

Nonadiabatic electron manipulation in quantum dot arrays

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A method of coherent manipulation of the electron tunneling in quantum dots is proposed, which utilizes the quantum interference in nonadiabatic double crossing of the discrete energy levels. In this method, we need only a smoothly varying gate voltage to manipulate electrons, without a sudden switching on and off. A systematic design of a smooth gate pulse is presented with a simple analytic formula to drive the two-level electronic state to essentially an arbitrary target state, and numerical simulations for complete transfer of an electron is shown for coupled double quantum dots and an array of quantum dots. Estimation of the manipulation time shows that the present method can be employed in realistic quantum dots.

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Quantum dots (QDs) have been attracting much attention recently because of the fundamental interest in the quantum-mechanical properties they show as man-made atoms and molecules.¹ The potential application of future quantum devices also stimulates the research interest in QDs. Especially, the recent success in the observation of a coherent oscillation of an electron in double quantum dots (DQDs) of semiconductors² enhanced the motivation to utilize QDs as elements of new information processing devices (i.e., qubits) based on the principle of quantum mechanics.³ Similar phenomena have been reported also for Josephson qubits.⁴ From a theoretical side, a number of proposals to control the tunneling coherence by applying time-dependent external fields have been presented, both for a double-well potential model⁵⁻⁷ and for a simplified two-level model.⁸⁻¹¹ See, Grifoni and Hänggi¹² for a review on driven tunneling systems.

The coherent manipulation of the electronic states is a prerequisite foundation for the realization of such quantum devices. Therefore, it is quite crucial to develop techniques on how to drive electronic states within the framework of quantum mechanics. The experimental realization of coherent manipulations of electronic states in QDs² and Josephson qubits⁴ has utilized the Rabi oscillation¹³ induced by a sudden switching of the gate voltage. For example, in order to transfer an excess electron from the left dot to the right dot, one applies a rectangular voltage pulse that brings the two-level system to the resonant condition suddenly, and after a half period of the Rabi oscillation, brings back to the off-resonant state suddenly again.² This requires high-frequency components in the gate fields, which in some cases may pose a difficulty in actual applications.

In the present work, we propose an alternative technique for the coherent manipulation. The process requires only a smoothly varying gate voltage in contrast to the manipulation by the Rabi oscillation. This is based on the nonadiabatic transition in level crossing systems. Therefore, we call it the nonadiabatic manipulation. It will be shown that, using a simple analytic formula, we can design a smooth temporal profile of the gate voltage that brings about desired transformation of the electronic state in coupled QDs. Our proposal here will extend the option for the coherent manipulation.

The principle of nonadiabatic manipulation is based on the celebrated Landau-Zener formula.^{14,15} The Landau-Zener mechanism has been recently studied as a possible implementation of qubit operations.¹⁶⁻¹⁸ With a suitable sequence of level crossing, interference effects and dissipative effects can be used to implement quantum memory devices.¹⁷ Interference patterns of qubits in a periodic controlling is useful to estimate a decoherence time of qubits.¹⁸ In the Landau-Zener mechanism, not only a transition probability but also a quantum phase is very important and useful for manipulation. In the present work, we use an asymptotically exact time evolution of the wave function to control a transition probability and a quantum phase. One aspect of controlling them is to use the transfer-matrix technique. The essence of the transfer matrix was derived by Zener¹⁵ as early as 1932, and its usefulness in elucidating the role of quantum coherence was shown in a periodically driven two-level system.⁹ In the following, we show some examples of nonadiabatic manipulation for a DQD and for an array of QDs.

First let us consider electronic states in a simple DQD, the Hamiltonian of which is given by

$$\mathcal{H}(t) = \varepsilon_1(t)|1\rangle\langle 1| + \varepsilon_2|2\rangle\langle 2| + \gamma(|1\rangle\langle 2| + |2\rangle\langle 1|), \quad (1)$$

where $|1\rangle$ and $|2\rangle$ describe the state in which the electron occupies a discrete level in the left and right dots, respectively. Without loss of generality, we can assume that only the energy of the state $|1\rangle$ is modulated by the gate voltage, since only the relative energy is relevant for the coherent dynamics. Let us take a gate voltage that drives $\varepsilon_1(t)$ from $t=0$ to $t=t_f$ as shown in Fig. 1: namely, the diabatic energy $\varepsilon_1(t)$ crosses ε_2 twice at t_1 and t_2 . Now we ask what the state vector $|\psi(t_f)\rangle$ for the two-level system is at the final state of the double crossing provided it starts from $|\psi(0)\rangle$. If the magnitude of the tunneling parameter γ is small enough, as is usually the case, the transition is localized around the level crossings. In such a situation, the time evolution of the two-level system can well be decomposed into a coherent succession of free propagations and the Landau-Zener-type impulsive transitions at around level-crossing times t_1 and t_2 .

