Spin and orbital effects in a 2*d* electron gas in a random magnetic field

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Using the method of *superbosonization* we consider a model of a random magnetic field (RMF) acting on both orbital motion and spin of electrons in two dimensions. The method is based on exact integration over one particle degrees of freedom and reduction of the problem to a functional integral over supermatrices $Q(\mathbf{r},\mathbf{r}')$. We consider a general case when both the direction of the RMF and the g factor of the Zeeman splitting are arbitrary. Integrating out fast variations of Q we come to a standard collisional unitary nonlinear σ model. The collision term consists of purely orbital, spin, and some mixed parts. For a particular problem of a fixed direction of RMF, we show that additional soft excitations identified with spin modes should appear. Considering δ-correlated weak RMF and putting $g=2$ we find the transport time τ_{tr}. This time is two times smaller than that for spinless particles.

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I. INTRODUCTION

Models of a random magnetic field (RMF) acting on electrons in two dimensions are intensively studied in mesoscopic physics. There are direct experiments on high-mobility heterostructures subjected to a magnetic field of randomly pinned flux vortices in a type-II superconducting gate,[1] type-I superconducting grains,[2] or a demagnetized ferromagnet.[3] For theoreticians, the RMF models are important as an example of systems with an interaction reduced to a gauge field. These models arise in theory of composite fermions in the fractional quantum Hall effect near half filling[4] as well as in a description of doped Mott insulators.[5]

From the theoretical point of view, one of the most interesting tasks in the study of any model with a disorder is to determine the large scale behavior of electrons and find universal properties of the model. In doing so, one may come either to a metal or insulating behavior. In the latter case electron wave functions should be localized. In the context of the RMF models the localizations have been discussed in many numerical works, which resulted in three different conclusions: (1) localization of all the states;[6–8] (2) existence of a band of delocalized states;[9–14] and (3) localization of all the states except those in the band center.[15–17]

Analytically, the RMF models were studied by the standard diagrammatic technique[18] as well as using different nonlinear σ models.[18–21] The final conclusion drawn by using the methods was that the RMF models belonged to the usual unitary class of universality with localization in two dimensions (provided the correlations of the RMF $\langle B_q B_{-q}\rangle$ at small $q\to 0$ increase slower than $1/q^2$, Ref. 20).

Usually, when deriving the proper σ model for the RMF problems one uses the standard scheme[22] based on the saddle-point approximation. Calculations presented in Refs. 18–20 are performed in this way. From the point of view of the conventional perturbation theory this approximation corresponds to the self-consistent Born approximation, which is not good for a long range disorder. The same approximation has been used in the diagrammatic approach of Ref. 18.

In addition to the difficulty of a description of the long range disorder, the standard scheme is not convenient for a generalization of the RMF model for the case when spin degrees of freedom are important. This is because the effect of the magnetic field on the orbital electron motion is accounted for by adding a vector potential in the Hamiltonian, whereas the interaction with the electron spin is described by the magnetic field itself. As the correlations of the vector potentials and the magnetic fields are different, it is not easy to consider both the effects on equal footing.

This problem has been partly resolved in a recent paper[23] for free electrons in 2*d* with the g-factor $g=2$ and a magnetic field perpendicular to the plane. Unfortunately, a mathematical trick of replacing the initial Hamiltonian by a dirac Hamiltonian used in that paper, which was the basis of the suggested calculation scheme, can be applied only for this particular system.

Another analytical method suggested in Ref. 21 enables one to avoid using the saddle-point approximation and to obtain a ballistic σ model applicable at all distances down to the Fermi wavelength λ_F. This method is based on using quasiclassical equations of motion which contain not the vector potential but the magnetic field only. As the vector potential itself does not enter the ballistic σ model one can include rather easily the Zeeman term without extra assumptions about the value of the g factor and the direction of the magnetic field. However, this is still not the most general method because it is essentially based on the quasiclassical approximation and therefore short range correlations of the magnetic field cannot be considered.

In the present paper we use a method of *superbosonization* suggested recently.[24] The method is based on the exact integration over the one particle motion and reformulation of the initial fermionic problem in terms of a functional integral over supermatrices $Q(\mathbf{r},\mathbf{r}')$. As it has been shown in Ref. 24, in the quasiclassical regime (smooth disorder and lengths larger than λ_F) the matrix $Q(\mathbf{r},\mathbf{r}')$ corresponds to the matrix $Q_{\mathbf{n}}(\mathbf{r})$ of Ref. 25. The method of the superbosonization has several advantages. First, it uses neither saddle-point nor

quasiclassical approximations and is exact. Both short and long range disorder can be considered on its basis. Second, integration over one particle motion is carried out before disorder averaging. The latter enables one to consider interaction effects related to the random scattering more carefully. Finally, the method results in a simple expression for the energy in terms of the matrices $Q(\mathbf{r},\mathbf{r}')$ and makes the disorder averaging a rather simple procedure.

The theory is invariant with respect to rotations of the matrix $Q(\mathbf{r},\mathbf{r}')$ in the superspace provided they commute with the Hamiltonian $\hat{H}$. This makes possible the separation of the massive modes from the soft ones and we can integrate them out. We show that this procedure is justified only in the regime when the scattering effects do not result in a strong coupling between electrons. For the short range disorder this is so over distances exceeding the correlation length of the disorder (or the Fermi wavelength λ_F) whereas for the smooth disorder the coupling becomes sufficiently weak beyond the *Lyapunov length*. The latter case was discussed in many details in Refs. 26 and 27.

The low energy theory found is described by a ballistic nonlinear σ model with a collision term. This term consists of three parts that can be related to the purely orbital and spin scatterings and some mixture of the first two. The last scattering originates from the interference between orbital and spin motions in the same configuration of RMF. The second scattering, similar to the model with magnetic impurities (see Ref. 22), results generally in the relaxation of all spin modes. Accordingly, the theory applicable in the diffusive limit is described by supermatrices $Q(\mathbf{r})$ proportional to the unit matrix in the spin space and corresponding to the unitary ensemble.

Further, we consider a particular model with a fixed direction of RMF. We show that an additional soft mode related to fluctuations of the spin along the field should appear in this case. Using a standard method of integrating out the angle modes we come to the diffusive σ model with both the charge and spin modes and find the transport time τ_{tr}. Due to the magnetic field the time reversal symmetry is broken and the supermatrix $Q(\mathbf{r})$ entering the obtained energy functional should be diagonal in the particle-hole space. At the same time, the presence of the additional spin mode changes the form of the supermatrix $Q(\mathbf{r})$ so that it partially preserves the spin structure. The latter means that the model discussed is not necessarily related to the usual unitary class of symmetry.

The paper is organized as follows. In Sec. II we discuss the superbosonization procedure and give an alternative derivation of superbosonized theory. At the end of the section we discuss symmetry properties of the obtained model and derivation of the nonlinear σ model.

In Sec. III we introduce the main definitions of the RMF model involved, derive the σ model valid in the collisional regime and discuss conditions of its applicability.

