Directional emission from a microdisk resonator with a linear defect

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Microdisk resonator with a linear defect at some distance away from the circumference is studied theoretically. We demonstrate that the presence of the defect leads to (i) enhancement of the output efficiency and (ii) directionality of the outgoing light. The dependence of the radiative losses and of the far-field distribution on the position and orientation of the defect are calculated. The angular dependence of the far field is given by Lorentzian with a width that has a sharp minimum for a certain optimal orientation of the defect line. For this orientation the whispering-gallery mode of a circular resonator is scattered by the extended defect in the direction normal to the disk boundary.

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I. INTRODUCTION

The idea to use a microdisk geometry as an alternative to the Fabry–Perot cavity in a resonator design for a semiconductor laser was introduced a decade ago.[1] The advantage of this geometry is that the losses for the whispering-gallery modes of a circular resonator are governed by evanescent leakage and, thus, can be very low. Namely, for a mode with a maximal angular momentum $M=nk_0R$, where the n is the effective refraction index, R is the resonator radius, and k_0 is the wave number of the radiation, the quality factor, Q, with exponential accuracy is given by

$$\ln Q = 2k_0R[n \ln(n+\sqrt{n^2-1}) - \sqrt{n^2-1}]. \quad (1)$$

The value of the effective refraction index is determined by the disk thickness and the indexes of an active and surrounding passive layers. In the pioneering paper Ref. 1 the effective index was $n\approx 2$, while k_0R for the smallest microdisk was ≈ 6. Then Eq. (1) yields $Q\approx 5\times 10^5$. Experimentally measured values of Q are much smaller,[2] $Q\sim 150$.The discrepancy is partially due to a prefactor neglected in Eq. (1), but primarily due to the absorption in the active layer.[1,3] With such a high Q-value the lasing threshold for a microdisk resonator is very low. For the same reason the output power is also low, which is not desirable. Another serious drawback of the microdisk geometry is that the angular dependence of the output intensity is $I(\psi)\propto \cos^2(M\psi)$, whereas applications require a directed emission. In order to remedy these drawbacks, i.e., to increase the net output without increasing the threshold, and to convert the outgoing light into a weakly divergent beam, two proposals were put forward:

(i) To extract the light out of the resonator by using two parallel disks.[4] The first disk with high Q contains a multiple quantum well structure in which the light is generated. The second passive disk coupled to the laser contains an opening serving as a leakage source. The shape of the opening determines the directionality of the output light.

(ii) To couple the light out by introducing either an indentation in the form of the "tip of the egg" [2] or corrugation[5] on the *circumference* of the disk. In the geometry,[2] the whispering gallery mode couples out efficiently upon traversing adiabatically the region of the "tip" with reduced radius of curvature. The idea underlying the calculation in Ref. 5 is that by choosing the angular period of the corrugation equal to $2\pi/M$ the whispering-gallery mode with angular momentum, M, can be partially redirected outwards. This is because the diffraction of the whispering-gallery mode on the corrugation creates a satellite with a zero angular momentum.

A radical solution for increasing the output, and, to a certain extent, directionality, by deforming the shape of the disk[6] seems to devaluate the attempts to extract light from a perfectly circular microdisk. This solution relied on the fact that deformation causes a qualitative change in the light-ray dynamics, so that the whispering-gallery trajectory of a ray becomes unstable. As a result, the ray eventually impinges on the boundary at an angle smaller than the critical angle, $\sin^{-1}(1/n)$. This leads to a refractive escape. The improvement of the directionality of the output light from a wave-chaotic resonator was studied theoretically in great detail.[7,8] The results of calculations for both "bouncing ball" and "bow-tie" modes and $nk_0R\approx 100$ can be roughly summarized as follows. In each 90°-quadrant the output light is concentrated within a total angular interval of about 60° with a strong peak of width $\sim$30° and a large number of narrow satellites.[7]