We expand $|\psi(t)\rangle$ as

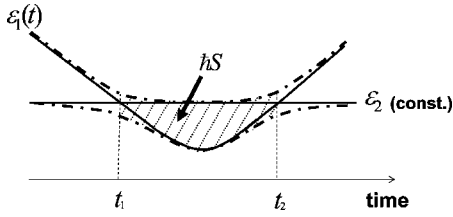


FIG. 1. Schematic picture of time dependence of energies of dots. The solid lines are the energies of two dots, and the dashed lines are the eigenvalues.

$$|\psi(t)\rangle = C_1(t)|1\rangle + C_2(t)|2\rangle, \quad (2)$$

and define a column vector $\mathbf{C}(t) \equiv [C_1(t), C_2(t)]^T$, where T means a transpose. Then, within the approximation of impulsive transitions, we have

$$\mathbf{C}(t_f) = U(t_f, t_2)T(t_2, t_1)U(t_1, 0)\mathbf{C}(0), \quad (3)$$

where $U(t'', t')$ is a free propagator,

$$U(t'', t') = \begin{pmatrix} e^{-\frac{i}{\hbar} \int_{t'}^{t''} du E_+(u)} & 0 \\ 0 & e^{-\frac{i}{\hbar} \int_{t'}^{t''} du E_-(u)} \end{pmatrix}, \quad (4)$$

in which $E_+(u)$ and $E_-(u)$ are the adiabatic eigenvalues for the upper and the lower branch, respectively, and are given by $E_{\pm} = \{\varepsilon_1(u) + \varepsilon_2 \pm \sqrt{[\varepsilon_1(u) - \varepsilon_2]^2 + 4\gamma^2}\}/2$. The matrix

$$K = \begin{pmatrix} qe^{i\frac{S}{2}} + (1-q)e^{-i(2\phi+\frac{S}{2})} & \sqrt{q(1-q)}[e^{-i(\phi+\frac{S}{2})} - e^{i(\phi+\frac{S}{2})}] \\ \sqrt{q(1-q)}[e^{-i(\phi+\frac{S}{2})} - e^{i(\phi+\frac{S}{2})}] & qe^{-i\frac{S}{2}} + (1-q)e^{i(2\phi+\frac{S}{2})} \end{pmatrix}, \quad (10)$$

where S is the relative phase,

$$S = \frac{1}{\hbar} \int_{t_1}^{t_2} \{E_+(u) - E_-(u)\} du,$$

which is proportional to the hatched area in Fig. 1. The matrix K plays an essential role to determine the population dynamics in our manipulation, while other factors only determine the relative phase. Note that the relative phase is easily controlled by the free propagation.

In order to realize the complete transfer, we tune the speed of passage to yield $q=1/2$, i.e., $v=2\pi\gamma^2/\hbar \log 2$. The K matrix is then reduced to

$$K = \begin{pmatrix} e^{-i\bar{\phi}} \cos \frac{\Theta}{2} & -i \sin \frac{\Theta}{2} \\ -i \sin \frac{\Theta}{2} & e^{i\bar{\phi}} \cos \frac{\Theta}{2} \end{pmatrix}, \quad (11)$$

where $\Theta \equiv S + 2\bar{\phi}$ is the phase factor which comes from the relative phase S between the double crossing and the Stokes