In Sec. IV the same problem is considered in the diffusive limit. We calculate with the help of the obtained diffusive σ model the transport time and the spin susceptibility.

II. SUPERBOSONIZATION AND DERIVATION OF THE σ MODEL

Below we consider a two-dimensional ($2d$) electron gas placed in a static inhomogeneous magnetic field. The field is assumed to act both on the orbital motion and electron spin. Our consideration is based on the method of *superbosonization* proposed in a recent paper.[24] This method uses exact integration over electron degrees of freedom and reformulation of the problem in terms of integrals over supermatrices with a rotational symmetry (nonlinear σ model).

Before starting the study of the RMF problem we would like to give an alternative derivation of the superbosonized model. Although, the scheme of the derivation presented in Ref. 24 is straightforward and exact, the final representation of the Green's function in terms of a functional integral over the supermatrices can be obtained even in a more simple way. Following the standard way (see the book[22]) we introduce first a generating functional $Z[a]$

$$Z[a] = \int \exp(-L_a[\psi])D\psi, \tag{2.1}$$

where the Lagrangian $L_a[\psi]$ has the form

$$L_a[\psi] = -i\int \overline{\psi}(\mathbf{r})\left(\hat{H}_{\mathbf{r}} - \epsilon + \frac{\omega}{2} + \frac{\omega + i\delta}{2}\Lambda\right)\psi(\mathbf{r})d\mathbf{r} + i\int \overline{\psi}(\mathbf{r})a(\mathbf{r},\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}d\mathbf{r}', \tag{2.2}$$

$\hat{H}_{\mathbf{r}}$ is Hamiltonian of the initial model (we write it below) and $\psi(\mathbf{r})$ is a supervector. The conjugated supervector $\overline{\psi}(\mathbf{r})$ is related to $\psi(\mathbf{r})$ according to the following definition:

$$\overline{\psi}(\mathbf{r}) = (C\psi)^T(\mathbf{r}). \tag{2.3}$$

The structure of the supervectors $\psi(\mathbf{r})$, $\overline{\psi}(\mathbf{r})$ as well as the form of the matrix C depends on the problem involved (for details see Ref. 22). For the problem of electrons with spin considered below $\psi(\mathbf{r})$ should be a 16-component supervector.

The Lagrangian $L_a[\psi]$, Eq. (2.2), differs from the conventional one by the presence of the source $a(\mathbf{r},\mathbf{r}')$. In general, the source term is for the problem considered an arbitrary 16×16 supermatrix depending on two coordinates $\mathbf{r}$, $\mathbf{r}'$. Differentiating in elements of this matrix one can obtain various correlation functions. For example, the level-level correlation function $R(\omega)$ can be written as

$$R(\omega) = \frac{1}{2} - \frac{1}{2(\pi\nu V)^2}\lim_{\alpha_1,\alpha_2=0}\mathrm{Re}\,\frac{\partial^2}{\partial\alpha_1\partial\alpha_2}Z[a] \tag{2.4}$$

provided the source $a(\mathbf{r},\mathbf{r}')$ is taken in the following form:

$$a(\mathbf{r},\mathbf{r}') = \hat{a}(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')$$

$$\hat{a}(\mathbf{r}) = \begin{pmatrix} \hat{\alpha}_1 & 0 \\ 0 & -\hat{\alpha}_2 \end{pmatrix}, \quad \hat{\alpha}_{1,2} = \frac{\alpha_{1,2}}{2}(1-k), \tag{2.5}$$

$k=\pm 1$ in fermionic and bosonic blocks, respectively. As supermatrix $\psi_\alpha(\mathbf{r})\overline{\psi}_\beta(\mathbf{r}')$ is self-conjugated, one may consider only self-conjugated sources

$$\overline{a}(\mathbf{r},\mathbf{r}') \equiv Ca^T(\mathbf{r}',\mathbf{r})C^T = a(\mathbf{r},\mathbf{r}'). \tag{2.6}$$

Sources with constraint Eq. (2.6) have a minimal number of elements required when finding an arbitrary correlation function.

The integral defined in Eqs. (2.1) and (2.2) is Gaussian and can be readily calculated

$$Z[a] = \exp\left(\frac{1}{2}\mathrm{Str}\ln\left[\hat{H} - \epsilon + \frac{\omega}{2} + \frac{\omega + i\delta}{2}\Lambda - a\right]\right). \tag{2.7}$$

The logarithmic derivative of the partition function $Z[a]$, Eq. (2.7), in a is a matrix Green's function $G(\mathbf{r},\mathbf{r}')$

$$\frac{\delta \ln Z[a]}{\delta a(\mathbf{r},\mathbf{r}')} = \frac{i}{2}G(\mathbf{r},\mathbf{r}') \tag{2.8}$$

satisfying the following equation:

$$\left(\hat{H}_{\mathbf{r}} - \epsilon + \frac{\omega}{2} + \frac{\omega + i\delta}{2}\Lambda - a\right)G(\mathbf{r},\mathbf{r}') = i\delta(\mathbf{r}-\mathbf{r}'). \tag{2.9}$$

Calculating the Green's function $G(\mathbf{r},\mathbf{r}')$ from Eq. (2.9) and using Eq. (2.8) one can find the partition function $Z[a]$, Eqs. (2.1) and (2.7).

What we want to show now is that the Green's function, Eq. (2.9), can be represented exactly as an integral over supermatrices $Q(\mathbf{r},\mathbf{r}')$ in the following form:

$$G(\mathbf{r},\mathbf{r}') = \tilde{Z}^{-1}[a]\int Q(\mathbf{r},\mathbf{r}')\exp(-F_a[Q])DQ, \tag{2.10}$$

where $\tilde{Z}[a]$ is a partition function

$$\tilde{Z}[a] = \int \exp(-F_a[Q])DQ \tag{2.11}$$

and the functional $F_a[Q]$ has the form

$$F_a[Q] = \frac{i}{2}\mathrm{Str}\int\left(\hat{H}_{\mathbf{r}} - \epsilon + \frac{\omega}{2} + \frac{\omega + i\delta}{2}\Lambda\right) \times \delta(\mathbf{r}-\mathbf{r}')Q(\mathbf{r},\mathbf{r}')d\mathbf{r}d\mathbf{r}' + \frac{1}{2}\mathrm{Str}\ln Q - \frac{i}{2}\mathrm{Str}\int a(\mathbf{r},\mathbf{r}')Q(\mathbf{r}',\mathbf{r})d\mathbf{r}d\mathbf{r}'. \tag{2.12}$$

Supermatrix $Q(\mathbf{r},\mathbf{r}')$ in the integral Eq. (2.10) is not arbitrary and should have a certain structure. First, it must be self-conjugated

$$\overline{Q}(\mathbf{r},\mathbf{r}') = Q(\mathbf{r},\mathbf{r}') \tag{2.13}$$

to provide the same for the result of the integration in Eq. (2.10).

