

Gradual decrease of conductance of an adiabatic ballistic constriction below $2e^2/h$

C.-T. Liang

Department of Physics, National Taiwan University, Taipei 106, Taiwan

O. A. Tkachenko, V. A. Tkachenko, and D. G. Baksheyev

Institute of Semiconductor Physics, Siberian Division, Russian Academy of Sciences, Novosibirsk, 630090 Russia

M. Y. Simmons

School of Physics, University of New South Wales, Sydney 2052, Australia

D. A. Ritchie and M. Pepper

Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE, United Kingdom

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We have performed four-terminal conductance measurements of a one-dimensional (1D) channel in which it is possible to modulate the potential profile using three overlaying finger gates. In such a 1D ballistic structure we have observed that the conductance steps show a gradual decrease from $2e^2/h$ to $0.97 \times 2e^2/h$ with increasing negative finger gate voltage in a short, clean 1D constriction. We suggest this phenomenon is due to differing shifts of 1D subbands with changing split-gate voltage. Both a simple analytical estimate for an adiabatic constriction and a realistic modeling of the device give the same magnitude of the conductance decrease as observed in our experiments.

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It is known that the conductance of a one-dimensional (1D) ballistic wire is quantized in units of $2e^2/h$. This was discovered experimentally¹ in split-gate induced constrictions in high-mobility two-dimensional electron gases (2DEGs). Both experimental² and theoretical results³ have shown that the accuracy of the observed quantization is sensitive to the detailed shape of the confining potential and the presence of impurities. The observation that a high number of conductance plateaus⁴ are exactly quantized, without the appearance of any resonant features, indicates that adiabatic transport in the constriction^{5,6} is attainable. However, regular deviations of plateaus from quantized values $n \times 2e^2/h$ have been observed in long 1D wires⁷⁻¹⁰ and have been associated with either nonadiabaticity of the constriction or electron-electron interactions in the 1D system. In the seminal work of Tarucha, Honda, and Saku,⁷ a few percent deviations were observed in long (5–10 μm) weakly disordered quantum wires.⁷ The deviation was observed to increase with increasing wire length and was explained in terms of electron-electron interactions in a “dirty Luttinger liquid.”¹¹ The reduction of conductance plateaus of up to 25% has also been observed in 5–6 μm quantum wires and was ascribed to backscattering of electrons due to impurities.¹⁰ Other structures with a more complicated geometry (T-shaped cleaved-edge-overgrowth quantum wires⁸ and V-groove quantum wires⁹) have shown a significant decrease of the value of the conductance steps that has been attributed to backscattering of electrons from the abrupt interface between the two-dimensional (2D) contacts and the 1D wire.

In this article, we show that a deviation from the quantized value $2e^2/h$ is possible even in a clean, short constriction in which residual scattering and effects of nonadiabaticity are negligible. It is well established¹² that the two-terminal conductance of a clean ballistic 1D system is

determined by noninteracting source and drain contacts. In such a clean system, deviations from exact quantization arising from the presence of electron-electron interactions are not expected,¹² since these occur only in weakly disordered quantum wires (“dirty Luttinger liquid”).¹¹ Therefore, the observed deviation in our system could not be explained by conductance renormalization due to electron-electron interactions.

We have studied constrictions electrostatically shaped in the 2DEG by a pair of split gates and three finger gates lying across the channel. This versatile device permits control of the electrostatic potential across and along the channel without changing the series resistance R_s . We have measured the conductance G as a function of split-gate voltage V_g at different center finger gate voltages, V_F with outer finger gates grounded to the 2DEG. With increasing negative voltage on the center finger gate, we observed in a series of $G(V_g)$ curves the transition from pronounced steps to smeared conductance steps. At the same time, the height of the first conductance step gradually decreased from $2e^2/h$ to $0.97 \times 2e^2/h$. The effect was reproduced in two- and four-terminal conductance measurements, whereas the series resistance $R_s = R(V_g = 0)$ is varied by an order of magnitude. We ascribe this deviation from the conductance quantum $2e^2/h$ to the interplay between reflection in the first transmission mode and tunneling via the second mode in the adiabatic constriction. Electrostatics calculations show that the tops of the first and second subbands move with different velocities in response to the change of the split-gate voltage. This leads to a slower decrease of the reflection as compared to simultaneous increase in the tunneling, and therefore, the onset of the conductance step occurs at a lower value than $G = 2e^2/h$. The effect disappears when the smeared conductance steps turn into pronounced ones.

