

# Slowing down of spin relaxation in two-dimensional systems by quantum interference effects

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The effect of weak localization on spin relaxation in a two-dimensional system with a spin-split spectrum is considered. It is shown that the spin relaxation slows down due to the interference of electron waves moving along closed paths in opposite directions. As a result, the averaged electron spin decays at large times as  $1/t$ . It is found that the spin dynamics can be described by a Boltzmann-type equation, in which the weak localization effects are taken into account as nonlocal-in-time corrections to the collision integral. The corrections are expressed via a spin-dependent return probability. The physical nature of the phenomenon is discussed and it is shown that the “nonbackscattering” contribution to the weak localization plays an essential role. It is also demonstrated that the magnetic field, both transversal and longitudinal, suppresses the power tail in the spin polarization.

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## INTRODUCTION

The relaxation of nonequilibrium spin polarization is the central phenomenon in spin-dependent transport in semiconductor nanostructures.<sup>1</sup> One of the most efficient mechanisms of electron spin relaxation in III–V semiconductors is the well-known Dyakonov-Perel mechanism<sup>2</sup> based on the classical image of angular diffusion of the spin vector in a random magnetic field. The field originates from the momentum-dependent spin-orbit splitting of the conduction band in the crystals with zinc-blend structure.<sup>3</sup> While passing through the crystal, the electron is scattered by impurities and its momentum changes randomly with time. As a consequence, the effective magnetic field also changes randomly with a correlation time of the order of the momentum relaxation time  $\tau$ . The spin relaxation time  $\tau_S$  is a characteristic time of the spin angular diffusion<sup>2</sup>  $1/\tau_S \sim \phi^2/\tau \sim \Omega^2\tau$ , where  $\phi \sim \Omega\tau \ll 1$  is the typical angle of the spin precession for the momentum relaxation time and  $\Omega$  is the frequency of precession in the effective magnetic field proportional to the conduction band splitting. For a two-dimensional (2D) case, when the electron motion in one direction is confined by the quantum well, the spin splitting and, hence, the precession frequency are proportional to the in-plane electron velocity<sup>4,5</sup>  $\Omega \sim v$ . So, at low temperatures, when inelastic processes can be disregarded, the spin relaxation rate for a 2D electron with a given energy  $E = mv^2/2$  is proportional to the particle diffusion coefficient

$$\frac{1}{\tau_S} \sim D, \quad D = v^2\tau/2. \quad (1)$$

The effects of localization on the particle diffusion have been discussed in a great number of publications. The first-order term in a series expansion of  $D$  in  $1/kl$  ( $l = v\tau$  is the mean free path,  $k$  is the electron wave vector) is known as the weak localization correction<sup>6</sup> (for a review, see Ref. 7), coming from the coherent enhancement of the backscattering amplitude. An important feature of this correction is the logarithmic divergence at low temperatures in the 2D case. Equation (1) implies a similar divergence of the spin relaxation rate.

Such a divergence was first predicted by Singh<sup>8</sup> in spin correlation functions for a system with spin-dependent impurity scattering. It was shown, however, that the quantum correction to the spin relaxation rate is not proportional to the quantum correction to the diffusion coefficient, as one might expect from Eq. (1). A similar result was obtained in Ref. 9 for a system with a spin-split spectrum. It was found in Ref. 10 that the weak localization slows down the spin relaxation of excitons in quantum wells, which leads to a  $1/t$  power tail in the spin orientation. A similar effect was also predicted for electrons in 2D semiconductors with a zinc-blend crystal structure.

In this paper, we consider the effects of localization on the spin relaxation for a 2D semiconductor with a spin-split spectrum. We show that the spin dynamics is described by a Boltzmann-type equation. In the first order in  $1/kl$ , the localization effects can be taken into account by a nonlocal-in-time correction to the Boltzmann collision integral. This correction is expressed in terms of the spin-dependent return probability. We discuss the role of coherent returns at different scattering angles and show that the “nonbackscattering” contribution to the collision integral plays a key role. We solve the generalized kinetic equation and demonstrate that, at large times, the spin polarization decays as  $1/t$ . The magnetic field, both transversal and longitudinal, is found to suppress the long-living tail in the spin-relaxation.

## DERIVATION OF THE KINETIC EQUATION

The Hamiltonian of a 2D with a spin-split spectrum is given by

$$H = \frac{\mathbf{p}^2}{2m} + \frac{\hbar}{2}\boldsymbol{\omega}\boldsymbol{\sigma} + U(\mathbf{r}). \quad (2)$$

Here  $\mathbf{p} = p\mathbf{n}$  is the in-plane electron momentum,  $m$  is the electron effective mass and  $\boldsymbol{\sigma}$  is a vector consisting of Pauli matrices. The spin-orbit interaction is described by the second term, in which  $\boldsymbol{\omega} = \boldsymbol{\omega}(\mathbf{n})$  depends on the direction of the electron momentum  $\omega_i(\mathbf{n}) = \sum_k n_k \Omega_{ki}$  ( $i = x, y, z; k = x, y$ ). The matrix  $\hat{\Omega} = \hat{\Omega}^{(1)} + \hat{\Omega}^{(2)}$  is the sum of two terms: the so-called

Bychkov-Rashba term<sup>4</sup>  $\hat{\Omega}^{(1)}$  (with nonzero components  $\Omega_{xy}^{(1)} = -\Omega_{yx}^{(1)} \sim p$ ) and  $\hat{\Omega}^{(2)}$ , which is the Dresselhaus term<sup>3</sup> averaged over the electron motion along the  $z$ -direction perpendicular to the quantum well plane. The Bychkov-Rashba coupling depends on the asymmetry of the quantum well confining potential. Its strength can be tuned by varying the gate voltage.<sup>11</sup> The Dresselhaus term is present in semiconductors with no bulk inversion symmetry. The components of the matrix  $\hat{\Omega}^{(2)}$  are also linear in the in-plane electron momentum  $\Omega_{ij}^{(2)} \sim p$  and vary with well plane orientation with respect to the crystallographic axes<sup>5</sup> (we neglect cubic Dresselhaus terms, assuming that the electron concentration is relatively small). We consider the scattering by the short-range impurity potential with the correlation function  $\langle U(\mathbf{r})U(\mathbf{r}') \rangle = \gamma \delta(\mathbf{r} - \mathbf{r}')$ , where the coefficient  $\gamma$  is related to the transport scattering time by  $\tau = \hbar^3/m\gamma$ .

