Electrical resistance of ballistic-electron transport through a finite disordered Fibonacci chain

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The average resistances of both a finite fully-random chain and a finite disordered Fibonacci chain are calculated as a function of the chain length. From these calculated average resistances, the localization lengths are computed and analyzed. The more the randomness of the system, the stronger the localization behavior will exhibit. The stronger the localization behavior, the smaller the localization length will be. A complete localization behavior for the fully-random chain with a small localization length is reproduced from our study as expected. However, only incomplete localization behavior is found for all the disordered Fibonacci chains with $p \neq 0.5$. Moreover, the values of localization lengths of all the disordered Fibonacci chains approach one larger common value due to suppression of the weak randomness by the strong self-similarity of the disordered Fibonacci chains. It turns out that the so-called disordered Fibonacci chain does not contain any more randomness than the pure Fibonacci chain. Our result agrees with the fact that the same Lyapunov exponent was found for both a pure Fibonacci chain and a disordered Fibonacci chain with random tiling that introduces phason flips at certain sites on the chain [see Phys. Rev. B **61**, 1043 (2000)].

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I. INTRODUCTION

Technical advances in submicron physics have made it possible for experimentalists to fabricate nearly ideal one-dimensional wires.[1] The connection between the electrical conductance at zero temperature and the transmission coefficient, provided by the well-known Landauer formula,[2] indicates that some experimentally measurable quantities can be adequately explained when a simple infinite one-dimensional array of short-range scatterers is considered. The discovery of quasicrystals[3] has stimulated interest in exploring the physical nature of quasiperiodic (e.g., Fibonacci and Thue-Morse) sequences[4] as well as commensurate-incommensurate systems.[5] Previous work on quasiperiodic sequences included, for example, plasmon excitation,[6] localization,[7] neutron polarization,[8] density-of-states,[9] optical-phonon tunneling,[10] nonlinear optical filters,[11] optical absorption in a random superlattice,[12] electric-field-induced localization,[13] and defect-assisted tunneling.[14] The quasiperiodicity in an infinite chain leads to a self-similar structure in the transmission of electrons as a function of their incident energy. The disorder (fully-random) chain introduced in this paper leads to the Anderson localization only when the chain becomes infinitely long. For a short disorder chain discussed in this paper, there is no complete localization, and only incomplete localization exists. However, when the chain length practically exceeds a threshold value, i.e., the localization length, the electron transport behaves very similar to that found when there is complete localization. In this case, the transmission of a long chain with disorder is extremely small.

The Fibonacci numbers, 1, 1, 2, 3, 5, 8, 13, … defined via the recurrence $F_{n+1}=F_n+F_{n-1}$, have been found to relate to the number of leaves, petals or seed grains in plants, and ancestors of a drone in nature. A straightforward stochastic modification of the Fibonacci sequence is to introduce both additions and subtractions. The random Fibonacci recurrence $x_{n+1}=x_n \pm x_{n-1}$ results in sequences which behave erratically for small generation index n. In the limit $n \to \infty$, however, exponential growth occurs with unit probability as was established by Furstenberg[15] in 1963. For the random Fibonacci recurrence where each $\pm$ sign is independent and occurs with probability $1/2$, the Lyapunov exponent $\lambda = 0.123\,975\,58\ldots$ is found.[16] Very recently, the renormalization-group method has been generalized[17] to study the local electronic properties of large disordered Fibonacci chains in which different generations of pure Fibonacci chains and inverted Fibonacci chains (see the definition in the next section) are randomly mixed (concatenation rule). The Lyapunov exponents for the weak and strong disordered cases can also be calculated using a perturbative expansion.[18] The Lyapunov exponents of both the pure and disordered Fibonacci chains were unexpectedly found to be the same.[19] An interesting question is whether the random mix of two different types of Fibonacci chains in a disordered Fibonacci chain will bring any additional randomness to electrical resistances of the system.

In this paper, we will calculate the transmission coefficients, resistances, and localization lengths for both the fully-random chain and the disordered Fibonacci chain. The more the randomness of the system, the stronger the localization behavior will exhibit. The stronger the localization behavior, the smaller the localization length will be. We will demonstrate how the localization is affected by the disorder and quasiperiodicity or a combination of two. From an analysis of the calculated localization lengths for the fully-random and disordered Fibonacci chains, we find that the so-called disordered Fibonacci chain with random tiling that introduces phason flips at certain sites on the chain does not bring any additional randomness to the electrical resistances of the system.