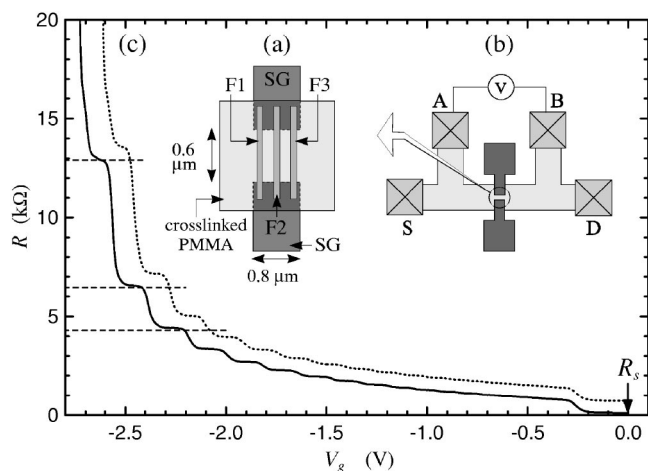


FIG. 1. (a) Schematic view of device. (b) Four-terminal measurement setup. (c) Measured dependences of resistance R on the split-gate voltage V_g , with all finger gates grounded to the 2DEG (the solid line). Horizontal dashed lines show values $R_n = h/2ne^2$ for $n=1, 2, 3$. For comparison two-terminal measurement of $R(V_g)$ is shown by the dotted line.

The multilayered gated 1D structure was lithographically defined 158 nm above the 2DEG, as shown in Fig. 1(a). There is a 30-nm-thick layer of polymethylmethacrylate (PMMA), which has been highly irradiated by an electron beam, to act as a dielectric between the split gate and three overlaying finger gates. The 2DEG has a carrier density of $2.5 \times 10^{15} \text{ cm}^{-2}$ with a mobility of $3.0 \times 10^6 \text{ cm}^2/\text{V s}$ after brief illumination with a red-light-emitting diode. Experiments were performed in a pumped ^3He cryostat at 300 mK and the four-terminal resistance was measured using an ac driving current of 10 nA at a frequency of 77 Hz with standard phase-sensitive techniques.

In our setup, the outer finger gates were grounded to the 2DEG, while the center finger gate voltage V_F and the split-gate voltage V_g were varied over a wide range. In this sample, potentiometric contacts were deliberately brought close to the constriction (four-terminal measurements) to reduce the contribution of series resistance of the 2DEG [Fig. 1(b)]. The measured resistance R_s between the contacts in the absence of the constriction was only 115 Ω , that is 0.9% of the quantum resistance $h/2e^2$. Thus, we were able to measure the conductance of the single-mode constriction precisely enough without resorting to subtraction of the series resistance [Fig. 1(c)]. Notice that the series resistance was 725 Ω in the two-terminal measurements.

Figure 2 shows the measured dependencies of conductance G on (a) the split-gate voltage V_g and (b) the derivative $dV_g(G)/dG$ for different finger gate voltages from 0 to -4.8 V. If we consider the first conductance step we can determine its height G_* by the value of G at a point of inflection, where the derivative dV_g/dG shown in Fig. 2(b) reaches a maximum. As the voltage on the center finger gate V_F is made more negative, the step becomes narrower and eventually transforms into a point of inflection. The reason for the observed smearing of the conductance steps lies in the decrease of the ratio ω_y/ω_x of the transverse and longitudinal frequencies that define the shape of saddle potential in the

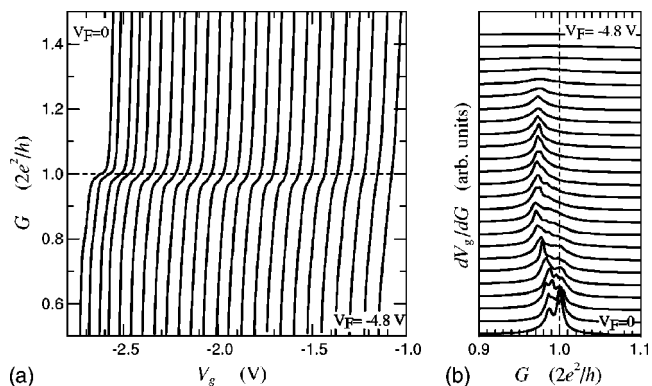


FIG. 2. (a) Measured four-terminal conductance $G(V_g)$ at different center finger gate voltages V_F . From left to right: $V_F=0$ to -4.8 V in 0.2 V steps. (b) Derivative $dV_g(G)/dG$ at different center finger gate voltages V_F . From bottom to top: $V_F=0$ to -4.8 V in 0.2 V steps.