The classical spin dynamics is described by the kinetic equation.<sup>2</sup> For a homogeneous case, this equation is

$$\frac{\partial \mathbf{s}}{\partial t} = \boldsymbol{\omega}(\mathbf{n}) \times \mathbf{s} + \hat{J}_0 \mathbf{s}. \quad (3)$$

Here  $\hat{J}_0$  is the Boltzmann collision integral and  $\mathbf{s} = \mathbf{s}(\mathbf{p}, t)$  is the spin density in the momentum space, related to the averaged spin by  $\mathbf{S} = \int \mathbf{s} d^2 \mathbf{p} / (2\pi\hbar)^2$ . We assume that the spin splitting is relatively small:  $\omega(\mathbf{n})\tau \ll 1$ . This inequality provides  $\tau \ll \tau_S$ . The relationship between  $\tau_S$  and the inelastic scattering time  $\tau_{in}$  varies with temperature. Here we focus on the case of low temperatures, assuming that  $\tau_{in} \gg \tau_S$ . Then spins with different energies do not correlate with each other, and the solution of Eq. (3) at  $t \gg \tau$  yields  $\mathbf{S}(t) = (m/2\pi\hbar^2) \int \mathbf{s}_i dE$ , where  $\mathbf{s}_i = \mathbf{s}_i(E, t) = e^{-t\hat{\Gamma}} \mathbf{s}_i(E, 0)$ ,  $\mathbf{s}_i(0, E) = \langle \mathbf{s}(\mathbf{p}, 0) \rangle$  is the initial spin density averaged over the momentum direction, and  $\hat{\Gamma} = \hat{\tau}_S^{-1}$  is the spin relaxation tensor (tensor of inverse relaxation times) given by<sup>5</sup>

$$\Gamma_{ik} = \left[ \delta_{ik} \sum_{s,l} \Omega_{sl}^2 - \sum_l \Omega_{li} \Omega_{lk} \right] \tau / 2. \quad (4)$$

The conventional approach to the calculation of the correlations functions in weakly localized systems is based on the Kubo formula.<sup>6,7</sup> An alternative approach<sup>12-14</sup> is to generalize the Boltzmann equation to include weak localization effects in the kinetic description. This approach may turn out to be more convenient when studying nonlinear and strongly nonequilibrium phenomena. To describe quantitatively the weak localization phenomenon in the kinetic picture, one has to modify the Boltzmann equation by introducing a nonlocal-in-time correction to the collision integral.<sup>12,13</sup> These corrections can be derived<sup>13</sup> from the diagrammatic structure of linear-response functions. Diagrammatically, the inclusion of a weak localization correction to the effective collision integral requires the consideration of the irreducible diagrams<sup>13,14</sup> in Figs. 1(b) and 1(c), in addition to the diagram for the Born scattering, shown in Fig. 1(a). The crossed-ladder diagrams Fig. 1(b) are usually considered to describe the coherent backscattering of the electron wave. A physical interpretation of the diagrams Figs. 1(b) and 1(c) in terms of a small change in the effective differential cross

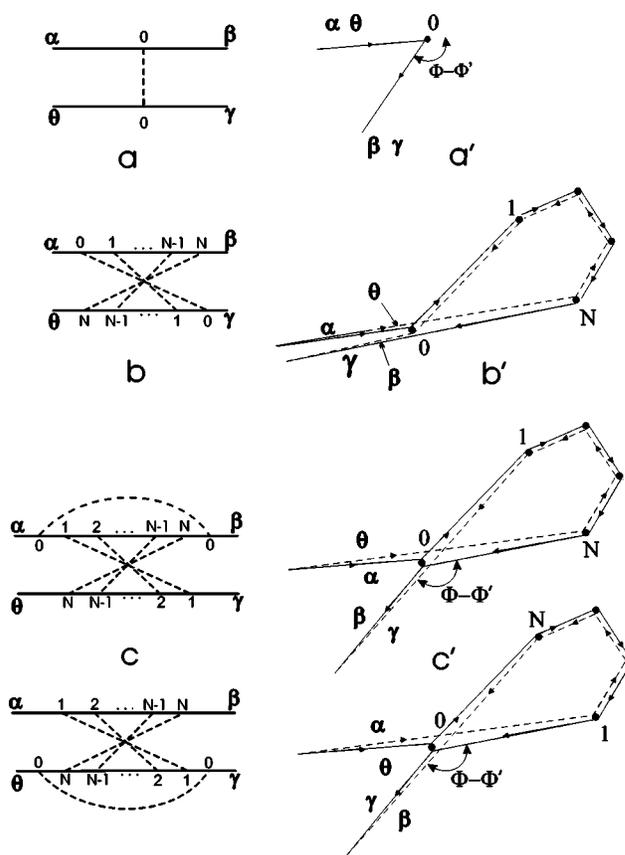


FIG. 1. Relevant irreducible diagrams  $a, b, c$  and the respective scattering processes  $a', b', c'$ . The Born collision process  $a'$  is independent of the electron spin (its contribution is proportional to  $\delta_{\alpha\beta}\delta_{\theta\gamma}$ ). Coherent backscattering  $b'$  ( $\Phi - \Phi' \approx \pi$ ), as well as the processes  $c'$  describing coherent scattering at an arbitrary angle ( $0 < \Phi - \Phi' < 2\pi$ ), are spin dependent due to rotation of the spin of an electron passing along the closed path.

section for a single impurity was suggested in Ref. 14. It was based on an analysis of the interference contribution of trajectories propagating in the opposite directions along closed paths in terms of the phase stationarity requirement. This analysis shows that the diagrams Fig. 1(b) correspond to a process shown in Fig. 1(b'), which is indeed a coherent backscattering. Diagrams 1(c) were found to describe coherent scattering processes with arbitrary scattering angles, shown in Fig. 1(c').

Next, we discuss the key points of a quantitative description of the weakly localized regime within the kinetic approach. We start with a brief discussion of the zero spin-orbit coupling. As can be seen from Figs. 1(b') and 1(c'), the relevant processes contain the same closed paths, so the effective change in the differential cross section of impurity 0, coming from both 1(b') and 1(c'), is expressed in terms of the return probability<sup>14,15</sup>

$$\frac{\delta S_{\omega}(\Phi)}{S_0} = \frac{\lambda l}{\pi} W_{\omega}(0) [\delta(\Phi - \pi) - 1/2\pi]. \quad (5)$$

Where  $S_0 = 1/N_i \nu \tau$  is the isotropic cross section in the Drude approximation,  $N_i$  is the impurity concentration,  $\lambda = 2\pi/k$  is

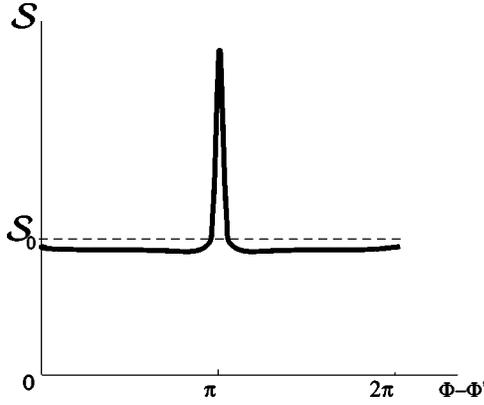


FIG. 2. Angular dependence of the effective cross-section modified by weak localization. The narrow peak at  $\Phi - \Phi' = \pi$  is due to the coherent backscattering shown in Fig. 1 (b'). The enhancement of the coherent backscattering is accompanied by a reduction of scattering at other angles [the process in Fig. 1 (c')], the total cross section being unchanged.

