Strong-coupling branching between edges of fractional quantum Hall liquids

V. V. Ponomarenko* and D. V. Averin
Department of Physics and Astronomy, Stony Brook University, SUNY, Stony Brook, New York 11794, USA
(Received 25 June 2004; published 15 November 2004)

We have developed a theory of quasiparticle backscattering in a system of point contacts formed between single-mode edges of several fractional quantum Hall liquids (FQHLs) with different filling factors ν_j and one common single-mode edge ν_0 of another FQHL. In the strong-tunneling limit, the model of quasiparticle backscattering is obtained by the duality transformation of the electron tunneling model. The new physics introduced by the multipoint-contact geometry of the system is coherent splitting of backscattered quasiparticles at the point contacts in the course of propagation along the common edge ν_0. The "branching ratios" characterizing the splitting determine the charge and exchange statistics of the edge quasiparticles that can be different from those of Laughlin's quasiparticles in the bulk of FQHLs. Accounting for the edge statistics is essential for the system of more than one point contact and requires the proper description of the flux attachment to tunneling electrons.

DOI: 10.1103/PhysRevB.70.195316 PACS number(s): 73.43.Jn, 71.10.Pm, 73.23.Ad

I. INTRODUCTION

Electron transport properties of the fractional quantum Hall liquids (FQHLs) continue to attract considerable interest motivated by the unusual properties of FQHL excitations which are characterized by fractional charge and nontrivial exchange statistics. Since excitations in the bulk of a typical FQHL are suppressed by the energy gap, the low-energy transport properties are determined by the gapless modes at the edges of the liquid.[1] In the simplest case of a FQHL with fillings factor $\nu=1/odd$, its sharp edge should support one bosonic mode that is described as a single branch of chiral Luttinger liquid.[2] Although the edge states are gapless and in principle can carry excitations of arbitrary charge, in the case of tunneling between the edges of FQHLs with equal filling factors, strong tunneling produces excitations which coincide[3] with Laughlin's quasiparticles in the bulk of the liquid. Experiments on resonant tunneling through a quantum antidot[4] and on the shot noise in quasiparticle backscattering in a single point contact[5] measure directly the fractional charge of these excitations. Edge-state tunneling should also make possible the measurements of quasiparticle exchange statistics[6] which can be used in the development of FQHE qubits[7] for solid-state quantum computation.

In the situation of tunneling between the edges of FQHLs with different filling factors, strong tunneling should produce quasiparticles that are different from bulk quasiparticles.[8] The charge of such "contact" quasiparticles coincides with the dc conductance of the point contact (if the two are measured in units of electron charge e and the free-electron conductance e^2/h, respectively). Theory up to now could predict only the properties of one point contact between different edges. The aim of this work is to develop a theory of strong tunneling between FQHL edges with different filling factors in a junction with more than one point contact. Such a model of multipoint-contact junction was introduced[9,10] to describe experiments[11] on tunneling between FQHL edge and external Fermi-liquid reservoir. Understanding of the strong-tunneling limit of this model is important for the description of the reservoir-edge equilibration that is responsible for correct quantization of the two-terminal FQHL conductance.[12] Multipoint-contact junctions with a well-controlled geometry similar to the one considered in this work are also studied experimentally.[13]

The main technical obstacle to the description of strong multipoint-contact tunneling between different edges is the fact that the bosonic fields in the tunneling operators of different contacts do not commute with each other and, at first sight, cannot be localized simultaneously at the minima of large tunnel potential, as can be done for one point contact. This problem is resolved, however, if the bosonic fields are modified[12] by the proper choice of the statistical phase of the tunneling electrons [see Eqs. (11) and (12) below]. The statistical phase preserves the Fermi statistics of electrons but accounts for the change of the number of flux quanta attached to them in the FQHLs[14] in the process of tunneling between the liquids with different filling factors. With the flux attachment taken into account, the total bosonic tunneling fields commute, and one can build the strong-coupling description of the multipoint-contact junction in close analogy to the case of one point contact. Here, we use this approach to develop complete description of strong tunneling in the multipoint-contact junction extending previous results[12] to quasiparticle backscattering.

Important elements of our approach can be summarized as follows. Specific junction model considered in this work is characterized by the existence of one edge ν_0 common to all point contacts (Fig. 1). The quasiparticle backscattering is produced by finite reflection coefficients of the contacts, and is described as instanton tunneling between the infinite set of the ground states of the original electron tunneling model that are degenerate in the absence of backscattering. Expansion of the junction dynamics in terms of instantons gives the model of quasiparticle tunneling which is dual to the electron tunneling model. Duality transformation relating the two models produces the quasiparticle exchange statistics from the electron statistical phases ascribed through the flux attachement. Correct description of quasiparticle statistics in our junction geometry enables one to combine sequentially the edge-state transformations at different point contacts in

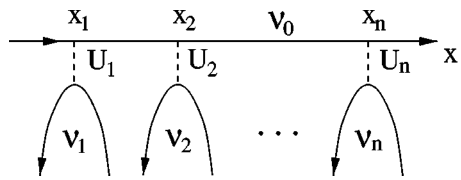

FIG. 1. Diagram of the model of a tunnel junction considered in this work: n edges of FQHLs with different filling factors ν_j tunnel into one FQHL edge with the filling factor ν_0 at points x_j along this edge. U_j are electron tunneling amplitudes. Each edge is assumed to support one branch of bosonic excitations with arrows indicating direction of propagation of these excitations.

such a way that the incoming edges are split coherently at each point contact in the course of propagation along the common edge ν_0. This process can be described with the "branching ratios" which define how the incoming currents (or charges) are split between the outgoing edges. Quasiparticles produced by backscattering at one point contact are split similarly at all point contacts downstream the edge ν_0 from the "contact of origin," and the edge-state branching ratios determine the charges generated in each edge by backscattering. As usual, these charges should manifest themselves through the intensity of the shot noise of outgoing currents. The charge splitting also provides natural interpretation of the quasiparticle exchange statistics obtained by the duality transformation, so that both the charge and statistics of backscattered quasiparticles can be viewed as being defined by the strong-coupling edge-state branching.