$T(t_2, t_1)$ represents the scattering by the double crossing. The time dependence of $\varepsilon_1(t)$ at the crossings is approximated by a linear function with the rate of change $v = |d(\varepsilon_1(t) - \varepsilon_2)/dt|$ measured at $t=t_1$ and t_2 . In the present work, we assume for simplicity that v is the same at t_1 and t_2 , although this is by no means a restriction. Then we find

$$T(t_2, t_1) = M^T V(t_2, t_1) M, \quad (5)$$

in which M is the transfer matrix

$$M = \begin{pmatrix} \sqrt{q} & -\sqrt{1-q}e^{i\phi} \\ \sqrt{1-q}e^{-i\phi} & \sqrt{q} \end{pmatrix}, \quad (6)$$

where $q \equiv \exp(-2\pi\delta)$ with $\delta \equiv \gamma^2/\hbar v$, and ϕ is the Stokes phase,

$$\phi = \pi/4 + \arg \Gamma(1 - i\delta) + \delta(\ln \delta - 1), \quad (7)$$

with the γ function $\Gamma(z)$. The propagator $V(t_2, t_1)$ is given by

$$V(t_2, t_1) = \begin{pmatrix} e^{-\frac{i}{\hbar} \int_{t_1}^{t_2} du E_-(u)} & 0 \\ 0 & e^{-\frac{i}{\hbar} \int_{t_1}^{t_2} du E_+(u)} \end{pmatrix}. \quad (8)$$

We find easily

$$T(t_2, t_1) = e^{-iS_1} K, \quad (9)$$

where $S_1 = 1/2\hbar \int_{t_1}^{t_2} du \{E_+(u) + E_-(u)\}$ and

phase $\bar{\phi}$. From Eq. (7), the Stokes phase is fixed to be $\bar{\phi} = 0.495039\dots$ If we introduce the Bloch vector \vec{p} defined for the density matrix $\rho \equiv |\psi(t)\rangle\langle\psi(t)|$ and the Pauli matrices $\vec{\sigma} \equiv (\sigma_x, \sigma_y, \sigma_z)$ by

$$\rho = \frac{1}{2}(1 + \vec{p} \cdot \vec{\sigma}), \quad (12)$$

the initial vector $\vec{p}_i = (0, 0, 1)$ which corresponds to $|\psi\rangle = |1\rangle$ is transformed by K into

$$\vec{p}_f = \left[\sin \Theta \cos \left(\bar{\phi} - \frac{\pi}{2} \right), \sin \Theta \sin \left(\bar{\phi} - \frac{\pi}{2} \right), \cos \Theta \right]. \quad (13)$$

The above formula indicates that we can drive a two-level system essentially to any desired state starting from the state $|1\rangle$ by controlling the relative phase S for the fixed value of v . If we set $\Theta = \pi(\text{mod } 2\pi)$, we have $\vec{p}_f = (0, 0, -1)$, which corresponds to the complete transfer from $|1\rangle$ to $|2\rangle$; the constructive interference between the two transition paths, $|1\rangle \rightarrow |1\rangle \rightarrow |2\rangle$ and $|1\rangle \rightarrow |2\rangle \rightarrow |2\rangle$ results in the complete trans-

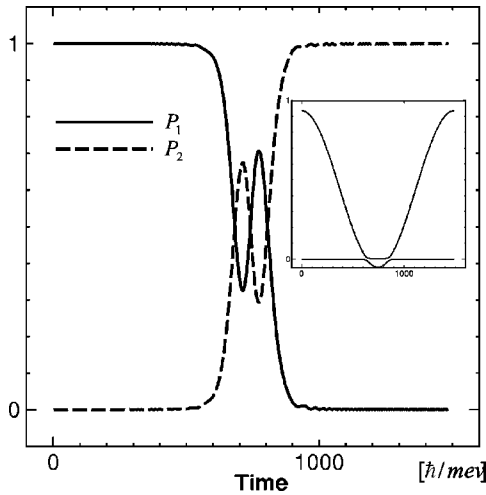


FIG. 2. The probabilities $P_1(t)$ and $P_2(t)$ are shown as a function of time. In the inset, the time dependences of eigenenergies are also shown. The unit of time is $\hbar/m\text{eV} \approx 4.14 \times 10^{-13}$ s.

fer. If we set, on the other hand, $\Theta = 2\pi(\text{mod } 2\pi)$, we have $\vec{p}_f = (0, 0, 1)$, which indicates the complete reflection to the initial state because of the destructive interference.¹⁹

Furthermore, if we choose $\Theta = \pi/2(\text{mod } 2\pi)$, the K matrix is reduced to

$$K = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\bar{\phi}} & e^{-i\pi/2} \\ e^{-i\pi/2} & e^{i\bar{\phi}} \end{pmatrix}. \quad (14)$$