the electron wavelength, and  $W_\omega(0)$  is given by

$$W_\omega(0) = \frac{1}{\tau} \int dt e^{i\omega t} W(0, t), \quad (6)$$

where  $W(0, t) = W(\mathbf{r}, t)|_{\mathbf{r}=0}$  is the probability density for a diffusing particle to return after the time  $t$  to the origin  $\mathbf{r}=0$ . The coefficient  $\lambda l / \pi$  in the cross-section correction was found in Ref. 14 by integration over small deviations of the electron trajectories from the trajectories Figs. 1(b') and 1(c') meeting the phase stationarity requirement. Physically,  $\lambda l$  is the characteristic area of the region around the origin into which the diffusing electron should return for the effective interference to occur. The calculations show that the contributions of Figs. 1(b') and 1(c') have different signs. The positive contribution represented in Eq. (5) by  $\delta(\Phi - \pi)$  comes from the process Fig. 1(b'), while the negative one (the term  $-1/2\pi$ ), from the process Fig. 1(c'). In other words, the enhancement of the differential cross section at the angle  $\pi$  due to the coherent backscattering is accompanied by a reduction of the scattering in other directions, the total cross-section remaining unchanged (see Fig. 2). We see that the correction to the effective impurity cross section is  $\omega$ -dependent. Therefore, the correction to the collision integral in the time representation turns out to be non-local in time:

$$\begin{aligned} \delta \hat{J}f(\mathbf{p}, t) &= (\lambda l / \pi \tau^2) \int_{-\infty}^t dt' W(0, t - t') \\ &\times \int d\Phi' [\delta(\Phi - \Phi' - \pi) - 1/2\pi] f(\mathbf{p}', t'). \end{aligned} \quad (7)$$

Here  $f(\mathbf{p}, t)$  is the electron distribution function (we consider a homogeneous case, assuming that  $f$  is independent of  $\mathbf{r}$ ) and  $\Phi, \Phi'$  are the angles of  $\mathbf{p}$  and  $\mathbf{p}'$ .

The probability density  $W_\omega(\mathbf{r})$  is found as a sum over the paths involving different numbers of scattering events (see, for example, Ref. 16)

$$W_\omega(\mathbf{r}) = \sum_N W_\omega^N(\mathbf{r}), \quad (8)$$

where

$$\begin{aligned} W_\omega^N(\mathbf{r}) &= \int P(\mathbf{r} - \mathbf{r}_N) P(\mathbf{r}_N - \mathbf{r}_{N-1}) \dots P(\mathbf{r}_2 - \mathbf{r}_1) \\ &\times P(\mathbf{r}_1) d\mathbf{r}_1 \dots d\mathbf{r}_N, \end{aligned} \quad (9)$$

and

$$P(\mathbf{r}) = \frac{1}{2\pi r l} e^{-r/l + i\omega r/v}. \quad (10)$$

At  $\omega=0$ , the function  $P(r)$  is the classical probability density for an electron starting to move from  $\mathbf{r}=0$  to experience the first collision around the point  $\mathbf{r}$ . In the framework of a diagrammatic approach, the function  $P(r)$  arises as the product of two spherical waves (retarded and advanced Green's functions)  $P(r) = \gamma G_{E+\hbar\omega}^R G_{E-\hbar\omega}^A$ . Here  $G_{R,A}(r) = (\mp i)^{3/2} (m/\hbar^2) e^{\pm ikr - r/2l} / \sqrt{2\pi kr}$ ,  $k = \sqrt{2mE}/\hbar$ , and we took into account that  $e^{ikE+\hbar\omega} e^{-ikE} \approx e^{i\omega r/v}$ . A path involving  $N$  scatterings contains  $(N+1)$  functions  $G_R$  for the clockwise propagation along the path and  $(N+1)$  functions  $G_A$  for the counterclockwise propagation. As a result,  $W_\omega^N$  contains  $(N+1)$  functions  $P(r)$ . Using Eqs. (6) and (8)–(10), one can show that in the diffusion approximation ( $\omega\tau \ll 1$ ), when the typical number of scattering events along a path is large ( $N \gg 1$ ), the function  $W(\mathbf{r}, t)$  obeys the diffusion equation

$$\frac{\partial W}{\partial t} - D\Delta W = \delta(\mathbf{r})\delta(t). \quad (11)$$

Solving Eq. (11) and taking  $\mathbf{r}=0$ , we find the return probability  $W(0, t) = 1/4\pi Dt$ , and  $W_\omega(0) \approx (1/2\pi l^2) \ln(1/\tau\omega)$ .

A generalization of the above results to a system with a spin-split spectrum is straightforward. Since the electron spin rotates while passing along a closed loop, electron Green's functions become operators with respect to the spin variables:

$$\hat{G}_{R,A}(\mathbf{r}) = (\mp i)^{3/2} \frac{m}{\hbar^2} \frac{e^{\pm ikr - r/2l}}{\sqrt{2\pi kr}} e^{-i\omega(\mathbf{n})\sigma r/2v}. \quad (12)$$

They are the products of spherical waves and spin-rotation matrices describing the electron spin precession. The precession frequency  $\omega(\mathbf{n})$  depends on the propagation direction  $\mathbf{n} = \mathbf{r}/r$ . Due to the spin precession, the quantum correction to the effective cross section becomes spin dependent:  $\delta S(\Phi) \rightarrow \delta S^{\beta\gamma\alpha\theta}(\Phi)$ , where the spin indices  $\alpha, \beta, \gamma, \theta$  correspond to the electron trajectories, as shown in Figs. 1(b') and 1(c')

$$\frac{\delta S^{\beta\gamma\alpha\theta}(\Phi)}{S_0} = \frac{\lambda l}{\pi} W_\omega^{\beta\gamma\alpha\theta}(0) [\delta(\Phi - \pi) - 1/2\pi], \quad (13)$$

The correction to the Boltzmann collision integral can be written as

$$[\delta\hat{J}f(\mathbf{p}, \omega)]_{\beta\gamma} = N_{i\nu} \int \delta S_{\omega}^{\beta\gamma\alpha\theta}(\Phi - \Phi') f_{\alpha\theta}(\mathbf{p}', \omega) d\Phi', \quad (14)$$

where  $f_{\alpha\theta}(\mathbf{p}', \omega)$  is the momentum-dependent spin-density matrix. To derive the expression for  $W_{\omega}^{\beta\gamma\alpha\theta}$ , we introduce the probability density for a diffusing electron to arrive after  $N$  collisions at the point  $\mathbf{r}$  with the spin rotated by an angle  $\phi$ :

$$W_{\omega}^N(\mathbf{r}, \phi) = \int \delta(\phi - \phi_N) P(\mathbf{r} - \mathbf{r}_N) \dots P(\mathbf{r}_2 - \mathbf{r}_1) \times P(\mathbf{r}_1) d\mathbf{r}_1 \dots d\mathbf{r}_N. \quad (15)$$