The paper is organized as follows. Section II defines the electron tunneling model considered in this work. Section III describes the bosonization procedure for the Klein factors of electron tunneling operators that implements the flux attachment. Section IV explains the instanton transformation, formulates the dual model of quasiparticle tunneling, and calculates the dc current and shot noise in outgoing edges of the junction. In Sec. V we apply the results of Sec. IV to the situation that corresponds to the junction between a Fermi-liquid reservoir and FQHL edge. Quasiparticle backscattering in this case gives rise to corrections to the Ohmic behavior of the junction: nonlinear current-voltage characteristic and shot noise, which limit the reservoir-edge equilibration obtained in the absence of backscattering. In the Appendix, we summarize for convenience the known results for strong tunneling in one point contact that are used in the main text.

II. MULTIEDGE MODEL OF WEAK ELECTRON TUNNELING

The model we consider consists of n edges of different FQHLs of the filling factors ν_j, $j=1,\ldots,n$, which form n tunnel point contacts with an edge of another FQHL of the filling factor ν_0 (see Fig. 1). All liquids are assumed to correspond to simple Laughlin states with the filling factors $\nu_l=1/odd$, $l=0,\ldots,n$, which in general can all be different. Since bulk excitations inside the liquids have energy gap, at energies below some common cutoff energy D, only the edges support excitations that propagate along them. In the weak tunneling regime, when each of the jth edges is well separated from the 0 edge, the tunnel currents between them should be carried by individual electrons. We assume that the tunnel contact formed by the jth edge is located at the point x_j along the 0 edge and its width is much smaller than the magnetic length. Tunneling in such a system of contacts is described by the Lagrangian

$$\mathcal{L}_{tunn}=\sum_{j=1}^{n}[U_j\psi_0^{+}(x_j,t)\psi_j(x_j,t)+\text{H.c.}], \tag{1}$$

where the tunnel amplitudes U_j are taken to be real and positive, and ψ_l is the electron operator of the lth edge.

If we assume that all edges are sharp enough to enable us to use simple bosonic single-mode description of the edge-state dynamics introduced by Wen,[2] the electron operator ψ_l can be expressed as

$$\psi_l=(2\pi\alpha)^{-1/2}\xi_l e^{i\phi_l(x,t)/\sqrt{\nu_l}}. \tag{2}$$

Here ϕ_l is the bosonic field of the edge l; Majorana fermions ξ_l account for mutual statistics of electrons in different edges, and a common factor $1/\alpha=D/v$ denotes momentum cutoff of the edge excitations. Since the spatial dynamics of the edges $j=1\ldots n$ does not affect the tunnel currents, propagation velocities of these edges are irrelevant, and we take them to be equal to the velocity v of the excitations of the edge ν_0. The bosonic fields ϕ_l are normalized in such a way that their free dynamics is governed by the standard Lagrangian density[2]

$$\mathcal{L}_0=\frac{1}{2}\sum_{l,p=0}^{n}\phi_l\hat{G}_{lp}^{-1}\phi_p\equiv\frac{1}{2}\phi\hat{G}^{-1}\phi,$$

$$\hat{G}_{lp}^{-1}=\frac{\delta_{lp}}{2\pi}\partial_x(\partial_t+v\partial_x), \tag{3}$$

which corresponds to the Hamiltonian

$$\mathcal{H}_0=\frac{v}{4\pi}\int dx(\partial_x\phi_l(x))^2 \tag{4}$$

and the commutation relations

$$[\phi_l(x),\phi_p(0)]=i\pi\delta_{lp}\text{sgn}(x). \tag{5}$$

In the following, we will need free correlator of the fields ϕ_l ordered in imaginary time τ:

$$\hat{G}_{lp}(x,\tau)=\delta_{lp}g(x,\tau),\quad g(x,\tau)=\langle T_\tau\{\phi_l(x,\tau)\phi_l(0,0)\}\rangle. \tag{6}$$

To find this correlator, we start with the retarded Green's function in real time t:

$$g^R(x,t)=-i\Theta(t)\langle[\phi_l(x,t),\phi_l(0,0)]\rangle, \tag{7}$$

where $\Theta(t)$ is the step function. Lagrangian (3) implies that dynamics of $g^R(x,t)$ is governed by the equation

$$\partial_x(\partial_t+v\partial_x)g^R(x,t)=2\pi\delta(x)\delta(t).$$

Solution of this equation that satisfies the symmetry of the Eq. (7) [the commutator part of Eq. (7) should be antisymmetric with respect to simultaneous change of sign of x,t] is

$$g^R(x,t) = \pi\Theta(t)\mathrm{sgn}(x - vt).$$

The standard steps: Fourier transformation of $g^R(x,t)$, $g^R(x,\omega)=\int dt e^{i\omega t} g^R(x,t)$, and analytical continuation to the upper half-plane of frequency, $\omega \to i\omega$, give the Fourier transform of the imaginary-time correlator for positive frequencies ω: $g(x,\omega) = -g^R(x,\omega)|_{\omega\to i\omega}$.

Similar calculations for the advanced Green's function

$$g^A(x,t) = i\Theta(-t)\langle[\phi_l(x,t),\phi_l]\rangle = -\pi\Theta(-t)\mathrm{sgn}(x-vt),$$

give $g(x,\omega)$ for negative frequencies, and it can finally be written for all Matsubara frequencies as

$$g(x,\omega) = \frac{2\pi}{\omega}\mathrm{sgn}(x)\left(-\frac{1}{2} + \Theta(\omega x)e^{-\omega x/v}\right). \tag{8}$$

The first part of this correlator does not depend on $|x|$ and characterizes singular behavior of the correlator at short times, $g(x,\tau) \sim i\pi\,\mathrm{sgn}(x\tau)/2$, which is responsible for the intraedge statistics of the electronic operators ψ_l. With the adopted normalization of ϕ_l, the operator of charge density of the lth edge at point x is

$$\rho_l(x,\tau) = (\sqrt{\nu_l}/2\pi)\partial_x\phi_l(x,\tau),$$

and the current in the edge is related to the charge density as $j_l = v\rho_l$.