The organization of the paper is as follows. In Sec. II, we introduce our model and theory for the calculation of resistance through a chain and localization length with distributed scatterers. Numerical results are displayed in Sec. III for the transmission coefficients, resistances, distributions of scatterers, and localization lengths of both the fully-random chain and the disordered Fibonacci chain. The paper is briefly concluded in Sec. IV.

II. MODEL AND THEORY

By taking $\hbar = 2m^* = 1$, the scaled one-dimensional Schrödinger equation in the presence of scattering potential $V_s(x)$ is written as

$$-\frac{d^2\psi(x)}{dx^2} + V_s(x)\psi(x) = E\psi(x), \tag{1}$$

where E is the incident energy of an electron and $\psi(x)$ is the one-dimensional electron wave function. The short-range scattering potential for N_s scatterers can be approximated by

$$V_s(x) = \sum_{j=1}^{N_s} v_j \delta(x - x_j), \tag{2}$$

where v_j represents the strength of the jth scatterer sitting at $x=x_j$. If the positions of the scatterers for the $(n-1)$th and nth generations of a pure Fibonacci chain are $\{y_1, y_2, \ldots, y_{F_{n-1}}\}$ and $\{z_1, z_2, \ldots, z_{F_n}\}$, respectively, we can obtain the positions of the scatterers for the $(n+1)$th generation at $\{x_1, x_2, \ldots, x_{F_{n+1}}\}$ from[7]

$$x_j = \begin{cases} z_j & \text{for } j = 1, \ldots, F_n \\ z_{F_n} + y_{j-F_n} & \text{for } j = (F_n+1), \ldots, F_{n+1}. \end{cases} \tag{3}$$

with $x_1 = b$ and $x_2 = b + a$, where a and b are real numbers, F_n is the nth number in a Fibonacci sequence $\{F_1, F_2, \ldots, F_{n-1}, F_n, F_{n+1}, \ldots\}$. The numbers in the Fibonacci sequence are determined through the recurrence $F_{n+1} = F_n + F_{n-1}$ starting from $F_1 = 1$ and $F_2 = 2$.

In this paper, we will further define an inverted Fibonacci chain. For this case, instead of using Eq. (3) we find the positions of scatterers for the $(n+1)$th generation through a concatenation rule[17]

$$x_j = \begin{cases} y_j & \text{for } j = 1, \ldots, F_{n-1}, \\ y_{F_{n-1}} + z_{j-F_{n-1}} & \text{for } j = (F_{n-1}+1), \ldots, F_{n+1}. \end{cases} \tag{4}$$

The disordered Fibonacci chain is based on the random mix of the pure and inverted Fibonacci chains. We introduce a random variable η which is uniformly distributed within the interval (0, 1). For any given real number $0 \leqslant p \leqslant 1$, we define a disordered Fibonacci chain as follows: the disordered Fibonacci chain becomes the pure Fibonacci chain if $\eta < p$, while the disordered Fibonacci chain becomes the inverted Fibonacci chain if $\eta \geqslant p$. Therefore, the disordered Fibonacci chain will simply reduce to the pure Fibonacci chain as $p=1$. On the other hand, the disordered Fibonacci chain will reduce to the inverted Fibonacci chain as $p=0$. When $p=1/2$, however, there is an equal probability for the disordered Fibonacci chain becoming either the pure or the inverted Fibonacci chain. In this case, the randomness of the system reaches a maximum. The strength of the jth scatterer is assumed to be distributed according to

$$v_j = \begin{cases} q_1 & \text{for } s_j = a, \\ q_2 & \text{for } s_j = b, \end{cases} \tag{5}$$

where q_1 and q_2 are real numbers, the separation between the jth and $(j-1)$th scatterers is $s_j = x_j - x_{j-1}$ with $s_1 = x_1 = b$. For a fully-random chain, we choose the separation between the jth and $(j-1)$th scatterers randomly from $0 < s_j < b$, and the strength of the jth scatter randomly from $0 < v_j < q_1$.