In the present paper we suggest an alternative approach for improving both the directionality and the output efficiency of a *circular* microdisk. This improvement can be achieved by introducing a properly oriented *linear* defect *away from the circumference*. Proposed geometry is illustrated in Fig. 1. The reason why the linear defect causes directional emission from a microdisk is the following. The field of a whispering-gallery mode "tunnels" *towards* the defect line, which then assumes a role of the secondary source. Since the source is extended, it emits a secondary light beam which is weakly divergent. The divergence is minimal when this secondary light beam is emitted in the radial direction, i.e., in the direction normal to the disk boundary. It is convenient to characterize the position and orientation of the defect by two parameters, namely $r_0 \gg k_0^{-1}$, radial distance from the edge to the circumference, and d, the minimal distance from the defect line to the disk center. As it will be shown below, the optimal orientation of the defect, for which

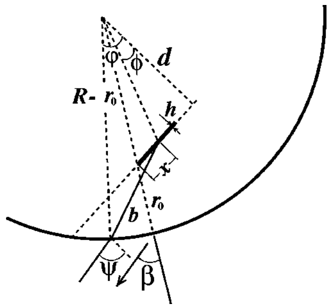

FIG. 1. Schematic illustration of a circular microdisk of a radius R with a linear defect. The defect position is characterized by r_0-the distance from the defect edge to the disk circumference along the radius; the defect orientation is fixed by the minimal distance, d, from the defect line to the disk center. The direction of the outgoing light is characterized by the angle β measured from the line connecting the edge of the defect to the center of the disk.

the direction of the secondary beam is radial, is determined by the condition $d=(R-r_0)/\sqrt{2}$. Under this condition the directionality of the output light is maximal. Below we will demonstrate that, with exponential accuracy, the radiative losses caused by the defect are given by

$$\ln Q = \frac{2^{5/2}}{3}\left(\frac{r_0}{R}\right)^{3/2}(nk_0R). \tag{2}$$

These losses dominate over the evanescent losses Eq. (1) if $r_0 \ll R$. The angular dependence of the defect-induced emission is Lorentzian, which under the optimal condition $d=(R-r_0)/\sqrt{2}$, has the form

$$I(\beta) = \frac{1}{\beta^2 + 2n^2(r_0/R)}, \tag{3}$$

with the width which is also governed by the ratio r_0/R. Note, that although Eqs. (2) and (3) apply only for $k_0 r_0 \gg 1$, this ratio can still be quite small as long as k_0R is large. Concerning the direction of the output light beam, the angle β is measured with respect to the line, connecting the edge of the defect and the center of the microdisk, as it is illustrated in Fig. 2.

The paper is organized as follows. In Sec. II we derive Eqs. (2) and (3) within the scalar diffraction theory. In Sec. III we discuss the limits of applicability of the theory. Numerical estimates are provided in Sec. IV.

II. ANGULAR DEPENDENCE OF THE OUTPUT LIGHT

Neglecting the difference between TE and TM polarizations, the field of a whispering-gallery mode in a microdisk represents a solution of the two-dimensional Helmholtz equation

$$\mathcal{E}_M(\rho,\phi) \propto \cos(M\phi) J_M(nk_0\rho), \tag{4}$$

where ρ and ϕ are the polar coordinates, M is the angular momentum, and J_M is the Bessel function. We assume that M is close to the maximal value nk_0R. Then the field Eq. (4) is localized at the boundary $\rho=R$ within a narrow ring of a

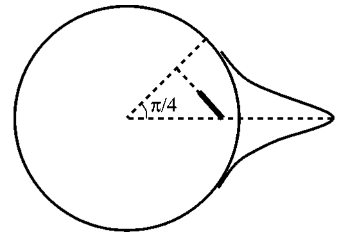

FIG. 2. Schematic illustration of the directional emission for the optimal orientation of the defect.

width $\delta\rho \sim R/(nk_0R)^{2/3} \ll R$. At smaller ρ the field falls off towards the center of the disk as

$$\mathcal{E}_M \propto \cos(M\phi)\exp\left[-\frac{2^{3/2}}{3M^{1/2}}(nk_0r)^{3/2}\right], \tag{5}$$

where $r=R-\rho$ is the distance from the boundary.