As in the case of a single point contact, the effective electron tunneling amplitudes are renormalized from their original values U_j in Eq. (1) by fluctuations of the fields ϕ_l. The effective tunneling amplitudes scale with energy in such a way that the weak-tunneling limit is always stable at sufficiently low energy. The main focus of this work is on the strong-tunneling regime realized at large temperatures or bias voltages V_j across the point contacts which make the effective tunneling amplitudes large. The description of the strong-tunneling limit presented below is based on the instanton expansion around the saddle-point solution.[12] As discussed in the Introduction, this solution requires appropriate bosonization of the Klein factors of the electron tunneling operators that takes into account the flux attachment.

III. BOSONIZATION OF THE KLEIN FACTORS

Substitution of the bosonic representation (2) of the electron operators ψ_l into Eq. (1) transforms the tunneling Lagrangian into

$$\mathcal{L}_{tunn} = \sum_{j=1}^{n}\left[\frac{U_j}{2\pi\alpha}F_j \exp\{i\lambda_j\varphi_j(t)\} + \mathrm{H.c.}\right]. \tag{9}$$

Here

$$\lambda_j\varphi_j(t) \equiv \frac{\phi_0(x_j,t)}{\sqrt{\nu_0}} - \frac{\phi_j(x_j,t)}{\sqrt{\nu_j}},$$

and the factors

$$\lambda_j = \left[\frac{\nu_0+\nu_j}{\nu_0\nu_j}\right]^{1/2} \tag{10}$$

are chosen in such a way that the normalization of the bosonic operator φ_j coincides with the normalization of the bosonic fields ϕ_l used before, so that the imaginary-time correlator of φ_j is given by the same Eq. (8) with $x=0$: $g(0,\omega)=\pi/|\omega|$.

In the Lagrangian (9), F_j represents the anticommuting statistical Klein factors, $F_j=\xi_0\xi_j$, which account for mutual statistics of electrons in different edges. These factors play an important role in the strong multiedge tunneling considered in this work. A convenient way of taking them into account is to express them through the zero-energy bosonic fields η_j:[12,15]

$$F_j = e^{i\eta_j}, \quad [\eta_i,\eta_j] = i\pi\gamma_{ij}, \tag{11}$$

where γ_{ij} are odd integers. Different choices of γ_{ij} correspond to different branches of the phase of the fermionic statistical factor -1 arising from interchange of F_i and F_j. Under the condition that the point contacts are well-separated, $|x_i - x_j| \gg \alpha$, minimization of energy of the strong-coupling ground state requires that the statistical phases are taken as[12]

$$\gamma_{ij} = \mathrm{sgn}(i-j)(1-\delta_{ij})\frac{1}{\nu_0}. \tag{12}$$

This equation assumes that the point contacts are numbered as in Fig. 1: $x_i < x_j$ for $1 \leqslant i < j \leqslant n$, and the coordinate x increases in the direction of the edge propagation.

The steps similar to those that lead to Eq. (8) give then the Fourier transform of the τ-ordered correlator of the fields η_j:[16]

$$\langle\eta_i\eta_j\rangle = \mathrm{sgn}(i-j)(1-\delta_{ij})\frac{\pi}{\omega\nu_0}. \tag{13}$$

Substitution of the bosonized form (11) of the Klein factors into Eq. (9) turns the tunneling Lagrangian into a function of n bosonic variables

$$\Phi_j = \lambda_j\varphi_j + \eta_j, \tag{14}$$

and one can reduce the kinetic Lagrangian (3) to a simplified form by integrating out all the fields except Φ_j. The resulting Gaussian action for the fields Φ_j can be obtained directly by noticing that it should be determined by the matrix of correlators $\hat{K}_{ij}(\omega)=\langle\Phi_i(-\omega)\Phi_j(\omega)\rangle$ as

$$\mathcal{S}_0 = \frac{1}{2}\Phi\hat{K}^{-1}\Phi = \frac{1}{2}\sum_{ij}\sum_{\omega}\Phi_i(-\omega)\hat{K}^{-1}_{ij}(\omega)\Phi_j(\omega), \tag{15}$$

where $\omega=2\pi mT$ denotes the Matsubara frequencies with $m=0\pm1,\pm2,\ldots$ and temperature T. The matrix of correlators $\hat{K}_{ij}$ can be found from the correlator (8) of the tunneling fields φ_j [determined by the original kinetic Lagrangian (3)] and the correlators (13) of the statistical fields. Combining the two we get

$$\hat{K}_{ij}(\omega) = \frac{2\pi}{|\omega|}\left[\frac{\lambda_j^2}{2}\delta_{ij} + \frac{\Theta(\mathrm{sgn}(i-j)\omega)}{\nu_0}e^{-|\omega t_{ij}|}(1-\delta_{ij})\right], \quad (16)$$

where $t_{ij}=t_i-t_j$ and $t_j \equiv x_j/v$. Important feature of the terms in Eq. (16) that are nondiagonal in i, j is that they do not contain the singular statistical parts. The singular part of the correlator (8) of the tunneling fields φ_j is cancelled by the statistical fields η_j, if the phases γ_{ij} (11) are chosen appropriately, as in Eq. (12).

IV. STRONG-TUNNELING LIMIT

A. Instanton expansion and duality transformation

The tunneling action (9) expressed through the bosonic fields Φ_j (14) has an infinite set of minima at $\Phi_j=2\pi \times$ integer, $j=1\ldots n$, that are degenerate in energy. In the strong-tunneling limit, when the electron tunneling amplitudes U_j are large, these minima are well-separated in a sense that the amplitude of Φ_j-tunneling between them is small. Such rare tunneling processes correspond physically to finite backscattering at each of the point contacts j and are described by the instanton tunneling solutions which on the long-time scale behave as $2\pi e_{lj}\Theta(\tau-\tau_{lj})$, where index l counts different instanton tunneling events, $e_{lj}=\pm 1$, and τ_{lj} are the times of tunneling of the Φ_j component. On the short time scales on the order of $1/D$, the variation of Φ_j is smoothed around τ_{lj}, with the exact behavior dependent on the form of the energy cutoff. For some special form of the cutoff,[17] the shape of the instantons for short times can be found from the equations of motion. It minimizes the instanton contribution to the action in the absence of the long-time interactions and determines the instanton tunneling amplitudes. The amplitudes are, in addition, modified by quantum corrections and we take them as some unspecified parameters $W_j/(2\pi\alpha)$.