Making use of the transfer matrix based on the boundary conditions, we can obtain the relationship between the wave functions $\psi_j^{(\mathrm{L,R})}(x)$ on the left (L) and right (R) sides of the jth scatterer. For electrons moving from left to right, this yields

$$\psi_j^{(\mathrm{R})}(x_j) = \mathcal{T}_j \psi_j^{(\mathrm{L})}(x_j), \tag{6}$$

where the (2×2) transfer matrix $\mathcal{T}_j$ is found to be

$$\mathcal{T}_j = \begin{bmatrix} 1 - iv_j/2k & -iv_j/2k \\ iv_j/2k & 1 + iv_j/2k \end{bmatrix}. \tag{7}$$

Here, $k = \sqrt{E}$ is the wave number of an incident electron along the chain. The propagation of the right-going plane wave between the jth and $(j+1)$th scatterers is related by

$$\psi_{j+1}^{(\mathrm{L})}(x_{j+1}) = \mathcal{D}_{j+1} \psi_j^{(\mathrm{R})}(x_j), \tag{8}$$

where $\mathcal{D}_j$ is the (2×2) displacement matrix, given by

$$\mathcal{D}_{j+1} = \begin{bmatrix} \exp(iks_{j+1}) & 0 \\ 0 & \exp(-iks_{j+1}) \end{bmatrix}. \tag{9}$$

The total (complex) transfer matrix $\mathcal{M}_{N_s}$ for the chain with N_s scatterers can be obtained from the product of a series of matrixes,

$$\mathcal{M}_{N_s} = (\mathcal{T}_{N_s}\mathcal{D}_{N_s}) \otimes (\mathcal{T}_{N_s-1}\mathcal{D}_{N_s-1}) \otimes \cdots \otimes (\mathcal{T}_2\mathcal{D}_2) \otimes \mathcal{T}_1 = \begin{bmatrix} M_{11}^{N_s} & M_{12}^{N_s} \\ M_{21}^{N_s} & M_{22}^{N_s} \end{bmatrix}, \tag{10}$$

where $N_s = F_n$ for the nth generation disordered Fibonacci chain. The total transmission coefficient $T_{N_s}(k)$ through the whole chain is given by[7]

$$T_{N_s}(k) = \frac{1}{|M_{22}^{N_s}|^2}. \tag{11}$$

With calculated transmission coefficient $T_{N_s}(k)$, the dimensionless resistance (in units of $h/2e^2$) $R_{N_s}(k)$ can be obtained by using the Landauer formula[2]

$$R_{N_s}(k) = \frac{1 - T_{N_s}(k)}{T_{N_s}(k)}. \tag{12}$$

By considering an incident electron wave packet with central wave number k_0 and broadening σ_0, we calculate the average

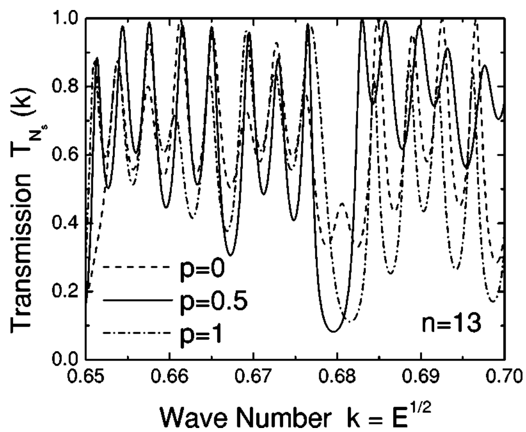

FIG. 1. Calculated transmission coefficients $T_{N_s}(k)$ as a function of electron wave number $k=\sqrt{E}$ for thirteenth-generation ($N_s=F_{13}$) disordered Fibonacci chains with $p=0$ (dashed curve), $p=0.5$ (solid curve), and $p=1$ (dashed-dotted curve), where E is the incident energy of an electron. The other parameters are given in the text.