Second, the structure of the supermatrix $Q(\mathbf{r},\mathbf{r}')$ should be defined such that the energy $F_a[Q]$, Eq. (2.12), would have a minimum. One can show that this takes place if $\mathrm{Str}(Q^2)$ is non-negative. The latter requirement is fulfilled for the supermatrices with the following constraint:

$$Q_\perp(\mathbf{r},\mathbf{r}') = KQ_\perp^+(\mathbf{r}',\mathbf{r})K, \quad K = \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}, \tag{2.14}$$

where $Q_\perp = (1/2)\Lambda[\Lambda, Q]$, k is the same as in Eq. (2.5). Relation (2.14) will be used below when formulating the structure of a σ model.

Equations (2.10)–(2.12) have been suggested in Ref. 24 and we want to demonstrate now a simpler way of their derivation. In order to prove Eq. (2.10) we write the following identity:

$$-2i\tilde{Z}^{-1}[a]\int\left[\int\frac{\delta\exp(-\frac{1}{2}\mathrm{Str}\ln \mathrm{Q})}{\delta Q(\mathbf{r}'',\mathbf{r})}Q(\mathbf{r}'',\mathbf{r}')d\mathbf{r}''\right] \times \exp\left(-\frac{i}{2}\mathrm{Str}\left[\hat{H} - \epsilon + \frac{\omega}{2} + \frac{\omega + i\delta}{2}\Lambda - a\right]Q\right)DQ = i\delta(\mathbf{r}-\mathbf{r}'), \tag{2.15}$$

and integrate over Q by parts. The derivative $\delta/\delta Q$ should act now on both Q and the exponential. At this point, the supersymmetry plays a crucial role. Differentiating first the supermatrix Q we obtain the supermatrix product $(\delta/\delta Q)Q$. As the number of the anticommuting variables in the sum over the matrix elements is equal to the number of the boson ones and the derivatives have the opposite signs, this matrix product vanishes. Differentiating the exponential only we come to the following equation:

$$\tilde{Z}^{-1}[a]\int d\mathbf{r}''\left(\hat{H} - \epsilon + \frac{\omega}{2} + \frac{\omega + i\delta}{2}\Lambda - a\right)(\mathbf{r},\mathbf{r}'') \times \int Q(\mathbf{r}'',\mathbf{r}')\exp(-F_a[Q])DQ = i\delta(\mathbf{r}-\mathbf{r}'). \tag{2.16}$$

Equation (2.16) proves immediately that the integral Eq. (2.10) does satisfy Eq. (2.9) and we really have the alternative representation of the Green's function in terms of an integral over the supermatrices Q.

Integrating Eq. (2.10) over the source $a(\mathbf{r},\mathbf{r}')$ we conclude that partition functions $Z[a]$, Eqs. (2.1) and (2.7) and $\tilde{Z}[a]$, Eq. (2.11), are proportional to each other with a coefficient that is independent of the source. Putting $a(\mathbf{r},\mathbf{r}')=0$ in the definitions of both functions and finding that $Z[a=0] = \tilde{Z}[a=0]=1$ due to supersymmetry we come to the equality

$$Z[a] = \tilde{Z}[a]. \tag{2.17}$$

We emphasize that the relation (2.17) is exact and does not depend on the form of the Hamiltonian $\hat{H}_{\mathbf{r}}$, Eq. (2.2), and the source $a(\mathbf{r},\mathbf{r}')$ provided both the partition functions are defined in Eqs. (2.1) and (2.11) in terms of the superintegrals.

In the low energy limit, the field theory specified by Eq. (2.12) can be reduced to an effective nonlinear σ model.[24] A possibility of this reduction is related to the invariance of the free energy functional F_a with respect to the rotations of the supermatrix Q

$$Q \to VQ\overline{V} \tag{2.18}$$

for the case when the unitary matrix $V\overline{V}=1$ commutes with the Hamiltonian $\hat{H}$, Eq. (2.2). This means that the low energy limit of the model is determined by rotations V with small values of commutator $[\hat{H},V]$. For different problems the smallness of the commutator can be provided by imposing on the rotations V different conditions. For the RMF model we discuss this point at the end of Sec. III.

In order to reduce the general formula, Eq. (2.12), to a σ model containing only the rotations V we represent the supermatrix Q in the form

$$Q = Vq\overline{V}, \quad V\overline{V}=1, \tag{2.19}$$

where q is a diagonal matrix $[q,\Lambda]=0$. One can impose the condition $V\Lambda=\Lambda\overline{V}$ to fix the gauge freedom related to the invariance of the definition, Eq. (2.19), with respect to the replacement $V\to Vv$, where $[v,\Lambda]=0$. The spectrum of fluctuations of the matrix q has a gap and they can be considered using the saddle-point approximation. Substitution of Eq. (2.19) into the free energy functional, Eq. (2.12), and integration over q carried out in the quadratic in fluctuations of q approximation results in the following expression for the effective free energy functional:

$$F[V] = \frac{i}{2}\mathrm{Str}\left(\overline{V}\left[\hat{H}+\frac{\omega+i\delta}{2}\Lambda, V\right]g\right) - \frac{1}{4}\mathrm{Str}(\overline{V}[\hat{H},V]g)^2, \tag{2.20}$$

where g is the Green's function, Eq. (2.9), in the absence of the source $a=0$. This expression has been obtained in Ref. 24 and is just the first two terms of the expansion of the free energy functional in powers of the commutator $[\hat{H},V]$. As it has been shown in Ref. 24, this approximation is sufficient if the scattering is rather weak. We use Eq. (2.20) as the starting point for the study of the RMF problem and consider the limit of a weak field.

III. RMF: FORMULATION OF THE PROBLEM. REDUCTION TO COLLISIONAL σ MODEL

In order to take into account the spin of the electrons one should double the number of variables and consider in Eqs. (2.1) and (2.2) 16-component supervectors $\psi(\mathbf{r})$. The most convenient choice for their structure is as follows (see also Ref. 22)

$$\psi = \begin{pmatrix} \psi^1 \\ \psi^2 \end{pmatrix}, \quad \psi^m = \begin{pmatrix} \vartheta^m \\ v^m \end{pmatrix},$$

$$\vartheta^m = \frac{1}{\sqrt{2}}\begin{pmatrix} \chi^{m*} \\ i\sigma_y\chi^m \end{pmatrix}, \quad v^m = \frac{1}{\sqrt{2}}\begin{pmatrix} S^{m*} \\ i\sigma_y S^m \end{pmatrix}, \tag{3.1}$$

where $m=1,2$ divides the supervector space into advanced (A) and retarded (R) subspaces; χ^m, S^m are anti- and commuting two-component vectors in spin space, σ_y is the second Pauli matrix. The matrix C that determines the charge conjugation is written now as

$$C = \Lambda \otimes \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix},$$

$$c_1 = \begin{pmatrix} 0 & i\sigma_y \\ i\sigma_y & 0 \end{pmatrix}, \quad c_2 = \begin{pmatrix} 0 & -i\sigma_y \\ i\sigma_y & 0 \end{pmatrix}, \tag{3.2}$$

where the matrix Λ is the third Pauli matrix in the advanced-retarded space. Below we also use the matrix $\tau_3=\pm 1$ in the time-reversal space and $\mathbf{\Sigma}=\tau_3\otimes\sigma$, $\sigma=(\sigma_x,\sigma_y,\sigma_z)$. Using these definitions we write the Hamiltonian $\hat{H}_\mathbf{r}$, Eq. (2.2), as follows:

$$\hat{H}_\mathbf{r} = \frac{1}{2m}\left[-i\nabla_\mathbf{r} - \frac{e}{c}\tau_3\mathbf{A}(\mathbf{r})\right]^2 - \epsilon_F - \frac{g}{2}\mu_B\mathbf{B}(\mathbf{r})\mathbf{\Sigma}, \tag{3.3}$$