It is an easy matter to see that, by an appropriate change of phase factors for the two-level system, the above K matrix is transformed into the Hadamard matrix,

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

For illustrative examples, we show some numerical simulations of electron transfer between coupled arrays of QDs. Our aim is to transfer an electron from QD to QD using smoothly varying gate voltages. In Fig. 2, the time dependence of the population in the left dot $P_1(t)$ and the right dot $P_2(t)$ are shown for the the initial condition $P_1(0)=1$ and $P_2(0)=0$. The parameter values are $\epsilon_1(0)=1.0$ meV, $\epsilon_2=0$, and $\gamma=0.01$ meV. The gate voltage is designed to modulate $\epsilon_1(t)$ as $\epsilon(t)=\epsilon_1(0)+A[\cos(\omega t)-1]$. The two parameters A and ω are adjusted to satisfy the conditions $v=2\pi\gamma^2/\hbar \log 2$ at $t=t_1, t_2$ and $\Theta=3\pi$. The time dependence of the adiabatic energies is shown in the inset. As shown in Fig. 2, the transfer of the electron is almost perfect. Error of manipulation can be estimated as the deviation of probability $P_2(t)$ from unity after one process. We found a quite small error $1-P_2(t) \sim 0.0011$. It should be noted that, although a large energy separation is required before and after the double-crossing events in order that the Landau-Zener theory works well, and that the stationary population in the respective dot is well defined, the energy separation in the intermediate state at $t \approx \pi/\omega$ need not be very large. This can be seen in the oscillation in $P_1(t)$ with a relatively large

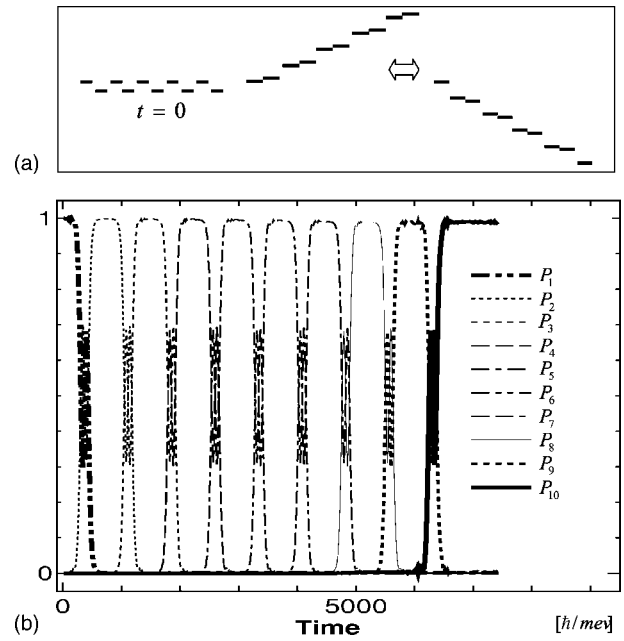


FIG. 3. Global control by an oscillating electric field. (a) Schematic picture of time dependence of the staggered dots model. (b) The probability finding electron in each dot is shown as a function of time.

amplitude in the intermediate state. Thus, we find that our recipe works well beyond our expectation, although it is based on the formalism of Zener which is asymptotically exact in the limit of scattering from $t=-\infty$ to $t=\infty$. We have also ascertained that the electron can be transferred from QD to QD one by one for an array of coupled QDs in which the energy levels are distributed randomly by a suitably designed temporal profile of the local gate voltages according to our method.

In contrast to the local control of the gate voltages, we may transfer an electron through an array of QDs by a global control using a time-dependent electric field, if the energy levels of the QDs are arranged regularly in a specific manner. For example, consider a *staggered-dots model* for which the Hamiltonian without the electric field is given by

$$\mathcal{H}_0 = \sum_{l=1}^n \epsilon_l^{(0)} |l\rangle\langle l| + \sum_{l=1}^{n-1} \gamma (|l\rangle\langle l+1| + \text{h.c.}), \quad (15)$$