The angle  $\phi_N$  changes with the coordinates of the scattering points  $\mathbf{r}_1, \dots, \mathbf{r}_N$ . One can find it from the matrix equation

$$e^{-i\phi_N\sigma/2} = e^{-i\epsilon(\mathbf{r}-\mathbf{r}_N)\sigma/2} e^{-i\epsilon(\mathbf{r}_N-\mathbf{r}_2)\sigma/2} \dots e^{i\epsilon(\mathbf{r}_2-\mathbf{r}_1)\sigma/2} e^{-i\epsilon(\mathbf{r}_1)\sigma/2}, \quad (16)$$

where  $\epsilon(\mathbf{r}) = \omega(\mathbf{n})r/v$ . The spin-dependent return probability is then expressed via the total probability density

$$W_{\omega}(\mathbf{r}, \phi) = \sum_N W_{\omega}^N(\mathbf{r}, \phi), \quad (17)$$

taken at  $\mathbf{r}=0$

$$W_{\omega}^{\beta\gamma\alpha\theta}(0) = \int \langle \beta | e^{-i\phi\sigma/2} | \alpha \rangle \langle \theta | e^{-i\phi\sigma/2} | \gamma \rangle W_{\omega}(0, \phi) d\Lambda, \quad (18)$$

where  $d\Lambda = g(\phi)d^3\phi$  and function  $g(\phi)$  is defined in Appendix A. (Note that, in the absence of the spin-orbit coupling,  $W_{\omega}(0, \phi) = \delta(\phi)W_{\omega}/g(\phi)$  and  $W_{\omega}^{\beta\gamma\alpha\theta}$  is expressed as  $W_{\omega}^{\beta\gamma\alpha\theta} = \delta_{\alpha\beta}\delta_{\gamma\theta}W_{\omega}$ .) What remains to be done is to find an equation for  $W_{\omega}(\mathbf{r}, \phi)$ . To this end, the probabilities  $W_{\omega}^N$  are related to each other by the recurrent equations:

$$g(\phi)W_{\omega}^{N+1}(\mathbf{r}, \phi) = \int P(\mathbf{r}')W_{\omega}^N(\mathbf{r} - \mathbf{r}', \phi') \times \delta(\phi - \phi' - \Delta) d\mathbf{r}' d\Lambda'. \quad (19)$$

The vector  $\Delta = \Delta_{\mathbf{r}', \phi'}$  describes the change of the spin rotation angle in a ballistic path between two scattering. One can find it from the following equation  $\exp[-i(\phi' + \Delta)\sigma/2] = \exp[-i\epsilon(\mathbf{r}')\sigma/2]\exp(-i\phi'\sigma/2)$ . Next, we sum Eq. (19) over  $N$  and take into account that  $\Delta \ll 1$ . After cumbersome but straightforward calculations, we find that  $W(\mathbf{r}, \phi, t) = \tau \int W_{\omega}(\mathbf{r}, \phi) \exp(-i\omega t) d\omega / 2\pi$  (the probability density to arrive after time  $t$  to the point  $\mathbf{r}$  with the spin rotated by the angle  $\phi$ ) is described by an equation similar to Eq. (11):

$$\frac{\partial W}{\partial t} - D \left( \frac{\partial}{\partial \mathbf{r}} - \frac{i\hat{\Omega}\hat{\mathbf{L}}}{v} \right)^2 W = \delta(t) \delta(\mathbf{r}) \delta(\phi) / g(0). \quad (20)$$

Here  $\hat{\mathbf{L}}$  is the angular momentum operator acting on the functions of vector  $\phi$  (see Ref. 17) and  $(\hat{\Omega}\hat{\mathbf{L}})_i = \Omega_{ik}\hat{L}_k$ . The explicit expressions for  $\hat{\mathbf{L}}$  and for the common eigenfunctions of  $\hat{L}^2$  and  $\hat{L}_z$  are presented in Appendix A. Expanding

$\delta(\phi)$  in a series over these functions, we keep the term with  $L=0$ . The corresponding eigenfunction  $\Psi_0$  is independent of the angle  $\phi$  and  $\hat{\mathbf{L}}\Psi_0=0$ . This is the only term which survives at  $t \gg \tau_S$ . The other terms decay exponentially with characteristic times of the order of  $\tau_S$ . (This statement is not true for a degenerate case, when  $\hat{\Omega}\hat{\mathbf{L}}$  depends on the component of  $\hat{\mathbf{L}}$  along a single axis. This case is discussed below.) As a result, we obtain the following expression for the asymptotical behavior of  $W(0, \phi, t)$ :

$$W(0, \phi, t) = \frac{1}{4\pi Dt}, \quad \text{for } t \gg \tau_S. \quad (21)$$

Now, we write the distribution function as

$$\hat{f} = \hat{I}f + \mathbf{s}\sigma, \quad (22)$$

where  $f$  is the particle density in the momentum space, related to the electron concentration by  $n = 2 \int f d\mathbf{p} / (2\pi\hbar)^2$ ,  $\hat{I}$  is the unit matrix, and  $\mathbf{s}$  is the spin density. Substituting Eq. (21) and Eq. (22) into Eq. (18) and Eq. (14), respectively, making a Fourier transform, and taking into account Eq. (13), we obtain the weak-localization-induced correction to the collision integral

$$(\delta\hat{J}f) = (\lambda l / \pi\tau^2) \int_{-\infty}^t dt' W(0, t-t') \times \int d\Phi' [\delta(\Phi - \Phi' - \pi) - 1/2\pi] f(\mathbf{p}', t'), \quad (23)$$

$$(\delta\hat{J}\mathbf{s})_i = (\lambda l / \pi\tau^2) \int_{-\infty}^t dt' W_{ik}(0, t-t') \times \int d\Phi' [\delta(\Phi - \Phi' - \pi) - 1/2\pi] s_k(\mathbf{p}', t'), \quad (24)$$

where  $W_{ik}(0, t)$  and  $W(0, t)$  are given by

$$W_{ik}(0, t) = \int \left( \delta_{ik} - 2e_i e_k \sin^2 \frac{\phi}{2} \right) W(0, \phi, t) d\Lambda, \quad (25)$$

$$W(0, t) = \int \cos \phi W(0, \phi, t) d\Lambda,$$

where  $\mathbf{e} = \phi/\phi$ . Using Eq. (21), we find the asymptotical behavior of these functions

$$W_{ik}(0, t) = \frac{\delta_{ik}}{8\pi Dt}, \quad W(0, t) = -\frac{1}{8\pi Dt}, \quad \text{for } t \gg \tau_S. \quad (26)$$

Note also that the spin-orbit coupling can be neglected in the time interval  $\tau \ll t \ll \tau_S$ . Hence,  $W(0, \phi, t) \sim \delta(\phi)$  and the expressions for  $W_{ik}$  and  $W$  become

$$W_{ik}(0,t) = \frac{\delta_{ik}}{4\pi Dt}, \quad W(0,t) = \frac{1}{4\pi Dt}, \quad \text{for } \tau \ll t \ll \tau_S. \quad (27)$$

We see that the difference between Eqs. (26) and (27) is in the numerical coefficients only.