The low-energy part of the action determines the instanton-instanton interaction and can be found by substitution of the long-time asymptotic form of $\Phi_j(\tau)$, i.e., $\Phi_j(\tau) = 2\pi\times$integer$+\Sigma_l e_{lj}2\pi\Theta(\tau-\tau_{lj})$ into the Φ_j action (15). Indeed, the constant part of $\Phi_j(\tau)$ is irrelevant, and as we will see later, only the trajectories with the same initial and final values of $\Phi_j(\tau)$ for each contact j, $\Sigma_l e_{lj}=0$, are important. The Fourier transform $\Phi_j(\omega)=\sqrt{T}\int_0^{1/T}d\tau e^{i\omega\tau}\Phi_j(\tau)$ of such "neutral" trajectories is

$$\Phi_j(\omega) = \sum_l e_{lj}\frac{2\pi i\sqrt{T}}{\omega}e^{i\omega\tau_l}, \quad (17)$$

and combining this equation with Eq. (15) we see that the low-energy part of the action has the form of the pair-wise instanton interaction with each pair of instantons with indices lj and ki contributing the term

$$\mathcal{S}_{int}(\tau_l,\tau_k) = e_{lj}e_{ki}T\sum_\omega \left(\frac{2\pi}{\omega}\right)^2\hat{K}_{ij}^{-1}(\omega)e^{i\omega(\tau_l-\tau_k)}. \quad (18)$$

It is convenient to formulate the instanton dynamics in terms of the fields Θ_j, $j=1,\ldots,n$, that are defined as dual to Φ_j, i.e., satisfy the commutation relations:

$$[\Theta_j,\Phi_{j'}] = 2\pi i\delta_{jj'}. \quad (19)$$

In terms of these fields, each instanton tunneling is generated by the operator

$$\frac{W_j}{2\pi\alpha}\exp\{ie_{lj}\Theta_j(\tau_l)\} \quad (20)$$

which shifts Φ_j by $2\pi e_{lj}$. The low-energy part of the action containing instanton-instanton interaction (18) can be understood then as arising from Gaussian fluctuations of the dual fields:

$$e^{-\mathcal{S}_{int}} = e^{-e_{lj}e_{ki}\langle\Theta_i(\tau_k)\Theta_j(\tau_l)\rangle/2},$$

and Eq. (18) gives the correlators of these fields:

$$\langle\Theta_i(-\omega)\Theta_j(\omega)\rangle = (2\pi/\omega)^2\hat{K}_{ij}^{-1}(\omega).$$

To find these correlators explicitly, we need to invert the matrix $\hat{K}$ defined in Eq. (16). This task is not difficult, since the matrix has a triangular form: for negative frequencies, $\hat{K}$ and $\hat{K}^{-1}$ are upper-triangular, while for positive frequencies they are lower-triangular. In both situations, one can find $\hat{K}^{-1}$ by writing equations for the matrix elements $\hat{K}_{ij}^{-1}$ as recurrence relations in terms of the "distance" from the diagonal and solving them starting with the diagonal elements. For negative frequencies, the result is

$$\langle\Theta_i(-\omega)\Theta_j(\omega)\rangle = (2\pi/\omega)^2\hat{K}_{ij}^{-1} = \frac{2\pi}{|\omega|}\left[\frac{2}{\lambda_i^2}\delta_{ij} - \Theta(\mathrm{sgn}(i-j)\omega)B_{ij}e^{-|\omega t_{ij}|}(1-\delta_{ij})\right], \quad (21)$$

$$B_{ij} = \frac{4}{\nu_0\lambda_i^2\lambda_j^2}\prod_{i<k<j}\left(1-\frac{2}{\nu_0\lambda_k^2}\right). \quad (22)$$

For positive frequencies, Eq. (21) remains valid if we define the matrix B_{ij} for $i>j$ (below the diagonal) by the condition $B_{ij}=B_{ji}$. Qualitatively, as will be discussed in more details below, the dual fields Θ_j describe quasiparticles backscattered at the point contacts, and the matrix B_{ij} (22) determines the exchange statistics of these quasiparticles.

Similarly to the case of electron tunneling model where the correlators (16) give the kinetic part (15) of the action, the correlators (21) determine the kinetic part of the action of the dual model of quasiparticle tunneling:

$$\overline{\mathcal{S}_0} = \frac{1}{2}\Theta(\omega/2\pi)^2\hat{K}\Theta. \quad (23)$$

One more remark is that the Θ_j-correlators (21) show that the condition of neutrality of instanton tunneling trajectories, $\Sigma_l e_{lj}=0$, that was used above, is satisfied at each point contact j. Indeed, the form (21) of the correlators implies that interaction between instantons diverges at low energies and only the trajectories that satisfy the neutrality condition for each j have finite action and contribute to the evolution of the system.

B. Dual chiral fields and Klein factors

One can see directly that the expansion of the system propagator in the number of instanton tunneling events can be generated as the expansion in the instanton tunneling Lagrangian composed of the tunneling operators (20). This means that the dual model of quasiparticle tunneling can be formulated in complete analogy to the direct model of electron tunneling. Comparing the correlators (21) with those from Eq. (16), one can notice that similarly to the Φ fields (14), Θ_j can be represented as the sum

$$\Theta_j = \frac{2}{\lambda_j}\theta_j + \overline{\eta_j} \tag{24}$$

of the fields with, respectively, chiral and pure statistical correlators. Equation (21) implies then that the chiral correlator of the fields θ_j for $i \neq j$ is

$$\langle \theta_i \theta_j \rangle = -(\lambda_i \lambda_j/4) B_{ij} g(x_i - x_j, \omega). \tag{25}$$