constriction.⁶ Figure 2(b) shows that $G_* = 2e^2/h$ at $V_F=0$ and $G_* = 0.97 \times 2e^2/h$ at a large negative voltage $V_F = -1.6$ V. In the voltage range $-1.6 \text{ V} < V_F < 0$, a continuous transition between these two values is observed. We note that the exact conductance quantization of $G_* = 2e^2/h$ at small negative V_F confirms the validity of the four-terminal measurement setup. Nonlocality of resistance in this setup can complicate measurements of the conductance of the constriction. However, by deliberately ensuring that the resistance of the potentiometric contacts is much smaller than that of the single-mode channel one can neglect this nonlocality.

It is also worth mentioning that the gradual decrease of the height of the first conductance step has been detected in two-terminal conductance measurements, where R_s has been subtracted. This decrease has been shown to be reproducible for different cool-downs, indicating that it cannot be explained by residual disorder scattering. In addition, the lack of resonant features and the pronounced conductance steps demonstrate that we have a clean 1D system in which impurity scattering is negligible. Therefore, we can exclude the possibility of “dirty Luttinger liquid” behavior in our case. Moreover, in contrast to previous observations,^{7–10} in which the deviation from exact quantization increased with the length of the quantum wire, in our case we observe an increase in the deviation as the 1D region shortens. Therefore, the deviation we observe must be of a different physical origin.

Let us now see if the observed decrease of the conductance steps can be explained in the model of quasi-1D ballistic transport for an adiabatic constriction. The transverse quantization of electrons allows us to view the scattering problem in terms of the local transverse modes. The conductance is then given by the Landauer formula: $G = (2e^2/h) \sum_{kj} |t_{kj}|^2$, where $|t_{kj}|^2$ is the probability that the flux input in subband j is traveling forward into subband k . The solution of the transverse Schrödinger equation at each value x along the constriction gives energies of 1D subbands $E_n(x)$. These subbands $E_n(x)$ have the shape of smooth potential barriers offset from each other by transverse quantum $\hbar\omega_{yn}(x)$. If $\hbar\omega_{yn}$ changes slowly along x then intersubband

scattering is suppressed and the total transmission coefficient is equal to the sum of contributions of all subbands:⁵ $T = \sum_n T_n$, $T_n = |t_{nn}|^2$. At $G \approx 2e^2/h$ the conductance can be expressed in the form

$$G = \frac{2e^2}{h}(1 - R_1 + T_2), \quad (1)$$

where R_1 is the reflection coefficient of the first subband and T_2 is the transmission coefficient through the second subband. When the plateaus of $G(V_g)$ appear smeared out, their height is determined by the interplay of two basic phenomena of quantum mechanics: above-barrier reflection and tunneling. If $R_1(V_g)$ reduces more slowly than $T_2(V_g)$ increases, then at the point of inflection of the curve $G(V_g)$, the height of the steps G_* can be less than the conductance quantum $2e^2/h$.

The effect is most easily seen in an approximation of a saddle-point potential $U(x, y) = V_0 + \frac{1}{2}m[\omega_y^2 y^2 - \omega_x^2(x - x_{\max})^2]$, for which the transmission coefficient via the n th subband $E_n(x) = V_0 + \hbar\omega_y(n - 1/2) - \frac{1}{2}m\omega_x^2(x - x_{\max})^2$ has a simple analytical expression:⁶ $T_n = (1 + e^{-\pi\epsilon_n})^{-1}$, where $\epsilon_n = 2[E - \hbar\omega_y(n - \frac{1}{2}) - V_0]/\hbar\omega_x$, $n = 1, 2, \dots$

With increasing negative voltage V_g on the split gate, the saddle potential goes up and becomes narrower. Modeling of the three-dimensional (3D) electrostatics shows that the change of ω_x is small compared to the change in ω_y and V_0 , as V_g varies. Thus, we can assume that ω_y and V_0 depend linearly on V_g , while ω_x remains constant. One can readily see that the second subband moves more rapidly than the first one:

$$\begin{aligned} E'_1 &= \frac{dE_1}{dV_g} = \frac{dV_0}{dV_g} + \frac{1}{2} \frac{\hbar d\omega_y}{dV_g}, \\ E'_2 &= \frac{dE_2}{dV_g} = \frac{dV_0}{dV_g} + \frac{3}{2} \frac{\hbar d\omega_y}{dV_g}. \end{aligned} \quad (2)$$