### SOLUTION OF THE KINETIC EQUATION: THE LONG-LIVING TAIL IN THE SPIN POLARIZATION

For the case with a spin polarization uniform in space, the generalized kinetic equation is

$$\frac{\partial \mathbf{s}}{\partial t} = \boldsymbol{\omega}(\mathbf{n}) \times \mathbf{s} + \frac{\mathbf{s}_i - \mathbf{s}}{\tau} + \delta \hat{J} \mathbf{s}, \quad (28)$$

where  $\delta \hat{J} \mathbf{s}$  is given by Eq. (24). This equation can be solved in the usual way.<sup>2</sup> Since  $\boldsymbol{\omega}(\mathbf{n})\tau \ll 1$ , the spin density can be represented as a sum of the isotropic part  $\mathbf{s}_i(E,t)$ , which depends on the electron energy only, and a small anisotropic correction  $\mathbf{s}_a(\mathbf{p},t)$ , which is linear in the electron momentum  $\mathbf{p}$ :

$$\mathbf{s} = \mathbf{s}_i + \mathbf{s}_a. \quad (29)$$

Substituting Eq. (29) into Eq. (28), using Eq. (26), and taking into account the equalities

$$\delta \hat{J} \mathbf{s}_i = 0, \quad \langle \delta \hat{J} \mathbf{s}_a \rangle = 0, \quad (30)$$

(here the angular brackets stand for averaging over the momentum direction) we obtain a closed relation for  $\mathbf{s}_i$ :

$$\frac{\partial \mathbf{s}_i}{\partial t} = -\hat{\Gamma} \left( \mathbf{s}_i - \frac{1}{2\pi kl} \int_{-\infty}^t dt' \frac{\mathbf{s}_i(t')}{t-t'} \right). \quad (31)$$

Assuming that the spin polarization was created at  $t=0$  with a density  $\mathbf{s}_i(E,0)$  and neglecting the quantum correction, we get the exponential relaxation  $\mathbf{s}_i(E,t) = \theta(t)e^{-\hat{\Gamma}t}\mathbf{s}_i(E,0)$  [here  $\theta(t)$  is the theta function]. This solution is valid until  $e^{-t/\tau_S} \sim 1/\pi kl$ . For larger times, the spin polarization should be found from the condition that the right-hand side of Eq. (31) equals zero:  $\mathbf{s}_i(E,t) \approx (1/2\pi kl) \int_0^t dt' e^{-\Gamma t'} \mathbf{s}_i(E,0)/(t-t')$ . So we find that the spin polarization has a long-living power tail at large times

$$\mathbf{s}_i(E,t) = \frac{1}{2\pi kl} \frac{\hat{\Gamma}^{-1}}{t} \mathbf{s}_i(E,0). \quad (32)$$

To conclude this section, we note that we neglected in our calculations the electron dephasing due to inelastic scattering. Such dephasing can be accounted for phenomenologically by introducing the factor  $\exp(-t/\tau_\varphi)$  into the right-hand side of Eq. (32). Here  $\tau_\varphi$  is the phase-breaking time.

### DEGENERATE CASE

Equation (32) is invalid for the degenerate case, when the spin precession frequency  $\boldsymbol{\omega}(\mathbf{n})$  is parallel to a certain vector  $\mathbf{u}$  for any electron momentum:  $\boldsymbol{\omega}(\mathbf{n}) \parallel \mathbf{u}$  for any  $\mathbf{n}$ . In the

classical limit, the component of the electron spin parallel to  $\mathbf{u}$  does not relax<sup>18</sup>  $\Gamma_{uu}=0$  and the two perpendicular components relax with equal rates  $\Gamma_1=\Gamma_2=\Gamma$ , the off-diagonal components of  $\hat{\Gamma}$  being equal to zero ( $\Gamma_{1v}=\Gamma_{2v}=\Gamma_{12}=0$ ). This happens in symmetric quantum wells grown in the [110] direction,<sup>5</sup> as well as in asymmetric quantum wells grown in the [001] direction, due to the interplay between Dresselhaus and Rashba couplings.<sup>19,20</sup> To find the long-time asymptotic of the return probability, we write the formal solution of Eq. (20) as

$$W(0,\phi,t) = \int \frac{d^2q}{(2\pi)^2} e^{-D(\mathbf{q} - \hat{\Omega}\hat{\mathbf{L}}/v)^2 t} \delta(\phi)/g(0). \quad (33)$$

In the degenerate case, each of the three operators  $(\hat{\Omega}\hat{\mathbf{L}})_x$ ,  $(\hat{\Omega}\hat{\mathbf{L}})_y$ , and  $(\hat{\Omega}\hat{\mathbf{L}})_z$  is proportional to  $\hat{L}_u$ , which is the component of  $\hat{\mathbf{L}}$  along the precession axis. Therefore, these three operators commute with each other. Changing the integration variables  $\mathbf{q} \rightarrow \mathbf{q} - i\hat{\Omega}\hat{\mathbf{L}}/v$  in Eq. (33) we obtain

$$W(0,\phi,t) = \frac{\delta(\phi)}{4\pi g(0)Dt}. \quad (34)$$

Thus, after traveling around a closed loop, the electron spin does not rotate at all. This can be interpreted as follows.<sup>19</sup> For the degenerate case, the spin rotation angle is simply given by  $\phi = \int \boldsymbol{\omega} dt \sim \int \mathbf{p} dt$ . For a closed loop, we have  $\int \mathbf{p} dt = 0$  and, as a consequence,  $\phi=0$ . After substituting Eq. (34) into Eq. (25) and using Eqs. (24) and (28), one can see that the weak localization does not affect the longitudinal component of the spin density  $\mathbf{u}\mathbf{s}_i(E,t) = \mathbf{u}\mathbf{s}_i(E,0)$ , while the relaxation of the perpendicular component is described by the following equation

$$\frac{\partial \mathbf{s}_i}{\partial t} = -\Gamma \left( \mathbf{s}_i - \frac{1}{\pi kl} \int_{-\infty}^t dt' \frac{\mathbf{s}_i(t')}{t-t'} \right), \quad \mathbf{s}_i \perp \mathbf{u}. \quad (35)$$

The integral in the right-hand side of Eq. (35) contains an additional factor 2 as compared with that in Eq. (31). It can be shown that this factor arises from the contribution of the eigenfunctions with  $L=1$  to the long-time asymptotic Eq. (34) of the return probability.<sup>21</sup> The long-time asymptotic of the spin polarization is given by

$$\mathbf{s}_i(E,t) = \frac{1}{\pi kl} \frac{1}{\Gamma t} \mathbf{s}_i(E,0), \quad \text{where } \mathbf{s}_i(E,0) \perp \mathbf{u}. \quad (36)$$