$\mu_B=e/(2mc)$, g is a factor that determines the Zeeman splitting. In Ref. 23 the authors put $g=2$ and assumed that the magnetic field $\mathbf{B}(\mathbf{r})$ was perpendicular to the plane of electron gas. This was the only case when the mathematical trick used in that paper and based on replacing the initial Hamiltonian (3.3) by the dirac Hamiltonian could be realized. In the present paper, using the scheme of the calculations we consider a more general case with an arbitrary value of g factor and the direction of the magnetic field.

As mentioned previously, we consider the limit of a weak magnetic field. This assumption implies that the corresponding radius R_H of the cyclotron motion $R_H=v_F(eB/mc)^{-1}$, v_F is Fermi velocity, is larger than other characteristic lengths. Then, considering electrons in a region with sizes smaller than R_H one can choose the vector potential $\mathbf{A}(\mathbf{r})$ such that it would be small everywhere in this region. The smallness of the vector potential allows one to represent the Hamiltonian Eq. (3.3) as the sum of the main part $\hat{H}_0$ and a small perturbation $\delta\hat{H}$

$$\hat{H}_\mathbf{r} = \hat{H}_{0\mathbf{r}} + \delta\hat{H}_\mathbf{r}, \quad \hat{H}_{0\mathbf{r}} = -\frac{\nabla_\mathbf{r}^2}{2m} - \epsilon_F,$$

$$\delta\hat{H}_\mathbf{r} = i\frac{e}{mc}\tau_3\mathbf{A}(\mathbf{r})\nabla_\mathbf{r} + \frac{e^2}{2mc^2}\mathbf{A}^2(\mathbf{r}) - \frac{g}{2}\mu_B\mathbf{\Sigma}\mathbf{B}(\mathbf{r}). \tag{3.4}$$

We choose for the vector potential the following gauge:

$$\mathrm{div}_\mathbf{r}\,\mathbf{A}(\mathbf{r},z=0) \equiv \partial_x A_x(\mathbf{r},z=0) + \partial_y A_y(\mathbf{r},z=0) = 0,$$

$$A_z(\mathbf{r},z=0) = 0,$$

$\mathbf{r}$ is the coordinate on the plane of electron gas. This gauge corresponds to the well-known London gauge in the superconductivity theory.

The next step in the calculation is to average the partition function $Z[a]=\int \exp(-F[V])DV$ with $F[V]$ from Eq. (2.20) over the random magnetic field. In the present paper we take a Gaussian distribution of the field with the pair correlation

$$\langle B_i(\mathbf{r})B_j(\mathbf{r}')\rangle = 2\left(\frac{mc}{e}\right)^2 w_{ij}(\mathbf{r}-\mathbf{r}'). \tag{3.5}$$

The free energy $F[V]$, Eq. (2.20), is written in a somewhat implicit way. In order to reduce it to a more explicit way we use the Wigner representation.

For any matrix O it is defined as

$$O(\mathbf{r}+\rho/2,\mathbf{r}'-\rho/2) = \int O_{\mathbf{p}}(\mathbf{r})e^{i\mathbf{p}\rho}d\mathbf{p}. \tag{3.6}$$

The product of two operators $\hat{O}_1$, $\hat{O}_2$ can be written in the Wigner representation in the following form:

$$O_{1\mathbf{p}}(\mathbf{r}) * Q_{2\mathbf{p}}(\mathbf{r}) = O_{1\mathbf{p}}(\mathbf{r})\exp\left[\frac{i}{2}(\overleftarrow{\nabla}_{\mathbf{r}}\overrightarrow{\nabla}_{\mathbf{p}} - \overleftarrow{\nabla}_{\mathbf{p}}\overrightarrow{\nabla}_{\mathbf{r}})\right]O_{2\mathbf{p}}(\mathbf{r}). \tag{3.7}$$

The operation $*$ introduced in Eq. (3.7) has all properties of the usual matrix product. In particular, it is associative.

Using the Wigner representation we expand Eq. (2.20) in $\delta\hat{H}$ and regard only terms up to the second order

$$F[V] = F_0[V] + F_1[V] + F_2[V], \tag{3.8}$$

where the right-hand side is the sum of terms of the zeroth, first, and second order in the field, respectively. Below we present an explicit calculation of the zeroth term only. Calculation of the other terms can be carried out in the same way.

The zeroth order term $F_0[V]$ takes the form

$$F_0[V] = \frac{i}{2}\mathrm{Str}(\overline{V}[\mathcal{H}_0,V]g_0) - \frac{1}{4}\mathrm{Str}(\overline{V}[\mathcal{H}_0,V]g_0)^2,$$

$$\mathcal{H}_0 = \frac{\hat{\mathbf{p}}^2}{2m} - \epsilon_F + \frac{\omega+i\delta}{2}\Lambda \tag{3.9}$$

$g_{0\mathbf{p}}$ is the Fourrier transformed Green's function Eq. (2.9) in the absence of the field, $\mathbf{A}=0$ and the source $a=0$. We find the commutator $[\mathcal{H}_0,V]$ and write it in the Wigner representation as

$$i[\mathcal{H}_0,V] = \frac{\mathbf{p}\nabla_{\mathbf{r}}}{m}V_{\mathbf{p}}(\mathbf{r}) + \frac{i(\omega+i\delta)}{2}[\Lambda,V_{\mathbf{p}}(\mathbf{r})]. \tag{3.10}$$

Equation (3.10) contains a small frequency ω and the gradient $\nabla_{\mathbf{r}}V_{\mathbf{p}}(\mathbf{r})$. Being interested in variations at distances much exceeding the wavelength we neglect the second term in Eq. (3.9). In the same approximation one may replace the "star product" $*$ Eq. (3.7) by the usual one and write the first term in Eq. (3.9) as follows:

$$F_0[V] = \frac{1}{2}\int \overline{V}_{\mathbf{p}}(\mathbf{r})\left[\frac{\mathbf{p}\nabla_{\mathbf{r}}}{m} + \frac{i(\omega+i\delta)}{2}\Lambda\right]V_{\mathbf{p}}(\mathbf{r})g_{0\mathbf{p}}d\mathbf{r}d\mathbf{p}. \tag{3.11}$$