Within the scalar diffraction theory the emitted field, caused by the presence of a defect, is determined by the Fresnel–Kirchhoff diffraction integral, in which the source is the field $\mathcal{E}_M(\rho,\phi)$ taken at $\rho=\rho(\phi)$, where $\rho(\phi)$ describes the defect profile. In the case of a linear defect (Fig. 1) we have $\rho(\phi)=d/\cos\phi$. It is convenient to introduce instead of ρ a variable x which is the distance along the defect (Fig. 1). The relation between ρ and x is the following:

$$\rho = [d^2 + (\sqrt{(R-r_0)^2 - d^2} - x)^2]^{1/2}$$
$$= R - r_0 - x\frac{\sqrt{(R-r_0)^2-d^2}}{R-r_0}. \tag{6}$$

In the second equality we have used the fact that $x \ll R$. Substituting Eq. (6) into Eq. (5) we obtain

$$\mathcal{E}_M(x) \propto \exp\left[-\frac{2^{3/2}}{3}\left(\frac{r_0}{R}\right)^{3/2} nk_0R\right]\exp(-ax)\cos[M\phi(x)], \tag{7}$$

where

$$a = 2^{1/2} nk_0R\left(\frac{r_0}{R^3}\right)^{1/2}\frac{\sqrt{(R-r_0)^2-d^2}}{R-r_0}. \tag{8}$$

In Eq. (7) we assumed that $x \ll r_0$. Indeed, the relevant values of x are $\sim a^{-1}$. Then the condition $x \ll r_0$ can be rewritten as $r_0 a \sim (nk_0R)(r_0/R)^{3/2} \gg 1$. We see that this condition is equivalent to the requirement that the asymptotics Eq. (5) is valid at $r=r_0$. The first x-independent factor in Eq. (7) determines the dependence of the output field on the defect position, r_0. The expression Eq. (2) immediately follows from this dependence.

The form of the function $\phi(x)$ in Eq. (7) can be easily established from Fig. 1:

$$\tan\phi = \frac{\sqrt{(R-r_0)^2-d^2} - x}{d}. \tag{9}$$

Now we are in position to write the expression for the intensity of the outgoing light in the direction β. It is given by the following integral along the defect

$$I(\beta) \propto \left| \int_0^\infty dx\, e^{-ax} \cos [M\phi(x)] \int_{-\pi}^{\pi} d\varphi \exp [ink_0 b(x,\varphi) - ik_0 R \cos (\psi - \varphi)] \right|^2. \quad (10)$$

The integral over φ is a standard Fresnel–Kirchhoff integral. Parameter b in the exponent is the distance from a point source, located on the defect, to the exit point (Fig. 1)

$$b^2(x,\varphi) = R^2 + \frac{d^2}{\cos^2[\phi(x)]} - \frac{2Rd \cos [\varphi - \phi(x)]}{\cos [\phi(x)]}. \quad (11)$$

It is convenient to express the distance b directly through x and φ, which can be done using Eq. (9):

$$b^2(x,\varphi) = R^2 + (R - r_0)^2 - 2R(d \cos \varphi + \sqrt{(R-r_0)^2 - d^2}\sin \varphi) + 2x(R \sin \varphi - \sqrt{(R-r_0)^2 - d^2}) + x^2. \quad (12)$$

Recall now that the values of x in the integral Eq. (10) are small, $x \sim a^{-1} \ll r_0$. It can also be seen from Fig. 1 that the outgoing ray is normal to the boundary when $\cos \varphi = d/(R - r_0)$. This suggests that the difference

$$\delta = \varphi - \cos^{-1}\left(\frac{d}{R - r_0}\right) \quad (13)$$

is a small parameter. In other words, the major contribution to the Fresnel–Kirchhoff integral comes from small $\delta \ll 1$.