### SUPPRESSION OF LONG-LIVING POLARIZATION BY THE MAGNETIC FIELD

Next, we consider the influence of the external magnetic field on the long-living power tail in the spin polarization. When an external field  $\mathbf{B}$  is present, the spin rotation matrices appearing in  $\hat{G}_R$  and  $\hat{G}_A$  become different:  $e^{-i\boldsymbol{\omega}(\mathbf{n})\sigma r/2v} \rightarrow e^{-i[\boldsymbol{\omega}(\mathbf{n})+\boldsymbol{\Omega}_0]\sigma r/2v}$  for  $\hat{G}_R$  and  $e^{-i\boldsymbol{\omega}(\mathbf{n})\sigma r/2v} \rightarrow e^{-i[\boldsymbol{\omega}(\mathbf{n})-\boldsymbol{\Omega}_0]\sigma r/2v}$  for  $\hat{G}_A$ . Here,  $\boldsymbol{\Omega}_0$  is the frequency of the spin precession in the field  $\mathbf{B}$ . So, Eq. (18) is modified as

$$W_{\omega}^{\beta\gamma\alpha\theta} = \int \langle \beta | e^{-i\phi\sigma/2} | \alpha \rangle \langle \theta | e^{-i\phi'\sigma/2} | \gamma \rangle W_{\omega}(0, \phi, \phi') d\Lambda d\Lambda', \quad (37)$$

where

$$W_{\omega}(\mathbf{r}, \phi, \phi') = \sum_N \int \delta(\phi - \phi_N) \delta(\phi' - \phi'_N) \times P_B(\mathbf{r} - \mathbf{r}_N) \dots P_B(\mathbf{r}_2 - \mathbf{r}_1) P_B(\mathbf{r}_1) d\mathbf{r}_1 \dots d\mathbf{r}_N. \quad (38)$$

Here,<sup>16</sup>

$$P_B(\mathbf{r} - \mathbf{r}') = P(\mathbf{r} - \mathbf{r}') \exp\left(i \frac{e\mathbf{B}_{\perp}}{\hbar c} [\mathbf{r} \times \mathbf{r}']\right), \quad (39)$$

and  $\mathbf{B}_{\perp}$  is the component of the magnetic field normal to the quantum well plane. The vectors  $\phi_N$  and  $\phi'_N$  should be found from Eq. (16) with  $\boldsymbol{\epsilon}(\mathbf{r}) = [\boldsymbol{\omega}(\mathbf{n}) + \boldsymbol{\Omega}_0]r/v$  for  $\phi_N$  and  $\boldsymbol{\epsilon}'(\mathbf{r}) = [\boldsymbol{\omega}(\mathbf{n}) - \boldsymbol{\Omega}_0]r/v$  for  $\phi'_N$ . Writing out the recurrent equations for the consecutive terms in Eq. (38), we can derive the following expression for the probability density  $W(\mathbf{r}, \phi, \phi', t) = \tau \int W_{\omega}(\mathbf{r}, \phi, \phi') \exp(-i\omega t) d\omega / 2\pi$ ,

$$\frac{\partial W}{\partial t} - i\boldsymbol{\Omega}_0(\hat{\mathbf{L}} - \hat{\mathbf{L}}')W - D \left( \frac{\partial}{\partial \mathbf{r}} - i \frac{2e\mathbf{A}}{c\hbar} - i \frac{\hat{\boldsymbol{\Omega}}(\hat{\mathbf{L}} + \hat{\mathbf{L}}')}{v} \right)^2 W = \delta(t) \delta(\mathbf{r}) \delta(\phi) \delta(\phi') / g^2(0). \quad (40)$$

Here the operators  $\hat{\mathbf{L}}$  and  $\hat{\mathbf{L}}'$  are given by Eq. (A1) (with the replacement  $\phi \rightarrow \phi'$  for  $\hat{\mathbf{L}}'$ ). The term  $-i2e\mathbf{A}/c\hbar$  in Eq. (40) is responsible for the magnetic field effect on the electron orbital motion.<sup>22</sup> Here  $\mathbf{A} = [\mathbf{B}_{\perp} \times \mathbf{r}]/2$  is the vector potential. We assume that the magnetic field is small

$$\Omega_0 \tau_S \ll 1, \quad \text{and} \quad DeB_{\perp} \tau_S / \hbar c \ll 1. \quad (41)$$

For  $\mathbf{B}=0$ , the solution of Eq. (40) is

$$W(\mathbf{r}, \phi, \phi', t) = W(\mathbf{r}, \phi, t) \delta(\phi - \phi') / g(\phi), \quad (42)$$

where  $W(\mathbf{r}, \phi, t)$  obeys Eq. (20). Using Eq. (42), we rewrite the long-living solution (21) (which corresponds to zero total angular momentum  $\hat{\mathbf{J}} = \hat{\mathbf{L}} + \hat{\mathbf{L}}'$ ) in terms of two angles  $\phi, \phi'$ :

$$\frac{1}{4\pi Dt} \frac{\delta(\phi - \phi')}{g(\phi)} = \int \frac{d^2q}{(2\pi)^2} e^{-\gamma(q)t} \frac{\delta(\phi - \phi')}{g(\phi)}, \quad \text{for } B=0. \quad (43)$$

In this equation  $\gamma(q) = Dq^2$  is the eigenvalue of the operator  $\hat{\gamma} = D(\mathbf{q} - \hat{\boldsymbol{\Omega}}\hat{\mathbf{J}}/v)^2$  at  $J=0$  and  $\delta(\phi - \phi')/g(\phi)$  is the respective eigenfunction. For  $B \neq 0$ , the long time dynamics of the spin relaxation is determined by the eigenvalues of the operator

$$\hat{\gamma} = -i\boldsymbol{\Omega}_0(\hat{\mathbf{L}} - \hat{\mathbf{L}}') + D(-i\partial/\partial\mathbf{r} - 2e\mathbf{A}/c\hbar - \hat{\boldsymbol{\Omega}}\hat{\mathbf{J}}/v)^2. \quad (44)$$

First, we neglect the term  $-i\boldsymbol{\Omega}_0(\hat{\mathbf{L}} - \hat{\mathbf{L}}')$ . In this approximation, the eigenvalues of  $\hat{\gamma}$ , corresponding to  $J=0$ , are given by

$$\gamma_n = \gamma_{\perp}(n + 1/2), \quad (45)$$

where  $\gamma_{\perp} = 4eB_{\perp}D/\hbar c$  and  $n=0, 1, 2$ , etc. Since the operator  $D(-i\partial/\partial\mathbf{r} - 2e\mathbf{A}/c\hbar)^2$  has a discrete spectrum, we have to make the replacement

$$\int \frac{d^2q}{(2\pi)^2} e^{-\gamma(q)t} \rightarrow \frac{\gamma_{\perp}}{4\pi D} \sum_n e^{-\gamma_n t} = \frac{\gamma_{\perp}}{4\pi D \sinh(\gamma_{\perp} t/2)} \quad (46)$$

in Eq. (43). The second step is to take into account the term  $-i\boldsymbol{\Omega}_0(\hat{\mathbf{L}} - \hat{\mathbf{L}}')$ , taking it to be a small perturbation. In the first order of the perturbation theory, this term leads to the mixing of the eigenfunctions with  $J=1$  to the eigenfunction with  $J=0$ . This mixing can be disregarded at  $\Omega_0 \ll \Gamma$ . Corrections to the eigenvalues  $\gamma_n$  arise in the second order only. They are calculated in Appendix B. Using Eq. (B5), we can show that the spin polarization dynamics at large times is described as

$$\mathbf{s}_i(E, t) = \frac{1}{2\pi k l} \frac{\gamma_{\perp} e^{-\Omega_0 \hat{\Gamma}^{-1} \Omega_0 t}}{2 \sinh(\gamma_{\perp} t/2)} \hat{\Gamma}^{-1} \mathbf{s}_i(E, 0). \quad (47)$$

Thus, we see that the magnetic field does suppress the long-living tail in the spin polarization.