The representation (24) of Θ_j allows us to write the instanton tunneling Lagrangian in the form analogous to Eq. (9):

$$\overline{\mathcal{L}}_{tunn} = \sum_{j=1}^{n} \left[\frac{W_j}{2\pi\alpha} \overline{F_j} \exp\left\{ i \frac{2}{\lambda_j} \theta_j(t) \right\} + \text{H.c.} \right], \tag{26}$$

where the Klein factors of the dual model are

$$\overline{F_j} = e^{i\overline{\eta_j}}, \quad [\overline{\eta_i}, \overline{\eta_j}] = -i\pi \,\text{sgn}(i-j)(1-\delta_{ij}) B_{ij}. \tag{27}$$

Before discussing the physical consequences of the dual model of quasiparticle backscattering, we present the derivation of this model that is less rigorous than the one that uses instanton expansion, but more direct. The approach is based on independent application of the known strong-tunneling solution for one point contact[8] to individual contacts of our multipoint-contact junction and matching the obtained solutions at successive contacts along the edge ν_0 common to all of them (see Fig. 1). Since the bosonic fields at different point contacts interact strongly at low frequencies, the possibility to simply match the solutions at successive contacts is by no means trivial and represents an important assumption. The instanton calculation described in the previous subsection can be viewed as the proof that this assumption is indeed valid provided one chooses correctly the statistical phases of electrons in different contacts, which after duality transformation determine the statistical phases of backscattered quasiparticles.

In more details, consider the jth point contact of our multicontact model. Strong-tunneling solution (reviewed in the Appendix) can be described as application of the Dirichlet boundary condition to the tunneling field $\varphi_j(x)$ which localizes this field at the minimum of the tunnel potential at the point of contact j. "Unfolded" form of this condition[18] implies free propagation of the two fields constructed from the

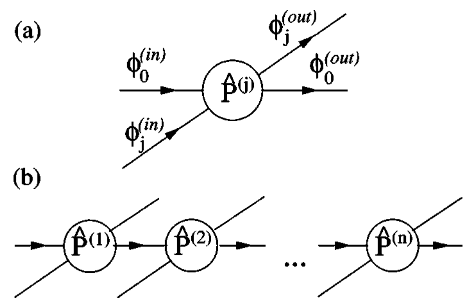

FIG. 2. Diagram of the strong-coupling scattering of the edge states in the multipoint-contact junction considered in this work: (a) transformation of the bosonic edge-state fields (30) in one individual point contact j; and (b) transformation of the fields in the n-point junction as a whole: the field ϕ_0 undergoes successive transformations in the course of propagation along the edge ν_0.

original bosonic modes $\phi_j(x)$ and $\phi_0(x)$ of the edges forming this contact. One is the field

$$\theta_j(x) = -\,\text{sgn}(x - x_j)\varphi_j(x) \tag{28}$$

dual to the tunneling field $\varphi_j(x)$, while the other, $\tilde{\varphi}_j(x)$, is the combination of $\phi_j(x)$ and $\phi_0(x)$ orthogonal to the tunneling field, i.e.,

$$\varphi_j = \frac{1}{\lambda_j}\left(\frac{\phi_0}{\sqrt{\nu_0}} - \frac{\phi_j}{\sqrt{\nu_j}} \right), \quad \tilde{\varphi}_j = \frac{1}{\lambda_j}\left(\frac{\phi_0}{\sqrt{\nu_j}} + \frac{\phi_j}{\sqrt{\nu_0}} \right). \tag{29}$$

[We use the same notation for the dual field θ_j defined by Eq. (28) as in Eq. (24) in anticipation of the fact, proven later, that these two fields indeed coincide.] Free evolution of the field $\theta_j(x)$ (28) is equivalent to the statement that the incoming tunneling field changes sign at the contact $x = x_j$. Combining this change of sign with Eq. (29), we see that the transformation of the incoming fields $\phi_0(x)$ and $\phi_j(x)$ at $x < x_j$ into outgoing fields at $x > x_j$ [Fig. 2(a)] can be described[8] with the 2-by-2 matrix $\hat{P}^{(j)}$:

$$(\phi_0^{(out)}, \phi_j^{(out)})^T = \hat{P}^{(j)} (\phi_0^{(in)}, \phi_j^{(in)})^T \tag{30}$$

with the matrix elements

$$\hat{P}_{00}^{(j)} = -\hat{P}_{jj}^{(j)} = \frac{\nu_0 - \nu_j}{\nu_0 + \nu_j}, \quad \hat{P}_{0j}^{(j)} = \hat{P}_{j0}^{(j)} = \frac{2\sqrt{\nu_0 \nu_j}}{\nu_0 + \nu_j}. \tag{31}$$

A natural way to combine transformations (30) at the neighboring contacts is to *assume* that the field $\phi_0^{(out)}$ coming out of each contact along the edge ν_0 serves as the incoming field $\phi_0^{(in)}$ for the next point contact downstream this edge [Fig. 2(b)]. Under this assumption (validity of which implies the nontrivial mutual statistics of backscattered quasiparticles), description of the overall transformation of the incoming bosonic modes in our multipoint-contact junction can be obtained by extending each of the matrices $\hat{P}^{(j)}$ trivially to the $n+1$ by $n+1$ matrix: the fields ϕ_i with $i \neq j$ are constant in $\hat{P}^{(j)}$, and multiplying this matrices. The full transformation of the incoming fields $\phi^{(in)} = (\phi_0, \phi_1, \ldots, \phi_n)^T$ into the outgoing fields [see Fig. 2(b)] is described then as

$$\phi^{(out)} = \hat{S}\phi^{(in)}, \quad \hat{S} = \hat{P}^{(n)}\hat{P}^{(n-1)}\cdots\hat{P}^{(1)}. \tag{32}$$

Such a combination of successive transformations of the fields at the point contacts together with the free propagation of the field ϕ_0 between the contacts implies that the correlator of the fields $\theta_j(x)$ defined by Eqs. (28) and (29) at different contacts is

$$\langle\theta_i(x_i)\theta_j(x_j)\rangle = -\frac{1}{\nu_0\lambda_i\lambda_j}g(x_i - x_j,\omega)\prod_{i<k<j}\hat{P}_{00}^{(k)}.$$