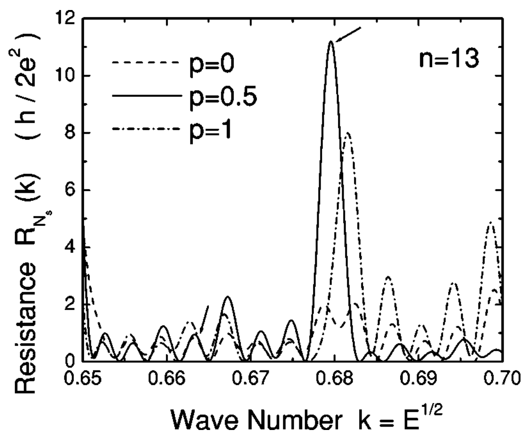

FIG. 2. Calculated resistance $R_{N_s}(k)$ as a function of electron wave number $k=\sqrt{E}$ for thirteenth-generation ($N_s=F_{13}$) disordered Fibonacci chains with $p=0$ (dashed curve), $p=0.5$ (solid curve), and $p=1$ (dashed-dotted curve). The other parameters are given in the text.

resistance over an interval $k\in[k_0-3\sigma_0,\ k_0+3\sigma_0]$, given by

$$\overline{\mathcal{R}}(L) = \frac{1}{I_{\max}+1}\sum_{i=0}^{I_{\max}} R(k_0 + (i - I_{\max}/2)\Delta k), \qquad (13)$$

where $I_{\max}=6\sigma_0/\Delta k$ with $\Delta k=5\times 10^{-5}$ in the following numerical calculations. The average resistance $\overline{\mathcal{R}}(L)$ depends on how scatterers are distributed in a chain. It also depends on the chain length L which equals $(F_{n-2}a+F_{n-1}b)$ (with $n\geqslant 2$) for the nth generation disordered Fibonacci chain and equals $\Sigma_{j=2}^{N_s}y_j$ for the fully-random chain with N_s scatterers. The localization length $\xi_{\rm loc}(L)$ can be calculated from $\overline{\mathcal{R}}(L)$ through[7]

$$\xi_{\rm loc}(L) = \frac{L}{\ln(1+\overline{\mathcal{R}}(L))}. \qquad (14)$$

For a complete localization, $\xi_{\rm loc}(L)$ should be independent of L. The more the randomness of the system, the stronger the localization behavior will exhibit. The stronger the localization behavior, the smaller the localization length will be. On the other hand, $\xi_{\rm loc}(L)$ depends on L in an oscillating way for an incomplete localization.[7]

III. NUMERICAL RESULTS AND DISCUSSIONS

In our numerical calculations, we have taken $q_1=0.5$, $q_2=\sqrt{2}/8$, $a=1$, and $b=\tau=(\sqrt{5}+1)/2$ for a disordered Fibonacci chain. For a fully-random chain, we choose $0<y_j<\tau$ and $0<v_j<0.5$. For the incident electron wave packet, we choose $k_0=0.675$ and $\sigma_0=1/12$. The values of p and the generation index n will be given in the captions.

Figure 1 displays the calculated transmission coefficients $T_{N_s}(k)$ as a function of electron wave number k for the thirteenth-generation ($n=13$) disordered Fibonacci chains with $p=0$ (dashed curve), $p=0.5$ (solid curve), and $p=1$ (dashed-dotted curve). Here, the total number of scatterers is $N_s=F_{13}$. From the figure, we find that $T_{N_s}(k)$ with $p=0.5$ resembles both the pure ($p=1$) and the inverted ($p=0$) Fibonacci chains. This fact is further supported by the calculated resistance $R_{N_s}(k)$ in Fig. 2, where the main peak of the disordered Fibonacci chain (indicated by an arrow) at $k=0.68$ resembles that of the pure Fibonacci chain nearby, while one of the side peaks of the disordered Fibonacci chain (indicated by another arrow) slightly above $k=0.66$ resembles that of the inverted Fibonacci chain at the same place. Moreover, the self-similarity, i.e., one main peak surrounded by multiple side peaks, can be seen from all the Fibonacci chains. The results presented in Fig. 1 with $n=13$ corresponds to a short chain, in contrast to those in a long chain with $n=17$ (not shown in this paper). There are many peaks of $T_{N_s}(k)$ close to unity as a function of k for several values of p. Here, electrons in the short chain are actually not localized.