## DISCUSSION

Next we discuss briefly the physical meaning of the results obtained. Our calculations were based on the interpretation of the weak localization phenomena in terms of two scattering processes: coherent backscattering [see Fig. 1(b)] and coherent scattering at an arbitrary angle [see Fig. 1(c')]. The existence of the long-living spin polarization can be explained as follows. Both coherent scattering processes were shown to be nonlocal in time. In other words, the transition of a spin  $\mathbf{s}$  to a spin  $\mathbf{s}'$ , caused by coherent scattering, takes a certain time  $t$  which may be relatively long:  $t \gg \tau_S$ . The coherent scattering events do not change the direction of the electron spin. Indeed, as seen from Eq. (26),  $W_{ik}(0, t) \sim \delta_{ik}$  and, as a consequence,  $\mathbf{s}' \parallel \mathbf{s}$ . Therefore, the electrons involved in such a scattering can keep memory about the initial spin polarization during the time much longer than  $\tau_S$ . The power law  $1/t$  is due to the proportionality of the probability of coherent scattering to the probability of diffusive return.

The collision integral accounting for both coherent processes does not change the total scattering cross section. Therefore,

$$\delta \hat{J}_i = 0, \quad \delta \hat{J}_s = 0. \quad (48)$$

Using Eqs. (23), (24), and (26), and taking into account that  $f_a(-\mathbf{p}, \omega) = -f_a(\mathbf{p}, \omega)$  and  $\mathbf{s}_a(-\mathbf{p}, \omega) = -\mathbf{s}_a(\mathbf{p}, \omega)$ , we obtain

$$\delta \hat{J}_a(\mathbf{p}, \omega) = \frac{\ln(1/\omega\tau)}{2\pi k l \tau} f_a(\mathbf{p}, \omega),$$

$$\delta \hat{J}_{s_a}(\mathbf{p}, \omega) = - \frac{\ln(1/\omega\tau)}{2\pi k l \tau} \mathbf{s}_a(\mathbf{p}, \omega), \quad \text{for } \omega\tau_S \ll 1. \quad (49)$$

Eqs. (48) and (49) imply that the effect of localization on the angular spin diffusion, as well as that on the particle diffu-

sion, can be accounted for by the  $\omega$ -dependent renormalization of the transport scattering time. However, the quantum corrections to this time have different signs for the particle and angular spin diffusion:

$$\frac{1}{\tau_{tr}} = \frac{1 - \ln(1/\omega\tau)/2\pi kl}{\tau}, \quad (50)$$

for the particle diffusion and

$$\frac{1}{\tau'_{tr}} = \frac{1 + \ln(1/\omega\tau)/2\pi kl}{\tau}, \quad (51)$$

for the spin diffusion. This implies that the Eq. (1) is invalid in the quantum case, or, more precisely, it relates  $\tau_S$  and  $\tau'_{tr}$  rather than  $\tau_S$  and  $\tau_{tr}$ .

An important comment should be made concerning the role of the coherent nonbackscattering contribution (processes  $c'$  in Fig. 1). Neglecting this contribution, we have  $\delta\hat{J}_s \neq 0$ . It can be easily shown that this leads to a physically meaningless result for the spin relaxation rate. Therefore, the correct treatment of the effect of weak localization on the spin relaxation is only possible when the nonbackscattering coherent processes are taken into account (the role of such effects for particle diffusion was discussed in Ref. 14). It is worth noting that the weak localization effects are usually considered to be due to the coherent backscattering only. The point is that the quantum correction to the conductivity is usually calculated by means of the Kubo formula, which expresses conductivity in terms of the current-current correlation function. This approach focuses on the calculation of the velocity correlation function, which depends on the anisotropic part of the distribution function  $f_a$ ; so, there is no need to know corrections to  $f_i$ . The situation is quite different for spin relaxation which is due to the relaxation of the isotropic part of the distribution function.<sup>23</sup>

To conclude, we have discussed the long-time dynamics of the spin polarization in a 2D disordered semiconductor. It is shown that, at large times, the spin relaxation slows down due to weak localization effects. An analytical expression for the long-living tail of the spin polarization has been derived. The magnetic field, both transversal and longitudinal, suppresses this tail, restoring the exponential relaxation.

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#### APPENDIX A

The analytical expression for the angular momentum operator  $\hat{\mathbf{L}}$  is

$$\hat{\mathbf{L}} = i \left\{ \frac{\partial}{\partial \phi} + \left( 1 - \frac{\phi}{2} \cot \frac{\phi}{2} \right) \left[ \mathbf{e} \times \left( \mathbf{e} \times \frac{\partial}{\partial \phi} \right) \right] - \left( \frac{\phi}{2} \times \frac{\partial}{\partial \phi} \right) \right\}, \quad (A1)$$

with  $\mathbf{e} = \phi/\phi$ . The components of this operator obey the usual commutation rules,  $[\hat{L}_i, \hat{L}_j] = i\epsilon_{ijl}\hat{L}_l$ . The common eigenfunctions of  $\hat{L}^2$  and  $\hat{L}_z$  are Wigner's rotation matrices (Ref. 17)  $D_{MM'}^L(\phi)$ :

$$\hat{L}^2 D_{MM'}^L(\phi) = L(L+1)D_{MM'}^L(\phi), \quad \hat{L}_z D_{MM'}^L(\phi) = MD_{MM'}^L(\phi). \quad (A2)$$

The values of  $L$  may be integer and half-integer  $L = 0, 1/2, 1, 3/2$ , etc. (as usual,  $M = -L, \dots, L$ ). There is also  $(2L+1)$  degeneracy with respect to  $M'$ . The eigenfunction corresponding to  $L=0$  is equal to unity

$$\Psi_0 = D_{00}^0 = 1, \quad \hat{\mathbf{L}}\Psi_0 = 0. \quad (A3)$$

The orthogonality conditions are<sup>17</sup>

$$\int d\Lambda D_{MM'}^L(\phi) D_{M_1 M_1'}^{L_1*}(\phi) = \delta_{LL_1} \delta_{MM_1} \delta_{M' M_1'} / (2L+1). \quad (A4)$$

Here the integration is made over all possible transformations of the SU(2) group. The integration measure is taken in the invariant form:

$$d\Lambda = g(\phi) d^3\phi, \quad \text{where } g(\phi) = \frac{1}{16\pi^2} \frac{\sin^2(\phi/2)}{(\phi/2)^2}. \quad (A5)$$