The Green's function $g_{0\mathbf{p}}$ has a sharp peak at the Fermi surface, whereas the function $V_{\mathbf{p}}$ is smooth. Therefore, one may replace in Eq. (3.11) the momentum $\mathbf{p}$ by its value at the surface: $\mathbf{p}\to p_F\mathbf{n}$, consider separately the integration over the absolute value $|\mathbf{p}|$, which is equivalent to integration over $\xi = \mathbf{p}^2/2m - \epsilon_F$, and unit vector $\mathbf{n}=\mathbf{p}/p$. Making the replacement $d\mathbf{p}\to\nu d\xi d\mathbf{n}$ and carrying out the integration over ξ only in the Green's function $g_{0\mathbf{p}}$ we have

$$\int g_{0\mathbf{p}}d\xi = i\int\left[\xi - \epsilon + \frac{\omega}{2}(1+\Lambda) + i\delta\Lambda\right]^{-1}d\xi = \pi\Lambda. \tag{3.12}$$

Using Eq. (3.12) we come to the following result for F_0, Eq. (3.11):

$$F_0[V] = \frac{\pi\nu}{2}\mathrm{Str}\int \Lambda\overline{V}_{\mathbf{n}}(\mathbf{r})\left(v_F\mathbf{n}\nabla_{\mathbf{r}} + \frac{i(\omega+i\delta)}{2}\Lambda\right)V_{\mathbf{n}}(\mathbf{r})d\mathbf{r}d\mathbf{n}. \tag{3.13}$$

This expression is just the usual kinetic term of the ballistic σ model with the matrix $Q_{\mathbf{n}} = V_{\mathbf{n}}\Lambda\overline{V}_{\mathbf{n}}$. In order to take into consideration the scattering one should calculate the rest terms $F_1[V]$, $F_2[V]$ of Eq. (3.8).

In the main in small commutator $[H_0,V]$, Eq. (3.10), approximation the term $F_1[V]$, Eq. (3.8), may be written in the following form:

$$F_1[V] = \frac{i}{2}\mathrm{Str}(\overline{V}[\delta H,V]g_0). \tag{3.14}$$

Using the Wigner representation and neglecting high gradients of the matrices $V_{\mathbf{p}}(\mathbf{r})$, $\overline{V}_{\mathbf{p}}(\mathbf{r})$ we find with the help of Eq. (3.4)

$$F_1[V] = i\frac{e}{2mc}\mathrm{Str}\int e^{i\mathbf{q}\mathbf{r}}[-\tau_3\mathbf{p}\mathbf{A}(\mathbf{q}) - (g/4)\boldsymbol{\Sigma}\mathbf{B}(\mathbf{q})] \times V_{\mathbf{p}-\mathbf{q}/2}(\mathbf{r})g_{0\mathbf{p}}\overline{V}_{\mathbf{p}+\mathbf{q}/2}(\mathbf{r})d\mathbf{r}d\mathbf{p}d\mathbf{q}. \tag{3.15}$$

In Eq. (3.15), we disregard the term $(e^2/2mc^2)\mathbf{A}^2(\mathbf{r})$ in $\delta H_{\mathbf{r}}$, Eq. (3.4), because it results in higher magnetic field corrections. Moreover, considering large distances and therefore assuming that the matrices $V_{\mathbf{p}}(\mathbf{r})$ vary in space slower than the vector potential $\mathbf{A}(\mathbf{r})$ and magnetic field $\mathbf{B}(\mathbf{r})$ we conclude that the functional $F_1[V]$ should be self averaging and vanish. Thus, only term $F_2[V]$ in the expansion Eq. (3.8) gives a contribution related to the scattering in the RMF. In the main order it reads

$$\begin{aligned}F_2[V] &= -\frac{1}{2}\mathrm{Str}(\overline{V}[\delta H,V]g_0\delta H g_0) - \frac{1}{4}\mathrm{Str}(\overline{V}[\delta H,V]g_0)^2\\ &\approx -\left(\frac{e}{2mc}\right)^2\mathrm{Str}\int[-\tau_3\mathbf{p}\mathbf{A}(\mathbf{q}) + (g/4)\boldsymbol{\Sigma}\mathbf{B}(\mathbf{q})]\\ &\times V_{\mathbf{p}-\mathbf{q}/2}(\mathbf{r})g_{0\mathbf{p}-\mathbf{q}/2}\overline{V}_{\mathbf{p}+\mathbf{q}/2}(\mathbf{r})\\ &\times[-\tau_3\mathbf{p}\mathbf{A}(\mathbf{q}') + (g/4)\boldsymbol{\Sigma}\mathbf{B}(\mathbf{q}')]\\ &\times V_{\mathbf{p}-\mathbf{q}'/2}(\mathbf{r})g_{0\mathbf{p}-\mathbf{q}'/2}\overline{V}_{\mathbf{p}+\mathbf{q}'/2}(\mathbf{r})e^{i(\mathbf{q}+\mathbf{q}')\mathbf{r}}d\mathbf{r}d\mathbf{p}d\mathbf{q}d\mathbf{q}'.\end{aligned} \tag{3.16}$$

Equation (3.16) can be easily averaged over the magnetic field. Noticing again that $g_{0\mathbf{p}}$ is a sharp function of the mo-

mentum **p** at the Fermi surface and using Eq. (3.5) we come to the σ model with the following effective energy:

$$F[Q_{\mathbf{n}}] = F_{\mathrm{kin}}[Q_{\mathbf{n}}] + F_{\mathrm{orb}}[Q_{\mathbf{n}}] + F_{\mathrm{sp}}[Q_{\mathbf{n}}] + F_{\mathrm{mix}}[Q_{\mathbf{n}}],$$

where $F_{\mathrm{kin}}[Q_{\mathbf{n}}]$ determines ballistic (or free) propagation and coincides with $F_0[Q_{\mathbf{n}}]$ Eq. (3.13), whereas the other terms are related to three types of scattering which can be called as orbital, spin, and some mixture of the first two, respectively,

$$F_{\mathrm{orb}}[Q_{\mathbf{n}}] = -\frac{(\pi\nu)^2}{8}\mathrm{Str}\int \frac{1+\mathbf{n}_1\mathbf{n}_2}{1-\mathbf{n}_1\mathbf{n}_2} w_{zz}(\mathbf{n}_1-\mathbf{n}_2) \times Q_{\mathbf{n}_1}(\mathbf{r})Q_{\mathbf{n}_2}(\mathbf{r})d\mathbf{r}d\mathbf{n}_1 d\mathbf{n}_2, \tag{3.17}$$

$$F_{\mathrm{sp}}[Q_{\mathbf{n}}] = -\frac{(\pi\nu)^2}{2}\mathrm{Str}\int \left(\frac{g}{4}\right)^2 w_{ij}(\mathbf{n}_1-\mathbf{n}_2) \times Q_{\mathbf{n}_1}(\mathbf{r})\sigma_i Q_{\mathbf{n}_2}(\mathbf{r})\sigma_j d\mathbf{r}d\mathbf{n}_1 d\mathbf{n}_2, \tag{3.18}$$