The integrand in Eq. (10) is a rapidly oscillating function. This allows one to expand the phase of the oscillations

$$\Phi(x,\varphi) = M\phi(x) + k_0[nb(x,\varphi) - R \cos (\psi - \varphi)] \quad (14)$$

in terms of x and δ:

$$\Phi(x,\varphi) = A_x x + A_{xx}x^2 + 2A_{x\delta}x\delta + A_{\delta\delta}\delta^2. \quad (15)$$

As it was already stated in the Introduction, the maximal directionality of the outgoing light is achieved for the position of the defect $d = (R - r_0)/\sqrt{2}$. To demonstrate this, we introduce a dimensionless deviation from the optimal defect position

$$\Delta(d) = \frac{d}{R} - \frac{R - r_0}{\sqrt{2}R}. \quad (16)$$

We will see that the width of the function $I(\beta)$ increases dramatically with Δ. Rather involved but straightforward calculations yield the following expressions for the coefficients in the expansion Eq. (15):

$$A_{x\delta} = \frac{nk_0R}{2r_0}\left(1 - \frac{4\Delta^2}{1 - 4\Delta^2}\right) \approx \frac{nk_0R}{2r_0}(1 - 4\Delta^2), \quad (17)$$

$$A_{\delta\delta} = -k_0R\left[1 - \frac{nR}{r_0}\left(1 - \frac{4\Delta^2}{1 - 4\Delta^2}\right)\right] \approx -k_0R\left[1 - \frac{nR}{r_0}(1 - 4\Delta^2)\right], \quad (18)$$

$$A_{xx} = \frac{nk_0}{4r_0}, \quad A_x = a\frac{\beta}{\delta_\psi}, \quad (19)$$

where the parameter δ_ψ in the expression for A_x is defined as

$$\delta_\psi = n\left(\frac{2r_0}{R}\right)^{1/2}\left[\frac{1 - 4\Delta^2}{1 - 4n^2\Delta^2}\right]^{1/2} \approx n\left(\frac{2r_0}{R}\right)^{1/2}[1 + 2(n^2 - 1)\Delta^2]. \quad (20)$$

With the use of the expansion Eq. (15), the Fresnel–Kirchhoff integral can be easily evaluated yielding

$$I(\beta) \propto \left| \int_0^\infty dx \exp \left[-x(a - iA_x) + ix^2\left(A_{xx} - \frac{A_{x\delta}^2}{A_{\delta\delta}}\right)\right]\right|^2. \quad (21)$$

The remaining integral over x is of the Fresnel-type. However, it cannot be reduced to the special functions $Ci(u)$ and $Si(u)$, which describe the diffraction from a semi-infinite plane.[9] This is because the linear term in the exponent contains a contribution $-ax$, which is *real*. For this reason, it is convenient to introducea new variable $z = ax$ in the integral (21). Upon substituting the coefficients (17)–(19) into Eq. (21) we arrive at the final result

$$I(\beta) \propto \left| \int_0^\infty dz \exp \left[-z\left(1 - i\frac{\beta}{\delta_\psi}\right) + iz^2\frac{n+1}{4n^2k_0r_0}F(\Delta)\right]\right|^2, \quad (22)$$

where the function $F(\Delta)$ is defined as

$$F(\Delta) = 1 + \frac{8nR\Delta^2}{(n+1)r_0}. \quad (23)$$

As in Eqs. (17)–(19), we kept only the leading Δ^2 term in the definition of F. Now we can substantiate the statement that the optimal directionality of the emission is achieved at $\Delta = 0$. Indeed, the z^2-term in the exponent of Eq. (22) leads to the broadening and oscillations of the angular dependence, $I(\psi)$. At small Δ we have $F \approx 1$; then the z^2-term is multiplied by a small factor $\sim (k_0r_0)^{-1} \ll 1$ and, thus, can be neglected. Then we immediately recover the Lorentzian Eq. (3). On the other hand, for a general position of the defect we have $\Delta \sim 1$, and $F \sim R/r_0$. Then the z^2-term acquires a much larger coefficient $2R/(nk_0r_0^2)$, resulting in the loss of the directionality of the output light. This is illustrated in Fig. 3. It is seen that significant broadening and sideback oscillations set in already at small values of Δ. In particular, for $\Delta = 0.3$ the broadening is 60%.