Note that SU(2) transformations are usually parameterized by Euler angles  $0 < \alpha < 2\pi$ ,  $0 < \beta < \pi$ ,  $-2\pi < \gamma < 2\pi$ . For such a parametrization, the invariant integration measure is given by<sup>17</sup>  $d\Lambda = \sin\beta d\alpha d\beta d\gamma / 16\pi^2$  (the expression for  $\hat{\mathbf{L}}$  via the Euler angles is also given in Ref. 17). It is convenient for us to use the components of vector  $\phi$  ( $0 < \phi < 2\pi$ ) instead of Euler angles. This gives<sup>24</sup>  $d\Lambda = g(\phi) d^3\phi$ . The expansion of the delta function  $\delta(\phi - \phi')$  in Wigner's functions is

$$\delta(\phi - \phi') = \sum_{L, M, M'} (2L+1) D_{MM'}^L(\phi) D_{MM'}^{L*}(\phi') g(\phi). \quad (A6)$$

The solution to Eq. (20) can be represented as a sum over all angular momenta  $L=0, 1/2, 1, 3/2$ , etc. However, as follows from Eq. (18) only two terms ( $L=0$  and  $L=1$ ) contribute to the spin-dependent return probability. Indeed, the action of the operator  $\mathbf{L}$  on the spin-rotation matrices is given by

$$\hat{\mathbf{L}} \langle \beta | e^{-i\phi\sigma/2} | \alpha \rangle = \frac{\sigma_{\beta\beta'}}{2} \langle \beta' | e^{-i\phi\sigma/2} | \alpha \rangle, \quad (A7)$$

$$\hat{\mathbf{L}}\langle\beta|e^{-i\phi\sigma^2}|\alpha\rangle\langle\theta|e^{-i\phi\sigma^2}|\gamma\rangle = \frac{\sigma_{\beta\beta'}\delta_{\theta\theta'} + \delta_{\beta\beta'}\sigma_{\theta\theta'}}{2} \times \langle\beta'|e^{-i\phi\sigma^2}|\alpha\rangle\langle\theta'|e^{-i\phi\sigma^2}|\gamma\rangle. \quad (\text{A8})$$

As follows from Eq. (A8), the projection of the operator  $\mathbf{L}$  onto the subspace formed by products of two rotation matrices is given by

$$\hat{\mathbf{L}}^* = \frac{\hat{\boldsymbol{\sigma}}^{(1)} + \hat{\boldsymbol{\sigma}}^{(2)}}{2}, \quad (\text{A9})$$

where  $\boldsymbol{\sigma}^{(1)}$  and  $\boldsymbol{\sigma}^{(2)}$  are the Pauli matrices acting on the first and second rotation matrices, respectively. Therefore, the angular momentum  $L^*$  can only be 0 or 1 (singlet and triplet contributions).

Using Eq. (20) and the property  $\int d\Lambda \Psi_1^* \hat{\mathbf{L}} \Psi_2 = -\int d\Lambda \Psi_2 \hat{\mathbf{L}} \Psi_1^*$ , valid for arbitrary functions  $\Psi_1(\phi)$  and  $\Psi_2(\phi)$ , we can see that the function

$$W^{\beta\gamma\alpha\theta}(\mathbf{r}, t) = \int \langle\beta|e^{-i\phi\sigma^2}|\alpha\rangle\langle\theta|e^{-i\phi\sigma^2}|\gamma\rangle W(\mathbf{r}, \phi, t) d\Lambda, \quad (\text{A10})$$

obeys the equation similar to Eq. (20)

$$\left[ \frac{\partial}{\partial t} - D \left( \frac{\partial}{\partial \mathbf{r}} + \frac{i\hat{\Omega}\hat{\mathbf{L}}^*}{v} \right)^2 \right]_{\beta\beta'\theta\theta'} W^{\beta\gamma\alpha\theta'} = \delta(t)\delta(\mathbf{r})\delta_{\alpha\beta}\delta_{\gamma\theta}, \quad (\text{A11})$$

where matrix elements  $\mathbf{L}_{\beta\beta'\theta\theta'}^*$  are given by Eq. (A8). Note that Eq. (A11) was used in Refs. 19, 25, and 26 for calculation of weak localization corrections to conductivity. The alternative derivation of the operator  $\hat{\mathbf{L}}^*$  was given in Ref. 27.

## APPENDIX B

Using Eq. (A6), we expand Eq. (43) as

$$\frac{\delta(\phi - \phi')}{g(\phi)} = \sum_L \sqrt{2L+1} \sum_{M_1=-L}^L \Psi_L^{M_1 M_1}, \quad (\text{B1})$$

where

$$\Psi_L^{M_1 M_2}(\phi, \phi') = \sqrt{2L+1} \sum_{M=-L}^L D_{MM_1}^L(\phi) D_{MM_2}^{*L}(\phi'), \quad (\text{B2})$$

is the full set of the eigenfunctions ( $L=0, 1/2, 1, 3/2$ , etc.;  $M_1=-L, \dots, L$ ;  $M_2=-L, \dots, L$ ) for zero total angular momentum  $\hat{\mathbf{J}}\Psi_L^{M_1 M_2}=0$ . In the second order of the perturbation theory, the term  $-i\Omega_0(\hat{\mathbf{L}}-\hat{\mathbf{L}}')$  leads to the corrections to the eigenvalues of the operator  $\hat{\gamma}$ . These corrections are different for the functions  $\Psi_L^{M_1 M_1}$  with different values of  $L$ . They are calculated as

$$D^{-1}\langle\Psi_L^{M_1 M_1}|\Omega_0(\hat{\mathbf{L}}-\hat{\mathbf{L}}')(\hat{\Omega}\hat{\mathbf{J}}/v)^{-2}\Omega_0(\hat{\mathbf{L}}-\hat{\mathbf{L}}')|\Psi_L^{M_1 M_1}\rangle = \Delta\gamma_L\delta_{M_1 M_1'}. \quad (\text{B3})$$

Next, we diagonalize the operator  $(\hat{\Omega}\hat{\mathbf{J}}/v)^2$  in the subspace formed by the following three functions  $(\hat{L}_x-\hat{L}'_x)\Psi_L^{M_1 M_1'}$ ,  $(\hat{L}_y-\hat{L}'_y)\Psi_L^{M_1 M_1'}$ , and  $(\hat{L}_z-\hat{L}'_z)\Psi_L^{M_1 M_1'}$ . Direct calculation gives

$$\Delta\gamma_L = \frac{4L(L+1)}{3}\Omega_0\hat{\Gamma}^{-1}\Omega_0. \quad (\text{B4})$$

The expression for the long-living solution becomes

$$W(0, \phi, \phi', t) \approx \frac{1}{4\pi D} \frac{\gamma_{\perp}/2}{\sinh(\gamma_{\perp}t/2)} \times \sum_L \sqrt{2L+1} e^{-\Delta\gamma_L t} \sum_{M_1=-L}^L \Psi_L^{M_1 M_1}, \quad (\text{B5})$$

where  $L=0, 1/2, 1, 3/2$ , etc. Substituting Eq. (B5) into Eq. (37) and using Eqs. (13), (14), and (22) we derive Eq. (47).

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