where the energy levels are arranged alternately, $\epsilon_{2l-1}^{(0)} = \Delta$, $\epsilon_{2l}^{(0)} = 0$, as shown in Fig. 3(a). If an oscillating electric field $\mathbf{E}(t)$ is applied along the direction of the dot array, the energy of the l th dot is modulated as $\epsilon_l(t) = \epsilon_l^{(0)} + (l-1)eaE(t)$, where $-e$ is the electric charge and a is the separation between the dots, i.e., the *lattice constant*. By setting $E(t) = E_0 \sin(\omega t)$, we can attain a succession of double crossing between neighboring QDs [see Fig. 3(a)]. Each level undergoes a double crossing with those in the left dot and the right dot once in a period of oscillation. The velocity of the energy change is given by $v = |d(\epsilon_l(t) - \epsilon_{l+1})/dt| = ea|dE(t)/dt|$ measured at crossing times. We have carried out the simulation of the population dynamics for an array of 10 dots for parameter

values $\gamma=0.01$ meV, $\Delta=0.5$ meV. As shown in Fig. 3(b), starting from the dot at $l=1$, the electron can be transferred from QD to QD successfully to the final target dot after five oscillations of the global field. We obtain an error of manipulation as $1-P_{10}(t)\sim 0.018$ after one process. If the dots array is isolated, the electron is reflected at the right end of the array, and is transferred back to the left end, thus repeating a back-and-forth motion driven by the oscillating field. It is interesting to note that the direction of the transfer depends on the phase of the oscillating field. If we start from the dot at say $l=5$, it moves to the right by the field $E(t)=E_0\sin(\omega t)$, but to the left by $E(t)=-E_0\sin(\omega t)$. In this way, we may carry the electron from an initial dot to the target dot at will by an appropriate design of the temporal profile of the external electric field.

In actual materials, the quantum system is always disturbed by various sources of decoherence. First of all, in order that our proposal is realizable, the coherence must be maintained during the Landau-Zener transition time $\Delta\tau$.⁹ This is given, in the order of magnitude, as

$$\Delta\tau \approx \frac{2\gamma}{v} = \frac{\hbar \log 2}{\gamma \pi}. \quad (16)$$

In addition, the coherence time should be longer than the period between the double crossing $\Delta T \approx t_2 - t_1$. This is roughly given for $\Theta = \pi$ as

$$\Delta T \approx 2 \sqrt{\frac{\hbar S}{v}} = \frac{2\hbar}{\gamma} \sqrt{\frac{\log 2(\pi - 2\bar{\phi})}{2\pi}}. \quad (17)$$

If we use the value $\gamma \sim 10 \mu\text{eV}$, $\Delta\tau$ and ΔT are estimated as $\Delta\tau \approx 1.4 \times 10^{-11}$ s and $\Delta T \approx 6.3 \times 10^{-11}$ s, respectively. Thus $\Delta T + \Delta\tau \sim 7.7 \times 10^{-11}$ s will be required for the coherent time. This is of the same order as the manipulation time by the Rabi oscillation $\hbar\pi/2\gamma$, and is much shorter than the reported decoherence time in the order of 10 nsec.²

To summarize, we proposed a unique method to manipulate electronic states in quantum dots, which utilizes the quantum coherence between transition paths in the Landau-Zener-type successive level crossings. Starting from a localized state to one of the levels, we can reach arbitrary configuration of the two-level system. This may be regarded as a Mach-Zehnder-type interference *in the time domain*, in which the dynamical phase between the crossings plays a role of the optical path length. We would like to stress here that the interference effect between nonadiabatic transition paths in driven systems will provide quantum phenomena, and should be exploited for tools of electron manipulation.²² The coherent destruction of tunneling found by Grossmann *et al.*,⁶ for example, can be regarded as a result of destructive interference between transition paths.⁹

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¹⁹In this case, the state vector $|1\rangle$ is transformed into $e^{-i(S_1+\bar{\phi}-\pi)}|1\rangle$ by the scattering matrix $T(t_1, t_2)$. Since $-S_1$ is nothing but the nonadiabatic dynamical phase (Ref. 20), the additional phase $\chi_g \equiv \pi - \bar{\phi}$ is the geometrical phase for this specific nonadiabatic cyclic evolution (Ref. 21). In fact, it can be ascertained that χ_g is equal to half of the solid angle of the trace of \vec{p} subtended at the origin in the Bloch sphere.

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