III. DISCUSSION

Let us first discuss the validity of the assumptions used in the above calculation

(a) $I(\beta)$ was calculated within the scalar diffraction theory using the Fresnel–Kirchhoff approach. Note that, for a circular geometry, $I(\beta)$ can be calculated exactly by solving the scalar wave equation and treating defect as a perturba-

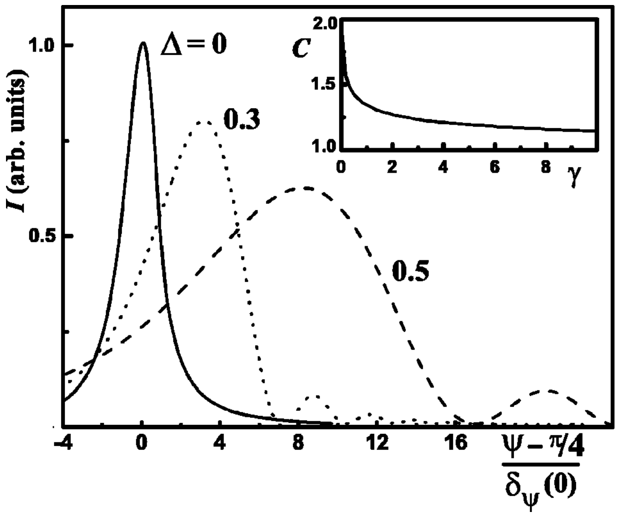

FIG. 3. (a) Schematic illustration of the directional emission for the optimal orientation of the defect. (b) Angular distribution of the far-field emission intensity is plotted for different deviations Δ [Eq. (16)] from the optimal condition $d=(R-r_0)/\sqrt{2}$. Inset: dimensionless broadening factor C is plotted versus the dimensionless parameter γ, defined by Eq. (25).

tion. Then the expression for $I(\beta)$ is given by a sum over angular momenta of the leaking modes. Fresnel diffraction corresponds to replacing this sum by an integral. The accuracy of such a replacement is determined by the next term in the Poisson expansion, which contains an exponential factor $\exp[-2^{3/2}\pi n k_0 (r_0 R)^{1/2}]$. Thus, the condition of validity of the Fresnel–Kirchhoff approach is $r_0 \gg 1/(k_0^2 R)$, which is not restrictive at all.

(b) The calculation based on the Fresnel–Kirchhoff integral Eq. (10) implies that the field on the defect is created only by the mode $\mathcal{E}_M(\rho,\varphi)$. In principle, the scattering by the defect leads to the excitation of the modes with *all* momenta. We have neglected the contribution of these excited modes to the integrand in Eq. (10). The question arises: what is the criterion for neglecting these contributions. There are two aspects to be addressed: (*i*) the validity of the perturbative approach for an arbitrary orientation of the defect and (*ii*) how the criterion of validity is modified for the optimal orientation $d=(R-r_0)/\sqrt{2}$. The answer to the question (*i*) is that the perturbative approach is valid when the decay $\mathcal{E}(R-r_0)/\mathcal{E}(R) \sim \exp[-(2^{3/2}/3)(r_0/R)^{3/2} n k_0 R]$ of the whispering-gallery mode Eq. (4) from the boundary to the defect position is much smaller than $(r_0 a)^{-1} \sim R^{1/2}/k_0 r_0^{3/2} \ll 1$. The meaning of this criterion, $[\mathcal{E}(R-r_0)/\mathcal{E}(R)](r_0 a) \ll 1$, is the following. The "dangerous" modes, i.e., the modes for which the corresponding rays intersect the defect, have the angular momenta $M < nk_0(R-r_0)$. Despite the microsdisk being an integrable system, the presence of the defect might transform these dangerous modes into chaotic. Thus, the above criterion requires that the coupling of the mode with $M \approx nk_0 R$ to the dangerous modes is weak. In this criterion the factor $\mathcal{E}(R-r_0)/\mathcal{E}(R)$ comes from the coupling "matrix element," while $(r_0 a)^{-1}$ originates from the "energy denominator."