Equations (10), (22), and (31) show that this correlator coincides with the correlator (25). According to Eq. (28), the correlator $\langle\theta_j\theta_j\rangle$ at the same contact is $\langle\theta_j\theta_j\rangle = \langle\varphi_j\varphi_j\rangle = \pi/|\omega|$ and also agrees with Eqs. (24) and (21). This means that the chiral fields θ_j introduced in Eqs. (24) and (26) as the fields describing the quasiparticles backscattered at the point contacts coincide with the dual fields $\theta_j(x_j)$ from Eq. (28) matched directly between the contacts. This coincidence is nontrivial and is only correct if the mutual statistics of quasiparticles backscattered at different point contacts is accounted for by inclusion of the Klein factors $\overline{F_j}$ (27). The statistics, however, does not affect the scaling dimensions of the quasiparticle tunneling operators, which have the same values $2/\lambda_j^2$ as for one independent point contact.[19] This means that the dual model of strong tunneling in the multipoint-contact junction constructed in this work is stable for large temperatures and/or voltages applied to all contacts, and the strong-tunneling limit is always reached at sufficiently large energies.

C. Interpretation of the quasiparticle statistics and calculation of the outgoing currents and shot noise

In this subsection, we use the approach developed above to calculate the transport properties of the multipoint-contact junction in the limit of strong tunneling. Application of the voltage V_l to the lth edge can be described simply as a shift of the incoming field (see, e.g., the Appendix)

$$\phi_l^{(in)} \to \phi_l^{(in)} - \sqrt{\nu_l}V_l t, \tag{33}$$

which can be understood as the evolution of the quantum-mechanical phase of the electron operator ψ_l (2) of the lth edge. (In this work, we discuss only the dc transport properties of the junction biased by constant voltages V_l.) Numerical shift of the fields ϕ_l does not affect their properties as quantum operators, and all the results for ϕ_l discussed above remain valid in the presence of finite bias voltages.

We consider first the situation with *no backscattering*. Since the current j_l in the lth edge is related to ϕ_l as $j_l(x,t) = -\sqrt{\nu_l}\partial_t\phi_l(x,t)/2\pi$, the outgoing currents are linear in the applied voltages and one can define the matrix of conductances through the current induced in the lth edge by the voltage applied to the kth edge: $\hat{G}_{lk} = j_l/V_k$. The transformation of the fields ϕ_l according to Eq. (32) means that the conductance matrix is

$$\hat{G}_{lk} = \sigma_0\sqrt{\nu_l\nu_k}\hat{S}_{lk},$$

where σ_0 is the universal free-electron conductance e^2/h equal to $1/2\pi$ in our units $e=\hbar=1$. Direct substitution of the matrices (31) into this expression gives

$$G_{00} = \nu_0\prod_1^n \frac{\nu_0 - \nu_l}{\nu_0 + \nu_l},$$

$$G_{jj} = \nu_j\frac{\nu_j - \nu_0}{\nu_0 + \nu_j}, \quad G_{ij} = 0 \quad \text{for } j > i \geqslant 1,$$

$$G_{ij} = \frac{4\nu_0\nu_i\nu_j}{(\nu + \nu_i)(\nu_0 + \nu_j)}\prod_{j<l<i}\frac{\nu_0 - \nu_l}{\nu_0 + \nu_l} \quad \text{for } 1 \leqslant j < i \leqslant n,$$

$$G_{0j} = \frac{2\nu_0\nu_j}{\nu_0 + \nu_j}\prod_{l>j}^n\frac{\nu_0 - \nu_l}{\nu_0 + \nu_l}, \quad G_{j0} = \frac{2\nu_0\nu_j}{\nu_0 + \nu_j}\prod_{l<j}\frac{\nu_0 - \nu_l}{\nu_0 + \nu_l}, \tag{34}$$

where G_{ij} denotes the conductance $\hat{G}_{ij}$ in units of σ_0. Charge conservation requires that $\Sigma_l G_{lj} = \nu_j$, the condition that is indeed satisfied by Eqs. (34).

Equations (34) describe a flow of charge along the edges in which the difference of currents in the edges 0 and j is redistributed at the jth point contact according to its tunneling conductance $\sigma_0(\nu_j - G_{jj}) = 2\sigma_0\nu_0\nu_j/(\nu_0+\nu_j)$. Such a description of current flow in the system of several point contacts was first obtained in Ref. 10 under the assumption that quantum coherence of electron propagation along the common edge is suppressed, and electron distribution is equilibrated between the successive point contacts. The calculations presented above show that this description of the current flow remains valid in the quantum-coherent regime, when any external decoherence mechanisms are absent.

We now turn to the regime of *finite backscattering*. As discussed above, each elementary process of backscattering can be described as instanton tunneling generated by the operator $\propto\exp\{\pm i2\theta_j/\lambda_j\}$ [see Eq. (26)]. If the tunneling occurs in the point contact j, the associated shift of the field ϕ_j corresponds to the transfer of charge $q_j = 2/\lambda_j^2 = 2\nu_0\nu_j/(\nu_0 + \nu_j)$ (see, e.g., the Appendix). In the multipoint-contact junction, quasiparticle of charge q_j produced in the 0th edge at the point x_j by instanton tunneling propagates along this edge and is split between edges i with $i > j$ and the edge 0 according to the conductance matrix. Distribution of charges in backscattering processes that results from such a "current branching" at the point contacts enables us to interpret the quasiparticle Klein factors (27) of the dual model as exchange statistics of charges generated in the edge 0. Indeed, the absence of the energy gap at the edge of a FQHL makes it possible to produce arbitrary edge excitations. For an isolated edge 0, the exchange statistics of two excitations with charges p_1 and p_2 is set by its filling factor ν_0 to be p_1p_2/ν_0. The two quasiparticles which tunnel in the nearest-neighbor contacts create charges q_j and q_{j+1} in the edge 0 and the statistical angle for these quasiparticles, $2\pi q_j q_{j+1}/\nu_0$ is easily seen to indeed coincide with the statistical angle $2\pi B_{j,j+1}$

determined by the Klein factors (27) through the statistical matrix B_{ij} (22). If the quasiparticles tunnel at the point contacts that are not nearest neighbors, one needs to take into account that the charge created in the edge 0 in the vicinity of the contact i by quasiparticle tunneling at the contact $j<i$ is smaller than q_j by the factor

$$r \equiv \prod_{j<l<i} [(\nu_0 - \nu_l)/(\nu_0 + \nu_l)]$$

due to charge splitting at the intermediate point contacts. Equation (22) shows then that the matrix B_{ij} can be written as $B_{ij}=rq_iq_j/\nu_0$, i.e., the statistics of quasiparticles can still be interpreted as the exchange statistics of charges in the common edge 0.