As a comparison, we present in Fig. 3 the separation s_j of the jth and $(j-1)$th scatterers as a function of the site index j for the thirteenth-generation ($n=13$) disordered Fibonacci chains (solid and dashed curves with ■ and ●, respectively) with $p=0.5$ and $p=1$, and the fully-random chain (dashed-dotted curve with ▲). Here, $N_s=F_{13}$ again. Although the results of disordered Fibonacci chains with $p=0.5$ and $p=1$ resemble each other except for a shift, the result of the fully-random chain looks much more randomized. This randomness enhancement can be seen very clearly from Fig. 4, where the localization lengths $\xi_{\rm loc}(L)$ are shown as a function of the chain length L for the disordered Fibonacci chains (dotted, dashed, thick solid, dashed-dotted, dashed-dotted-dotted curves) with $p=0$, 0.25, 0.5, 0.75, 1 and for the fully-random chain (thin solid curve with upper and right scales indicated by two arrows). From Fig. 4 we find that $\xi_{\rm loc}(L)$ increases with L in a strong oscillating way (without localization) for the disordered Fibonacci chains, and the values of these localization lengths are bundled very well. On the other hand, the localization of the fully-random chain exhibits an approximate localization behavior with $\xi_{\rm loc}(L)$ becoming approximately independent of L when L becomes large.

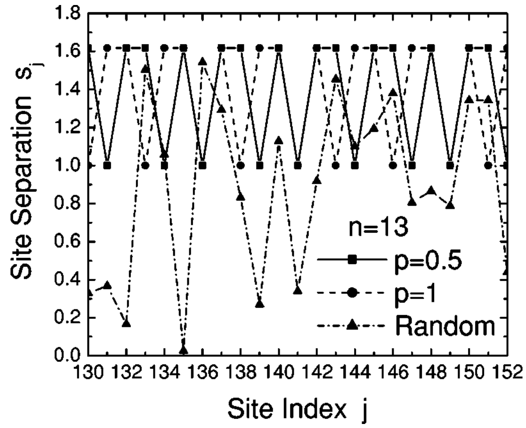

FIG. 3. Calculated separation s_j between the jth and $(j-1)$th scatterers as a function of scatterer position index $130 \leqslant j \leqslant 152$ for thirteenth-generation $(N_s = F_{13})$ disordered Fibonacci chains with $p = 0.5$ (solid curve with ■) and $p=1$ (dashed curve with ●). The result for a fully-random chain (dashed-dotted curve with ▲) is also shown for a comparison. The other parameters are given in the text.

Moreover, $\xi_{\text{loc}}(L)$ of the fully-random chain is more than an order of magnitude smaller than those of the disordered Fibonacci chains.

In order to demonstrate the complete localization behavior, we show in Fig. 5 the calculated localization lengths $\xi_{\text{loc}}(L)$ as a function of the chain length L for the seventeenth-generation $(n=17)$ disordered Fibonacci chains (dotted, dashed, thick solid, dashed-dotted, dashed-dotted-dotted curves) with $p=0$, 0.25, 0.5, 0.75, 1 and a longer fully-random chain (thin solid curve with upper and right scales indicated by two arrows). Here, the total number of scatterers is $N_s = F_{17} \gg F_{13}$. It is evident from this figure that the complete localization behavior can be seen for the fully-random chain with $\xi_{\text{loc}} \approx 14$ when L exceeds 600. On the other hand, the values of these localization lengths ξ_{loc} for all

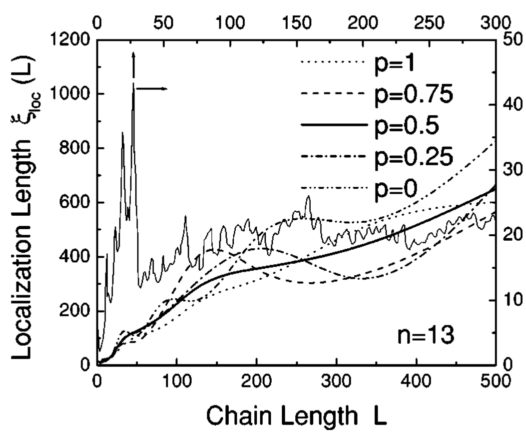

FIG. 4. Calculated localization length $\xi_{\text{loc}}(L)$ as a function of chain length L for thirteenth-generation disordered Fibonacci chains with $p=1$ (dotted curve), $p=0.75$ (dashed curve), $p=0.5$ (thick solid curve), $p=0.25$ (dashed-dotted curve), and $p=0$ (dashed-dotted-dotted curve). The result for a fully-random chain (thin solid curve with upper and right scales indicated by two arrows) is also shown for a comparison. The other parameters are given in the text.