Remarkably, for the optimal orientation, the criterion of validity of the perturbative treatment is much weaker, and is given by $[\mathcal{E}(R-r_0)/\mathcal{E}(R)](a/nk_0) \ll (r_0 a)^{-1}$. The additional small factor, (a/nk_0), in the matrix element originates from the fact that, for the optimal orientation, the field of the mode Eq. (4) with $M \approx nk_0 R$, upon scattering by the defect, is directed primarily normally to the boundary of the disk. On the other hand, the dangerous modes have the momenta, $M < nk_0(R-r_0)$. As can be seen from Eq. (10), the excitation of the modes with $(nk_0 R - M) \gtrsim nk_0 r_0$ is suppressed by a factor $(nk_0/a)^2 \sim R/r_0$. Thus, the final criterion of validity of the perturbative treatment can be presented as

$$\gamma \gg \frac{3}{2} \ln (nk_0 r_0^2/R), \tag{24}$$

where the parameter γ is defined as

$$\gamma = 2^{1/2} n k_0 R \left(\frac{r_0}{R}\right)^{3/2}. \tag{25}$$

The condition Eq. (24) can be also rewritten as $\gamma \gg \ln[\gamma^2/(nk_0 R)^{1/2}]$, i.e., it is satisfied practically for any $\gamma > 1$.

(c) According to Eq. (3), the full width at half maximum (FWHM) is equal to $2\delta_\psi = 2n(2r_0/R)^{1/2}$. This equation was derived under the assumption that the defect is located far enough from the circumference of the disk, i.e., $r_0 \gg \delta\rho \sim R/(nk_0 R)^{2/3}$. It is possible to derive a more general expression for $I(\beta)$, that is valid for $r_0 \sim \delta\rho$, when the asymptotics Eq. (5) is not yet applicable. Derivation is based on the integral representation of the Bessel function and yields

$$I(\beta) \propto \frac{2}{(\pi\gamma)^{1/2}} \int_0^\infty ds \frac{e^{-\gamma s^2}}{(1+s)^2 + (\beta/\delta_\psi)^2}, \tag{26}$$

where γ is defined by Eq. (25). It is seen that the condition $r_0 \gg \delta\rho$ corresponds to $\gamma \gg 1$. Then we immediately recover the Lorentzian Eq. (3). At moderate γ, the FWHM is given by $2C(\gamma)\delta_\psi$, where the function $C(\gamma)$ is plotted in Fig. 3, inset. It is seen that within a wide interval $1 \lesssim \gamma \lesssim 10$ the broadening factor $C(\gamma)$ changes very slowly. Then the FWHM can be expressed in terms of γ as $2^{4/3} n C(\gamma)(\gamma/nk_0 R)^{1/3}$, which is also a slow function of γ. Choosing for concreteness $\gamma=1$, we find for FWHM a concise expression $3.35n/(nk_0 R)^{1/3}$.

IV. CONCLUSION

A. Numerical estimates

We now turn to the numerical estimates. Four types of microdisk semiconductor lasers have been described in the literature so far:

(i) The lasers for wavelengths $\lambda \approx 1.5\ \mu$m have M-values, reported in Refs. 1–4 and 10–15, that are rather low $(10 \lesssim M \lesssim 70)$, and $n \approx 2.5$. For this n and maximal $M=70$ the FWHM is 116°.

(ii) Lasers for $\lambda \approx 0.8\ \mu m$[16–18] have $n \approx 3.1$ and also rather small M ($30 \lesssim M \lesssim 300$). With maximal $M = 300$ we get 89° for FWHM.

(iii) Nitride-based lasers operating at $\lambda \approx 0.4\ \mu m$[19–21] have much higher M-values ($200 \lesssim M \lesssim 600$) and $n \approx 2.8$. This yields the FWHM of 64°.

(iv) Microdisk lasers based on noncrystalline materials (polymer[22] and dye solution[23]) have also been reported. For these materials $n \approx 1.8$ is smaller, and the values of M (930 in Ref. 22 and 3000 in Ref. 23) are high. Both factors tend to narrow $I(\beta)$. Namely, for $M = 1000$ the FWHM of 34° can be achieved.