This interpretation remains valid if some of the filling factors ν_l are equal to each other. For instance, if the edges j and $j+1$ have the same filling factor ν_0 as the edge 0, the charge and statistics of the backscattered quasiparticles coincide with those of Laughlin's quasiparticles: $q_j=B_{j,j+1}=\nu_0$, in agreement with previous results.[3] Our discussion here shows that in general the strong coupling between non-identical edges should produce quasiparticles with both charge and statistics that are different from those of Laughlin's quasiparticles in the bulk.

To find the backscattered currents quantitatively, we calculate first the charges injected into each edge by quasiparticle tunneling at the jth contact. Combining the two facts, that the fraction of the incoming current in the jth edge that goes out into the ith edge is equal to G_{ij}/ν_j, and that the quasiparticle charge q_j is created in the process of backscattering directly in the 0th edge, we see from Eqs. (34) that each backscattering event in the contact j creates the charge equal to G_{ij} in the ith edge, $i\neq j$. This means that the backscattering current $I_l^{(bsc)}$ in the lth outgoing edge ($l=0,\ldots,n$) is related to the rate $I_j^{(0)}$ of quasiparticle backscattering in the jth contact as

$$I_l^{(bsc)} = \sum_{j=1}^{n} \Delta G_{lj} I_j^{(0)}, \quad \Delta G_{lj} \equiv G_{lj} - \nu_j \delta_{lj}. \tag{35}$$

To find the rates $I_j^{(0)}$, we need to calculate the distribution of bias voltages for quasiparticles. In contrast to finding the bias (33) for electron tunneling, this task is not completely trivial, since in the strong tunneling limit, the bias voltage V_l applied to the lth edge affects the quasiparticle bias $\overline{V_j}$ in all point contacts. The bias $\overline{V_j}$ can be obtained as the voltage-induced shift of the dual fields θ_j in the arguments of exponents in the tunneling Lagrangian (26). We find this shift from the linear relation between the incoming fields $\phi_l^{(in)}$ and the fields θ_j (28). Extending the set of n fields θ_j to the ($n+1$)-component vector $\theta \equiv (\phi_0^{(out)}, \theta_1, \theta_2, \ldots, \theta_n)^T$ we can write this linear relation similarly to Eq. (32):

$$\theta = \hat{S}' \phi^{(in)}, \quad \hat{S}' = \hat{T}^{(n)} \hat{T}^{(n-1)} \cdots \hat{T}^{(1)}, \tag{36}$$

where the matrices $\hat{T}^{(j)}$ describe transformations of the incoming fields into $(\phi_0^{(out)}, \theta_j)$ in the individual point contacts. The part of $\hat{T}^{(j)}$ related to $\phi_0^{(out)}$ is the same as in $\hat{P}^{(j)}$ (30), while the part related to θ_j follows directly from Eqs. (28) and (29):

$$\hat{T}_{j0}^{(j)} = \left(\frac{\nu_j}{\nu_0+\nu_j}\right)^{1/2}, \quad \hat{T}_{jj}^{(j)} = -\left(\frac{\nu_0}{\nu_0+\nu_j}\right)^{1/2},$$

$$\hat{T}_{00}^{(j)} = \hat{P}_{00}^{(j)}, \quad \hat{T}_{0j}^{(j)} = \hat{P}_{0j}^{(j)}. \tag{37}$$

Since the voltages V_l shift the incoming fields according to Eq. (33), the transformation (36) shows that the voltage-induced shift of the quasiparticle fields θ_j is

$$\theta_j \to \theta_j - \sum_l \hat{S}'_{jl} \sqrt{\nu_l} V_l t.$$

Obtaining the matrix elements $\hat{S}'_{jl}$ explicitly from Eqs. (37) and (31) we see that the voltage bias in the quasiparticle tunneling operator in Eq. (26) is

$$\frac{2\theta_j}{\lambda_j} \to \frac{2\theta_j}{\lambda_j} - \overline{V_j} t, \quad \overline{V_j} = \sum_l \Delta G_{jl} V_l. \tag{38}$$

Inserting this result into the expression for the rate of quasiparticle tunneling from the 0th edge at the jth contact that follows from Eq. (26) we get

$$I_j^{(0)} = -\frac{W_j}{\pi\alpha} \sin\left(\frac{2}{\lambda_j}\theta_j + \overline{\eta_j} - \overline{V_j} t\right). \tag{39}$$

Finally, combining the strong-coupling branching of the currents according to the conductance matrix (34) with the backscattering contribution to the current, we see that the average *outgoing current* $\langle I_l \rangle$ in the lth edge is

$$\langle I_l \rangle = \sigma_0 \sum_k G_{kl} V_l + \langle I_l^{(bsc)} \rangle. \tag{40}$$

In the limit of strong tunneling, the backscattering is weak and the average quasiparticle tunneling rate $\langle I_j^{(0)} \rangle$ can be evaluated in the lowest order of perturbation theory in backscattering (26). For large quasiparticle bias voltages $\overline{V_j}$ one gets

$$\langle I_j^{(0)} \rangle = \frac{W_j^2 D}{2\pi v^2 \Gamma(4/\lambda_j^2)} \mathrm{sgn}(V_j) |\overline{V_j}/D|^{(2/\lambda_j)^2-1}.$$

This expression combined with Eqs. (35) and (40) describes how the average outgoing current in the lth edge approaches its linear large-voltage asymptotics.