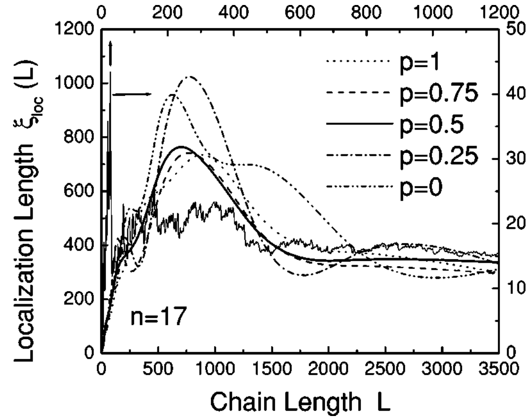

FIG. 5. Calculated localization length $\xi_{\text{loc}}(L)$ as a function of chain length L for seventeenth-generation disordered Fibonacci chains with $p=1$ (dotted curve), $p=0.75$ (dashed curve), $p=0.5$ (thick solid curve), $p=0.25$ (dashed-dotted curve), and $p=0$ (dashed-dotted-dotted curve). The result for a fully-random chain (thin solid curve with upper and right scales indicated by two arrows) is also shown for a comparison. The other parameters are given in the text.

values of $p \neq 0.5$ are bundled into one larger value around 300 in a weak oscillating way (incomplete localization) for the range of chain length shown here. The localization length of the disordered Fibonacci chain with $p=0.5$ (maximized randomness) also approaches the same value 300 as L exceeds 2000 in a monotonic way. As a comparison between Figs. 4 and 5, electrons in a short chain with $n=13$ are not completely localized. This is reflected in the oscillating $\xi_{\text{loc}}(L)$ as a function of chain length $L \leqslant 500$ in Fig. 4. However, the localization of electrons is nearly accomplished in a long chain with $n=17$, as shown in Fig. 5, when $L=3500$ is approached. This is reflected as a suppressed oscillation of $\xi_{\text{loc}}(L)$ for large L in Fig. 5. There is, however, no phase transition from metallic to insulator. When the chain length approaches infinity, all the disorder Fibonacci chains with $0 \leqslant p \leqslant 1$ have the same localization length. On the other hand, the infinite fully-random chain will have a much smaller localization length, implying a higher degree of randomness in the system. This unexpected behavior implies that the weak randomness in all the disordered Fibonacci chains are mostly suppressed by the strong self-similarity of them. This discovery qualitatively agrees with the fact that the same Lyapunov exponent λ was found for both the pure Fibonacci chain and disordered Fibonacci chain[19] with random tiling that introduces phason flips at certain sites on the chain since $\xi_{\text{loc}} \sim 1/\lambda$.

IV. CONCLUSIONS

In conclusion, we have calculated the average resistances of both the fully-random chain and the disordered Fibonacci chain as a function of the chain length, from which the localization lengths have been obtained and compared. From our study, we have found a complete localization behavior for the fully-random chain as expected. The more the randomness of the system, the stronger the localization behavior will exhibit. The stronger the localization behavior, the smaller the localization length will be. On the other hand, we have found only incomplete localization behavior for all the disordered Fibonacci chains with $p \neq 0.5$ when the generation index is large. Moreover, we have discovered that the values of localization lengths of all the disordered Fibonacci chains approach one common value. This has proved the unexpected suppression of the weak randomness by the strong self-similarity in the disordered Fibonacci chains. From this point of view, it seems that the so-called disordered Fibonacci chain does not contain any more randomness than the pure Fibonacci chain. This discovery agrees with the fact that the same Lyapunov exponent was found for both pure Fibonacci chain and disordered Fibonacci chain with random tiling that introduces phason flips at certain sites on the chain.[19]

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