B. Qualitative interpretation

Let us now discuss the physical meaning of the optimal condition, $d = (R - r_0)/\sqrt{2}$. As it is seen from Eq. (7), the phase of the whispering-gallery mode changes along the defect. As the defect plays a role of a source of the outgoing light, this change, $\phi(x)$, is equivalent to the rotation of the line of the constant phase by an angle $\sin^{-1}[d/(R - r_0)]$. Then, under the optimal condition, the line of the constant phase is *perpendicular* to the radial line drawn through the edge of the defect (Fig. 1). In other words, under the optimal condition, the defect can be replaced by a constant phase line at distance r_0 from the circumference that is *parallel* to the circumference. Clearly, the angular width of the far field emitted by this line is minimal for this parallel orientation. Note that the condition $d = (R - r_0)/\sqrt{2}$ corresponds to the angle between the line of the defect and the line, connecting the defect edge to the center of the microdisk, being equal to 45° (see Fig. 2). The meaning of this condition becomes immediately transparent if the defect was located not within the "tunneling" *tail* of the whispering-gallery mode, but rather near the circumference. For this defect position, the whispering-gallery mode can be viewed as a plane wave, propagating along the perimeter. Correspondingly, the defect can be viewed as a *beamsplitter* for this plane wave. Then it is obvious that one of the splitted beams would be directed normally to the circumference precisely for the 45° orientation shown in Fig. 2. As follows from Eq. (3), moving the defect away from the circumference by r_0 leads to the divergence of the output beam. On the other hand, such moving is necessary to maintain a high quality factor, as follows from Eq. (2).

Let us mention three points that are not directly related to the optimal orientation:

(a) The extension of the whispering-gallery mode in the direction perpendicular to the plane of the disk is equal to $D_{eff} = D + [k_0\sqrt{n^2 - n_s^2}]^{-1} + [k_0\sqrt{n^2 - n_a^2}]^{-1}$, where D is the thickness of the disk, and n_s, n_a are the refractive indices of the substrate and air, respectively. Then the angular divergence of the output beam in the vertical direction can be estimated as $2\pi/k_0 D_{eff}$.

(b) Normally a whispering-gallery mode can be viewed as a standing wave resulting from superposition of two counterpropagating waves along the circumference. This gives rise to the $\cos^2(M\psi)$ angular distribution of intensity in the absence of a defect.[1] From the analogy to the beamsplitter it is easy to see that only one of two counterpropagating waves is redirected by the defect towards the perimeter of the disk, while the second wave is redirected towards the center. This means that the defect-induced output *does not* contain angular modulation $\propto \cos^2(M\psi)$.

(c) The scattering efficiency of the linear defect constitutes a prefactor in the defect-induced quality factor, given by Eq. (2). If the thickness, h, of the defect is small, this prefactor can be also small. In Eq. (2) for ln Q this strength constitutes an additive term. It is straightforward to see that this term is h/r_0 times smaller than the main term. If the material of the defect does not differ from the material of the disk, so that the defect differs only in thickness from the rest of the disk, then the scattering strength acquires an additional small factor $\delta D_{eff}/D_{eff}$, where δD_{eff} is the change of the effective thickness within the defect.

C. Concluding remarks

(i) In Ref. 6 the improvement of the output characteristics of microdisk laser, achieved by introducing the deformation, is due to the fact that, when the disk is deformed, the light rays are unable to stay within a whispering-gallery trajectory, and experience refractive escape in the course of the chaotic motion.[24] In the present paper we considered a perfectly circular microdisk with a defect. A *pointlike* defect at some distance away from the boundary, which is the geometry similar to that considered in Ref. 25, would be unable to couple out all the whispering-gallery modes. Namely, such a defect would not affect the modes having a *node* at the defect position. Naturally, a microdisk would lase in one of these high-Q modes "decoupled" from the defect. Our main message here is that no whispering-gallery mode can evade an *extended* defect and will be directed out of the resonator as a result of scattering by this defect.

(ii) An alternative example of an extended defect in a perfectly circular microdisk, an annulus, was studied in great detail as a convenient example of a system exhibiting the wave chaos.[26–28] However, from the perspective of the emission from the disk, the annulus cannot provide directionality. This can be seen from a simple geometrical consideration. If the linear defect with optimal orientation is approximated by segment of a circle, so that the defect line is tangent to the circle, then this approximating circle would contain another region, which is *closer* to the circumference than a defect. Therefore, the leakage would be governed by this *closest* region. But the closest region has a "wrong" (nonoptimal) orientation. Hence, there will be no directionality of the output.

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