Hence, at the minimum of dT/dV_g , it holds that $R_1 > T_2$. Indeed, at $T \approx 1$, the coefficients can be approximated as

$$\begin{aligned} R_1 &\approx 1 - T_1 \approx \exp\left(-2\pi \frac{E - E_1(x_{\max})}{\hbar\omega_x}\right), \\ T_2 &\approx \exp\left(-2\pi \frac{E_2(x_{\max}) - E}{\hbar\omega_x}\right). \end{aligned} \quad (3)$$

At the point of inflection, $d^2T/dV_g^2 = 0$, we have $T_2 \approx R_1(E'_1/E'_2)^2$, $E = V_0 + \hbar\omega_y + \hbar\omega_x \ln(E'_1/E'_2)/2\pi$, and

$$T_* \approx 1 - e^{-\pi\omega_y/\omega_x} \left(\frac{E'_2}{E'_1} - \frac{E'_1}{E'_2} \right). \quad (4)$$

Let us estimate the deviation $\Delta = 1 - T_*$. If the steps on the plot $T(V_g)$ are to be observed, it is necessary that that $\omega_y/\omega_x > 1.25$.⁶ Thus, the exponent in Eq. (4) does not exceed 2%. On the other hand, from computation of the electrostatics (see below), $dV_0/dV_g \sim \frac{1}{2}\hbar d\omega_y/dV_g$ and $E'_2/E'_1 \approx 2$. Therefore, the conductance step can decrease by 3%. When the negative voltage on the center finger gate approaches

zero, ω_x decreases, and the effect vanishes exponentially. Thus, the simple equation (4) predicts behavior that qualitatively agrees with our experimental results (Fig. 2).

Notice that the effect is absent if $E'_2 = E'_1$, i.e., when V_0 has a linear dependence on the gate voltage V_g , and frequencies ω_y and ω_x remain constant.⁶ Essentially, this is the same as if energy changes within the saddle potential are kept constant. In this case the quantization is exact ($T = n$) because at the inflection points $E = V_0 + n\hbar\omega_y$, the reflection coefficient R_n is equal to the transmission coefficient T_{n+1} . However, if the electrostatic potential deforms with varying gate voltage, then $G(V_g)$ dependencies for fixed Fermi energy and $G(E)$ for a fixed potential can be qualitatively different.

Realistic modeling of the electrostatics of the devices allows us to check the validity of this prediction without making any assumptions about the shape of the potential profile. The basis of the computations is given by the structural data: thickness and material of the layers, concentration of doping impurities, and 3D geometry of the gates. The electrostatic potential $U(x, y, z)$ was calculated self-consistently along with the electron density $\rho(x, y, z)$ and the statistical exchange-correlation potential $U_{\text{exc}}(x, y, z) = U_{\text{exc}}(\rho)$ in the GaAlAs/GaAs/GaAlAs quantum well.¹³ The function $U_{\text{exc}}(\rho)$ was chosen to be the same as that used recently for modeling a 2DEG (Ref.14) and a 1D quantum wire.¹⁵ It is important that exchange and correlation are taken into account, particularly at low electron densities.¹⁶

In the middle part of the channel, ρ was computed using a quasiclassical approximation from the 1D density of states, and the temperature was chosen to imitate tunneling through the potential barrier along x . We have found that the threshold voltages for the finger gates are different in the modeling of the electrostatics of the structures and those found experimentally. However, we note that experimentally these voltages are sensitive, for example, to the charge in PMMA layer. As a result, we chose the gate voltages to fit computed curves $G(V_g)$ to the experimental curves. The conductance was determined by solving the multiple-mode transmission problem with an effective potential $U_{\text{eff}}(x, y)$ in the plane of 2DEG calculated using 3D-electrostatic modeling for a given V_g , and V_F .¹⁷ We considered 15 transverse modes in the calculations. As a result, if nonadiabaticity of the constriction is responsible for the reduction in quantized values, then we would expect to find evidence for intersubband scattering. In fact, intersubband mixing is absent for the modeled constrictions and transport is adiabatic.

We also studied the effect of the transverse finger gates on the electrostatic potential in the constriction. When the voltage on the outermost finger gates is equal to zero and the negative voltage on the center finger gate is large, a potential of a triangular shape is formed along the channel, with a smoothed top and the base width defined by the separation between the outermost finger gates [solid line in Fig. 3(a)]. If there were only one finger gate in the center then the longitudinal potential would resemble a triangle with a wider base (dotted line). Finally, in the case in which there are no overlapping finger gates, the potential would look like a smooth wide hump [dashed line in Fig. 3(a)]. The presence of the finger gates increases ω_x and in addition decreases ω_y , permitting the ratio ω_y/ω_x to approach 1.