$$F_{\mathrm{mix}}[Q_{\mathbf{n}}] = -\frac{(\pi\nu)^2}{2}\mathrm{Str}\int \left(i\frac{g}{4}\right)\frac{[\mathbf{n}_1\times\mathbf{n}_2]_z}{1-\mathbf{n}_1\mathbf{n}_2} w_{zj}(\mathbf{n}_1-\mathbf{n}_2) \times \sigma_j Q_{\mathbf{n}_1}(\mathbf{r})Q_{\mathbf{n}_2}(\mathbf{r})d\mathbf{r}d\mathbf{n}_1 d\mathbf{n}_2, \tag{3.19}$$

where $w_{ij}(\mathbf{n}_1-\mathbf{n}_2)$ is the Fourier transformation of the correlation function Eq. (3.5) for the momentum difference **q** at the Fermi surface $\mathbf{q}=p_F(\mathbf{n}_1-\mathbf{n}_2)$ and the sum over i, j is implied in Eqs. (3.18) and (3.19). The last *mixed term*, Eq. (3.19), takes into account the effect of interference between the orbital and spin scattering in the same configuration of RMF. The functional $F_{\mathrm{orb}}[Q_{\mathbf{n}}]$, Eq. (3.17), has been previously obtained in Ref. 20. Let us be reminded that $Q_{\mathbf{n}} = V_{\mathbf{n}}\Lambda\overline{V_{\mathbf{n}}}$ in Eqs. (3.17)–(3.19) is still a 16×16 supermatrix with an additional spin structure.

The functionals, Eqs. (3.17)–(3.19), have a form typical for σ model in the regime that can be specified as collisional. From this point of view $\mathbf{n}_1$, $\mathbf{n}_2$ in Eqs. (3.17)–(3.19) can be considered as momenta of a particle before and after collision whereas the integrands determine corresponding transition probability. Previously we assumed that the matrices $V_{\mathbf{n}}(\mathbf{r})$ vary in space sufficiently slowly. Now we can clarify this assumption more carefully.

We notice that its formulation is directly related to the conditions under which the collisional regime described by Eqs. (3.17)–(3.19) should be used. These conditions depend on the kind of disorder. If disorder is short ranged (δ correlated) the scattering may be considered as a collision on the length scales exceeding the Fermi wavelength λ_F. The situation becomes more complicated when the disorder is long ranged. This case for the potential disorder was first studied in Refs. 26 and 27 and then, with using results of these papers and applying the ballistic σ model, in Ref. 25. The model with a long ranged magnetic field was directly considered in Ref. 21. It was shown that collisional regime corresponds to lengths exceeding the Lyapunov length $l_L = v_F\lambda_L^{-1}$, λ_L is the rate of divergency of the classical trajectories.

Summing up, the model found in Eqs. (3.17)–(3.19) should be valid for matrices $V_{\mathbf{n}}(\mathbf{r})$ slowly varying over the Fermi wavelength λ_F in the case of short ranged RMF or over the Lyapunov length l_L in the case of a long ranged field.

IV. DIFFUSIVE MODEL

In this section we study the diffusive limit of the model obtained in Eqs. (3.17)–(3.19) and also calculate the spin susceptibility in this limit.

We start the reduction of the theory given by Eqs. (3.17)–(3.19) to the effective σ model applicable in the diffusive limit noticing that the functionals, Eqs. (3.17)–(3.19), are invariant with respect to rotations $U(\mathbf{r})$

$$Q_{\mathbf{n}}(\mathbf{r}) \to U(\mathbf{r})Q_{\mathbf{n}}(\mathbf{r})\overline{U}(\mathbf{r}), \quad U(\mathbf{r})\overline{U}(\mathbf{r}) = 1 \tag{4.1}$$

provided $U(\mathbf{r})$ is the δ function in the spin space. This means that only the charge mode related to fluctuations of the electron density and described by supermatrices with the usual unitary symmetry remains gapless in the diffusive limit.

The situation changes if one considers a random magnetic field with a fixed direction **h**, $\mathbf{h}^2=1$, replacing the pair function, Eq. (3.5), by the following one:

$$\langle B(\mathbf{r})B(\mathbf{r}')\rangle = 2\left(\frac{mc}{e}\right)^2 w(\mathbf{r}-\mathbf{r}'), \tag{4.2}$$

where $B(\mathbf{r})$ is the absolute value of the magnetic field. Function $w(\mathbf{r}-\mathbf{r}')$ being considered in $3d$ should be independent of the coordinate along **h** due to the condition $\mathrm{div}_{\mathbf{r}}\mathbf{B}=0$. One can readily see that in this case the energies Eqs. (3.17)–(3.19) are invariant with respect to the transformation, Eq. (4.1), if $U(\mathbf{r})$ commutes with the matrix $\sigma_h \equiv (\sigma\mathbf{h})$ and, consequently, an additional soft mode identified with the spin mode along **h** appears. Considering only charge and spin modes, $[Q_{\mathbf{n}}, \sigma_h]=0$, and simplifying Eqs. (3.17)–(3.19) we come to the following form of collision term:

$$F_{\mathrm{scatt}}[Q_{\mathbf{n}}] = -\frac{(\pi\nu)^2}{8}\mathrm{Str}\int\left[\cos^2\theta\frac{1+\mathbf{n}_1\mathbf{n}_2}{1-\mathbf{n}_1\mathbf{n}_2} + \left(\frac{g}{2}\right)^2\right] \times w(\mathbf{n}_1-\mathbf{n}_2)Q_{\mathbf{n}_1}(\mathbf{r})Q_{\mathbf{n}_2}(\mathbf{r})d\mathbf{r}d\mathbf{n}_1 d\mathbf{n}_2, \tag{4.3}$$

where $\cos\theta=(\mathbf{e}_z\mathbf{h})$, $\mathbf{e}_z$ is z ort. Because of the usual space symmetry the term $F_{\mathrm{mix}}[V]$, Eq. (3.19), vanishes and does not contribute to the energy Eq. (4.3).

From Eq. (4.3) one can come to the diffusive σ model singling out the angle modes $\widetilde{Q}_{\mathbf{n}}(\mathbf{r})$

$$Q_{\mathbf{n}}(\mathbf{r}) = U(\mathbf{r})\widetilde{Q}_{\mathbf{n}}(\mathbf{r})\overline{U}(\mathbf{r}),$$

that describe the fluctuation of the particle momentum **n** and integrating them out. Calculation of integrals over the angle modes $\widetilde{Q}_{\mathbf{n}}$ is standard and can be found in details, e.g., in Ref. 20. The final expression for the effective energy has the usual form

$$F[Q(\mathbf{r})] = \frac{\pi\nu}{8}\mathrm{Str}\int[D(\nabla_{\mathbf{r}}Q)^2 + 2i\omega\Lambda Q(\mathbf{r})]d\mathbf{r}, \tag{4.4}$$

where $D=v_F^2\tau_{\mathrm{tr}}/2$, τ_{tr} is the transport time

$$\tau_{\rm tr}^{-1} = \pi\nu \int \left[\cos^2\theta(1+\mathbf{n}_1\mathbf{n}_2) + \left(\frac{g}{2}\right)^2 (1-\mathbf{n}_1\mathbf{n}_2) \right] \times w(\mathbf{n}_1-\mathbf{n}_2) d\mathbf{n}_1 d\mathbf{n}_2. \quad (4.5)$$

This time determines the relaxation of the angle modes and, hence, of the particle momentum $\mathbf{n}$. The supermatrices $Q(\mathbf{r})$ entering into Eq. (4.4) are diagonal in the particle-hole space but not necessarily proportional to the unity in the spin space. The only requirement is that they should commute with the matrix σ_h. The presence of the spin structure of $Q(\mathbf{r})$ leads to a difference between the discussed and conventional unitary models and means that the former may not be related to one of the usual types of ensembles.

