To demonstrate this, in the bottom panel of Fig. 3, we compute the bandstructure of an *infinitely long* chain of quantum dots at $B=0$ and 0.2 T (a is the period of the crystal and is equal to 0.4 μm in this case). These results were obtained by solving the discretized Schrödinger equation for a single dot, applying Dirichlet boundary conditions at its upper and lower walls, and periodic boundary conditions at its left- and right-leads. To solve the resulting sparse-matrix eigenvalue problem, we use the publicly available ARPACK software package.[18] To illustrate the connection to the ideal behavior, the calculations are actually performed for a clean dot. Comparison of the two bandstructures shows that the effect of the magnetic field is to open several forbidden gaps, which occur over a range of energy. This can be seen more clearly in the density-of-states plots in the same figure, obtained by integrating the bandstructure over the full Brillouin zone. While at 0 T there are no clear regions where the density of states vanishes, at 0.2 T a number of such gaps (indicated by arrows) can be seen.

The idea that the magneto-resistance peaks are associated with dynamic Bragg reflection of electrons, and transmission via evanescent states, suggests that the peak magnitude should be extremely sensitive to variation of the array length. Indeed, we expect a strong connection in this case to the problem of tunneling through the gap states of single molecules, whose conductance can be strongly (even exponentially) dependent on molecular length.[19–21] In Fig. 4, we

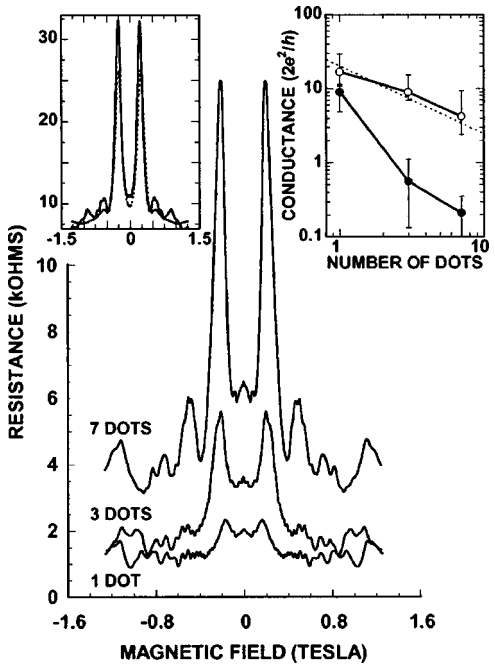

FIG. 4. Main panel: comparison of the magneto-resistance of the single dot and the three- and seven-dot arrays. Left inset: Magneto-resistance of the seven-dot array at 30 mK (solid line) and 8 K (dotted line). Right inset: Scaling of the conductance with dot number at 0 T (open symbols) and 0.2 T (filled symbols).

show the scaling of the magneto-resistance peaks with array length. The three curves in the figure correspond to similar values of the split-gate voltage, and the resulting scaling behavior is typical of such comparisons. The magneto-resistance peaks grow rapidly, and at a clearly faster rate than the increase of the background resistance, as the number of dots in the array is increased. To illustrate this, in the right inset of Fig. 4, we compare the scaling of the zero-field conductance (open symbols) with that at 0.2 T (filled symbols). The figure is plotted using double-log axes, and each data point was obtained by averaging the results of magneto-resistance measurements at many gate voltages. The error bars in the figure thus represent the range of values contributing to the average, rather than an experimental error. The data indicate that, at zero magnetic field, the conductance decreases linearly as the number of dots of increased, which is just the behavior expected for the Ohmic addition of uncorrelated, individual, dot resistances (the dotted line in the inset indicates a conductance variation $G\propto N^{-1}$, where N is the dot number). At 0.2 T, however, the conductance shows a much more rapid, superlinear decrease as the length of the array is increased. While one expects exponential damping for tunneling through a single energy gap,[19] the situation is likely more complicated here, since *several* gaps may influence transmission (see below). Unfortunately, we have insufficient data points to determine the precise functional form of the decay in Fig. 4. Nonetheless, the rapid decrease of the conductance at 0.2 T is consistent with the coherent exponential damping of the wave function shown in Fig. 3.

The minigaps in Fig. 4 are ~ 20 μeV wide, so that they should only be resolved at very low temperatures ($\leqslant 0.2$ K).

Nonetheless, experiment reveals only a weak decay of the magneto-resistance peaks with increasing temperature. In the left inset of Fig. 4, we show the magneto-resistance of the seven-dot array at 0.01 and 8 K. The resistance peaks are only slightly damped at 8 K, in spite of the fact that the thermal energy significantly exceeds the expected size of the forbidden gaps at this temperature. This can be understood, however, if we recall that the effect of the magnetic field is to open a *series* of gaps in the energy spectrum, over a wide range of energy (Fig. 4). While the effect of increasing temperature is to increase the effective energy range that contributes to the conductance,[9] in the present case this will result in the sampling of additional forbidden gaps that contribute to the superlinear decay effect. As such, the magneto-resistance peaks should be attributed to a general reduction in the density of states, which occurs at the resonant magnetic field, rather than to the signature of any specific gap. We believe that a similar argument can also be made to explain the robustness of the magneto-resistance peaks to the presence of disorder; while disorder should give rise to the mixing of states, the appearance of a series of gaps, over a wide range of energy, in the unperturbed density of states, should allow the survival of the magneto-resistance peaks.

In conclusion, we have demonstrated a novel manifestation of a dynamic Bragg reflection in artificial quantum-dot crystals that is driven by the application of a magnetic field. This backscattering is progressively cascaded in a coherent manner as the number of dots in the structure is increased, causing a superlinear damping of the electron wave function and the appearance of a series of gaps in the miniband spectrum. The dynamic evolution of the miniband structure as the magnetic field is varied gives rise to behavior analogous to a metal-insulator transition, which is manifest as a dramatic resonance in the magneto-resistance of the structures.

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