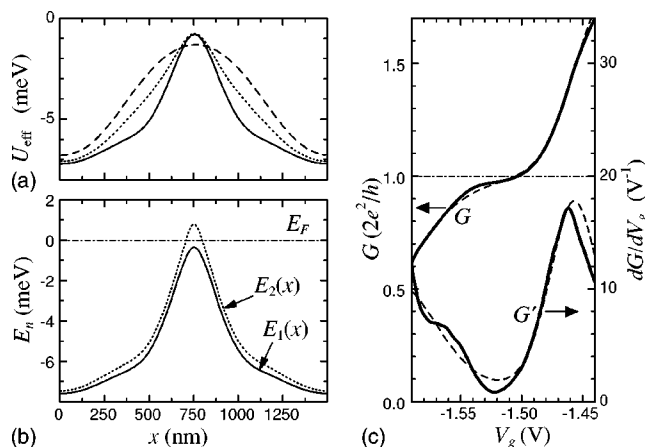


FIG. 3. (a) Effective potential $U_{\text{eff}}(x, y=0)$ computed for structures without finger gates (dashed line), with one center finger gate (dotted line), and with three finger gates (solid line). The gate voltages were chosen to adjust the transmission coefficient to unity. (b) 1D subbands $E_{1,2}(x)$ for a case characterized by surface density of charge in PMMA layer $n_i=10^{11} \text{ cm}^{-2}$ and gate voltages $V_g=-1.47 \text{ V}$ and $V_F=-2.1 \text{ V}$. (c) Experimental and theoretical curves of $G(V_g)$, dG/dV_g versus V_g . The computation (dashed line) was performed for the 1D subbands shown in (b). Measured curves (solid) refer to $V_F=-3.6 \text{ V}$. Theoretical curves were shifted by -0.055 V for better alignment.

Figure 3(b) shows the computed 1D subbands $E_{1,2}(x)$ for $V_g=-1.47 \text{ V}$ and $V_F=-2.1 \text{ V}$ [in this case, $T(E_F)=T_1+T_2=0.96$]. The Fermi level $E_F=0$ is located between the tops of curves $E_1(x)$ and $E_2(x)$. Interestingly, the quantization of the conductance $G(E)$ is exact for computed subbands $E_{1,2}(x)$.

The computed $G(V_g)$ is shown in Fig. 3(c) together with the corresponding experimental curve. The computation was done as follows. The two first subbands $E_1(x)$ and $E_2(x)$ were shifted with changing gate voltage with velocities $dE_1/dV_g=-4.8$ and $dE_2/dV_g=-11 \text{ meV/V}$, respectively ($E_2'/E_1'=2.3$). The values of these velocities were determined from the electrostatics at adjacent values of gate voltage. The transmission coefficient was computed as the sum of transmission coefficients of the first and second 1D subbands. By

computing $G(V_g)$ in such a way, there is no need to compute the electrostatic potential for every value of V_g . As a result, we find that near the conductance quantum we get results very close to the more cumbersome and time-consuming computations of 3D electrostatics followed by 2D transmission. From Fig. 3(c) we see that the predicted theoretical curve is very close to the experimental curve. The same can also be said about the curves $dG(V_g)/dV_g$. The computations demonstrate that the quantization steps are smeared out as they are in the experiment and reproduce, with 1% precision, the observed reduction in the height of the quantized conductance steps.

In conclusion, we have demonstrated that it is possible to observe deviations from exact conductance quantization in short adiabatic constrictions. In our devices we are able to control the potential profile in the 1D constriction using overlaying finger gates, and find a gradual decrease of the height of the first conductance step from $2e^2/h$ to $0.97 \times 2e^2/h$ as we increasing apply a negative voltage to an overlaying finger gate. The effect can be explained using a saddle-point potential model taking into account the deformation of the potential with varying split-gate voltage V_g . Here we find that the dependence of conductance on gate voltage is qualitatively different from that of energy such that the deviation from exact quantization appears only in $G(V_g)$. We find that the top of the first subband moves two times slower than the second subband in response to the change of the split-gate voltage. This leads to a slower decrease of the reflection compared to simultaneous increase in the tunneling, such that when the conductance step height is smeared out it is also reduced. Realistic numerical modeling of the electrostatics of the devices and transmission through the constriction produces results that are very similar to our experimental data.

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