If the magnetic field is δ correlated in space the transport time can be explicitly calculated and expressed through the time $\tau'_{\rm tr}$ in the model of spinless electrons and perpendicular magnetic field

$$\tau_{\rm tr}^{-1} = \tau_{\rm tr}'^{-1}[\cos^2\theta + (g/2)^2]. \quad (4.6)$$

In the case of the magnetic field perpendicular to the plane, $\cos\theta=1$, and free electrons, $g=2$, the transport time $\tau_{\rm tr}$ turns out two times smaller than the transport time for the spinless electrons. The same result for this particular case has been established in Ref. 23.

Using Eqs. (3.17)–(3.19), and (4.4) one can calculate various physical quantities. Below we find the spin magnetic susceptibility relying on the diffusive model Eq. (4.4).

First, we use the Kubo formula and write the magnetic moment due to the spin of electrons as a sum of two terms

$$\mathbf{m}(\mathbf{r},\omega) = \mathbf{m}_{\rm sp}(\mathbf{r},\omega) + \mathbf{m}_{\rm so}(\mathbf{r},\omega),$$

where $\mathbf{m}_{\rm sp}$ is a purely spin contribution

$$\begin{aligned}\mathbf{m}_{\rm sp}(\mathbf{r},\omega) = {} & i\mu_B^2 \frac{g}{2} \int_{-\infty}^{+\infty} \frac{d\epsilon}{2\pi} [n(\epsilon-\omega) - n(\epsilon)] \\ & \times {\rm Tr} \int [G^R_\epsilon(\mathbf{r},\mathbf{r}')[\boldsymbol{\sigma}\cdot\mathbf{B}(\mathbf{r}',\omega)] G^A_{\epsilon-\omega}(\mathbf{r}',\mathbf{r})\boldsymbol{\sigma}] d\mathbf{r}' \\ & + i\mu_B^2 \frac{g}{2} \int_{-\infty}^{+\infty} n(\epsilon) {\rm Tr} \int [G^A_\epsilon(\mathbf{r},\mathbf{r}') \\ & \times [\boldsymbol{\sigma}\cdot\mathbf{B}(\mathbf{r}',\omega)] G^A_\epsilon(\mathbf{r}',\mathbf{r})\boldsymbol{\sigma} - G^R_{\epsilon+\omega}(\mathbf{r},\mathbf{r}') \\ & \times [\boldsymbol{\sigma}\cdot\mathbf{B}(\mathbf{r}',\omega)] G^R_\epsilon(\mathbf{r}',\mathbf{r})\boldsymbol{\sigma}] d\mathbf{r}', \end{aligned} \quad (4.7)$$

Tr corresponds to the trace in the usual spin space. The other term $\mathbf{m}_{\rm so}$ is given by the same formula taken after the replacement $(g/4)\sigma\mathbf{B}(\mathbf{r}',\omega) \to -i\mathbf{A}(\mathbf{r}',\omega)\nabla'_{\mathbf{r}}$. This contribution is related to both orbital and spin motions. It is determined by the angle modes and interaction between the charge and spin modes. Both factors are irrelevant in diffusive limit and the contribution is small.

So, we neglect $\mathbf{m}_{\rm so}$ and calculate $\mathbf{m}_{\rm sp}$. The contribution to this quantity given by the second term in Eq. (4.7) can be easily found and corresponds to the usual Pauli susceptibility. In order to calculate the first term we express the product of two Green's functions G^R and G^A through the partition function, Eqs. (2.1) and (2.11)

$${\rm Tr}[G^R_\epsilon(\mathbf{r}_1,\mathbf{r}_2)\sigma_j G^A_{\epsilon-\omega}(\mathbf{r}_2,\mathbf{r}_1)\sigma_i] = -\left.\frac{\partial^2 Z[a]}{\partial\alpha_1\partial\alpha_2}\right|_{\alpha_1,\alpha_2=0}, \quad (4.8)$$

where the source $a(\mathbf{r},\mathbf{r}')$ is taken as follows:

$$\begin{aligned} a(\mathbf{r},\mathbf{r}') &= \begin{pmatrix} 0 & a_{12}(\mathbf{r}) \\ a_{21}(\mathbf{r}) & 0 \end{pmatrix} \delta(\mathbf{r}-\mathbf{r}'), \\ a_{12}(\mathbf{r}) &= \frac{1-k}{2} \otimes \begin{pmatrix} -\alpha_2\sigma_j^T\delta(\mathbf{r}-\mathbf{r}_2) & 0 \\ 0 & \alpha_1\sigma_i^T\delta(\mathbf{r}-\mathbf{r}_1) \end{pmatrix}, \\ a_{21}(\mathbf{r}) &= \frac{1-k}{2} \otimes \begin{pmatrix} \alpha_1\sigma_i^T\delta(\mathbf{r}-\mathbf{r}_1) & 0 \\ 0 & -\alpha_2\sigma_j^T\delta(\mathbf{r}-\mathbf{r}_2) \end{pmatrix}. \end{aligned} \quad (4.9)$$

Substitution of the partition function $Z[a]$ taken in diffusive limit into Eq. (4.8) results in the following expression for the susceptibility:

$$\chi_{ij}(\mathbf{q},\omega) = g\mu_B^2\nu \Big[\delta_{ij} - i\omega\frac{\pi\nu}{16} \langle {\rm Tr}(\sigma_i^T[Q_{84}(\mathbf{q}) + Q_{37}(\mathbf{q})]) \times {\rm Tr}(\sigma_j^T[Q_{73}(-\mathbf{q}) + Q_{48}(-\mathbf{q})]) \rangle_Q \Big]. \quad (4.10)$$

The first term in the brackets is the usual Pauli susceptibility. The other term determines the correction to this result related to RMF. The angular brackets $\langle\ldots\rangle_Q$ in Eq. (4.10) imply the averaging of the quantity inside them over the matrix Q with the free energy functional, Eq. (4.4).

We calculate this average using the parametrization

$$Q(\mathbf{r}) = \Lambda[1+iP(\mathbf{r})][1-iP(\mathbf{r})]^{-1}, \quad (4.11)$$

where the nondiagonal matrix $P(\mathbf{r})$, $\overline{P}(\mathbf{r}) = -P(\mathbf{r})$, $[P,\Lambda]=0$ is the sum of the charge and spin modes

$$P(\mathbf{r}) = P_{\rm ch}(\mathbf{r}) + (\boldsymbol{\Sigma}\mathbf{h})P_{\rm sp}(\mathbf{r}). \quad (4.12)$$

$P_{\rm ch}(\mathbf{r})$, $P_{\rm sp}(\mathbf{r})$ are δ functions in the spin space. Their structure should be found in correspondence with Eq. (2.14) and condition $[Q,\hat{\tau}_3]=0$. This results in the following representation:

$$\begin{aligned} P_{\rm ch(sp)}(\mathbf{r}) &= \begin{pmatrix} 0 & B_{\rm ch(sp)}(\mathbf{r}) \\ \overline{B}_{\rm ch(sp)}(\mathbf{r}) & 0 \end{pmatrix}, \\ B_{\rm ch(sp)}(\mathbf{r}) &= \begin{pmatrix} \hat{a}_{\rm ch(sp)}(\mathbf{r}) & i\hat{\rho}_{1\rm ch(sp)}(\mathbf{r}) \\ \hat{\rho}^+_{2\rm ch(sp)}(\mathbf{r}) & i\hat{b}_{\rm ch(sp)}(\mathbf{r}) \end{pmatrix}, \end{aligned} \quad (4.13)$$

where $\overline{B}_{\rm ch(sp)}(\mathbf{r}) = C_0 B^T_{\rm ch(sp)}(\mathbf{r}) C_0^T$, $\hat{a}_{\rm ch(sp)}(\mathbf{r})$, $\hat{b}_{\rm ch(sp)}(\mathbf{r})$ are usual and $\hat{\rho}_{1,2\rm ch(sp)}$-Grassman matrices 4×4 unit, in spin and particle-hole spaces

$$\hat{a}_{\rm ch(sp)}(\mathbf{r}) = \begin{pmatrix} a_{\rm ch(sp)}(\mathbf{r}) & 0 \\ 0 & a^*_{\rm ch(sp)}(\mathbf{r}) \end{pmatrix},$$

$$\hat{b}_{\mathrm{ch(sp)}}(\mathbf{r}) = \begin{pmatrix} b_{\mathrm{ch}}(\mathrm{sp})(\mathbf{r}) & 0 \\ 0 & b^{*}_{\mathrm{ch(sp)}}(\mathbf{r}) \end{pmatrix},$$

$$\hat{\rho}_{1,2\mathrm{ch(sp)}}(\mathbf{r}) = \begin{pmatrix} \rho_{1,2\mathrm{ch(sp)}}(\mathbf{r}) & 0 \\ 0 & -\rho^{*}_{1,2\mathrm{ch(sp)}}(\mathbf{r}) \end{pmatrix}. \quad (4.14)$$

In Gauss approximation the charge and spin modes give independent contributions to the energy Eq. (4.4). Standard calculation of the average in Eq. (4.10) in the same approximation gives

$$\chi_{ij}(\mathbf{q},\omega) = g\mu_B^2\nu(\delta_{ij} - h_i h_j) + g\mu_B^2\nu\frac{D\mathbf{q}^2}{D\mathbf{q}^2 - i\omega}h_i h_j. \quad (4.15)$$

In the particular case when RMF has only the perpendicular component, this expression is in agreement with the result of Ref. 23.

V. DISCUSSION

In the present work we consider a two-dimensional electron gas placed in a nonuniform (random) magnetic field (RMF) using a method of superbosonization (Ref. 24). The problem is studied in a quite general formulation with taking into account interaction of the magnetic field with the spin of electrons. The direction of the magnetic field is assumed random in space and the value of g factor—arbitrary.

In the present paper we use a method proposed in Ref. 24. This method is based on the exact transformation of the initial field theory written in terms of a functional integral over supervectors ψ to a theory described by a functional integral over supermatrices Q and, by the analogy with quantum field theory, can be called superbosonization. As the method is exact it enables us to deal with both the short and long range disorder.

The found theory turns out to be invariant with respect to the rotations in the superspace $Q \to VQ\overline{V}$, V are unitary supermatrices $V\overline{V}=1$, provided that they commute with Hamiltonian: $[\hat{H},V]=0$. This means that the low energy limit of the theory is described by the rotations V with a small commutator $[\hat{H},V]$. Conditions which should be imposed on the supermatrices V to provide the smallness of the commutator depend on the character of the terms contained in Hamiltonian H, e.g., on the kind of disorder. Singling out the massive modes related to the fluctuations of the advanced and retarded blocks q in the representation $Q=Vq\overline{V}$ where q-arbitrary diagonal supermatrices $[q,\Lambda]=0$ and integrating them out we come to an effective energy functional in terms of the rotations V only and obtained some nonlinear σ model.

The derivation of the σ model is carried out for an arbitrary Hamiltonian H and does not depend on its explicit form. Particularly, it may be used for both short and long range disorder. Explicit form of the model as well as the limits of its applicability depend on the character of terms contained in Hamiltonian H. In this paper we deal only with a weak RMF without an additional potential disorder and consider the theory in the regime that may be called collisional. This regime is achieved differently depending on correlation length of RMF. If the field is δ correlated the model is valid at distances larger than wavelength λ_F. In the case of long range RMF they should be larger than the Lyapunov length l_L.

After averaging the free energy functional over RMF we obtain the σ model with a collision term consisting of three contributions that could be called orbital, spin, and mixed one. The first contribution comes from the part of the Hamiltonian acting on the orbital motion, the second one—from the part acting on spin, whereas the last one comes as a result of joint averaging of them and describes correlations between the spin and orbit scattering.

The collisional term turns out to be invariant with respect to the rotation $Q_{\mathbf{n}} \to UQ\overline{U}$ provided that U is δ function in the spin space. This allows us to conclude that in general only the mode related to the charge transport or charge mode should survive in the diffusive limit. At the same time, we show that in the particular case of RMF with a fixed direction $\mathbf{h}$ one more spin mode should be considered as well. The existence of the additional mode changes the form of the used supermatrices $Q(\mathbf{r})$ so that they, remaining diagonal in the particle-hole space, may have a non-trivial spin structure. This contrasts the symmetry of the supermatrices Q for, e.g., the model with magnetic impurities when all spin blocks are proportional to the unit matrices. In this particular case we obtain the σ model in the standard diffusive form and find the transport time τ_{tr}. Due to the symmetry with respect to the inversion of the coordinates the collision term describing the correlations between the spin and orbit contributions vanishes in the model with a non-fluctuating fixed RMF and does not contribute to τ_{tr}.

Finally, in diffusive limit, we calculate the spin magnetic susceptibility. The part of the susceptibility transversal to the direction $\mathbf{h}$ is given by the usual Pauli expression whereas the longitudinal one has an additional contribution proportional to the diffusion propagator.

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