In this regime of weak backscattering, the quasiparticle tunneling produces regular *shot noise*, which at low temperatures is the only source of noise of the outgoing currents.[20] This mean that the correlators of the outgoing currents $\langle I_l \rangle$ at zero frequency can be expressed as

$$\langle \{I_l, I_k\} \rangle|_{\omega\to 0} = 2\sum_{j=1}^{n} \Delta G_{lj} \Delta G_{kj} |\langle I_j^{(0)} \rangle|, \tag{41}$$

where $\{\ldots,\ldots\}$ denotes the anticommutator. In general, the current noise (41) is not proportional to the average backscattered current, unless the contribution to backscattering of one point contact is dominant. If the contact m does dominate, the shot noise of the outgoing current in the lth edge is characterized by the fractional charge equal to ΔG_{lm},

$$q* \equiv \left| \frac{\langle\{I_l,I_l\}\rangle|_{\omega\to 0}}{2\langle I_l^{(bsc)}\rangle} \right| = |\Delta G_{lm}|, \quad (42)$$

i.e., the charge is proportional to the fraction of the quasiparticle charge which reaches the lth edge in the current branching process. The shot noise with the charge (42), generated at one point contact and fractionally split into another contact, is direct manifestation of coherent propagation of electrons between the point contacts. Equilibration of electron distribution between the contacts would leave in each edge only the shot noise generated by the backscattering at the point contact formed by this edge.

V. TUNNELING BETWEEN MULTIMODE FERMI-LIQUID RESERVOIR AND FQHL

In this section, we consider the situation with one specific choice of the filling factors, $\nu_j=1$, for $j=1\ldots n$, in the general model of the multipoint-contact junction (Fig. 1). The multi-point-contact junction in this regime has been used before[9,10,12] to model an interface between the multimode Fermi-liquid reservoir and the FQHL. In the limit of strong tunneling, the reservoir-FQHL interface has all the features of a regular Ohmic contact: linear current-voltage characteristics and no shot noise, and establishes equilibrium between the reservoir and the edge of the FQHL when the number n of reservoir modes is large. Results of this section describe corrections to the Ohmic behavior caused by the weak quasiparticle backscattering which manifests itself through the shot noise and nonlinear corrections to current.

In the case of reservoir-FQHL interface, the $\nu=1$ edges of the general model of Fig. 1 represent one-dimensional free-electron scattering modes of the n-mode Fermi-liquid reservoir. When a voltage V is applied to the FQHL edge, the tunneling current I_T that flows between the reservoir and the edge can be found as the sum of the outgoing currents [see Eqs. (35) and (40)] in the free-electron modes $j=1\ldots n$:

$$I_T = -\sigma_0\Delta G_{00}V - \sum_{j=1}^{n} G_{0j}I_j^{(0)}(G_{j0}V). \quad (43)$$

The argument of the jth rate of backscattering $I_j^{(0)}$ here represents the quasiparticle bias voltage (38) in the jth contact.

Perturbation theory in quasiparticle tunneling (26) of the dual model is justified when either the temperature or all quasiparticle bias voltages are larger than the energy scale $T_X \simeq \max_j (W_j/v)^{1/(1-2/\lambda^2)}D$ of the crossover to strong tunneling in all point contacts. The condition on the bias voltages is always satisfied if the applied voltage V is sufficiently large: $|V|[(1-\nu_0)/(1+\nu_0)]^n > T_X$.

Tunneling at large temperatures can be characterized by the *linear conductance* G of the reservoir-edge interface. We calculate this conductance by first obtaining the average rates $\langle I_j^{(0)}\rangle$ of backscattering in each contact in the first nonvanishing order of perturbation theory in tunneling (26) and then summing them according to Eq. (43). We find that with increasing temperature, conductance G approaches its saturation value G_n,

$$G_n = \sigma_0\nu_0[1-(q-1)^n], \quad q = \frac{2\nu_0}{1+\nu_0} = \frac{2}{\lambda^2}, \quad (44)$$

reached in the absence of backscattering, as

$$G = G_n - \frac{q^2(q-1)^{n-1}}{4\sqrt{\pi}}\frac{\Gamma(q)}{\Gamma(q+1/2)}\left(\frac{\pi T}{D}\right)^{2(q-1)}\sum_j \frac{W_j^2}{v^2}. \quad (45)$$

Since $q<1$, the first-order perturbation correction to G_n (45) vanishes with increasing temperature as a power of temperature, $G-G_n \propto 1/T^{2(1-q)}$. In the limit of large number of point contacts, $n\to\infty$, this power-law correction is, however, suppressed by the same small factor $(1-q)^n$ which characterizes how G_n approaches its equilibration value $\sigma_0\nu$ at $n\to\infty$. One can show that correction to G_n in the next nonvanishing order of the perturbation theory in backscattering is also suppressed by this factor. This suggests that G approaches the saturation value faster than any negative power of T.

Following the same step as for the linear conductance (45) we obtain the average *tunneling current* in the large-voltage regime:

$$\langle I_T\rangle = G_nV + \frac{(-1)^n q^{2q}V}{2\pi\Gamma(2q)}(V/D)^{2(q-1)}(1-q)^{(n-1)(2q-1)} \times \sum_j (W_j/v)^2(1-q)^{2(n-j)(1-q)}. \quad (46)$$

For large n, the main contribution to the sum over j in this equation comes from the contacts at the end of the junction, where $j\simeq n$. The overall magnitude of the average backscattered current (46) in the perturbative regime of weak backscattering is proportional to $(1-q)^{(n-1)(2q-1)}$, and for $\nu_0 \geqslant 1/3$, does not vanish when $n\to\infty$. If $\nu_0=1/3$, the exponent in this dependence is zero: $2q-1=0$, while for smaller ν_0 the exponent is negative and makes the absolute value of the backscattered part of current (46) a growing function n. Equations (45) and (46) show also that the sign of the backscattering contribution to the tunnel current oscillates with the parity of the number n of point contacts, with backscattering increasing the tunnel current for even n.

The *shot noise* of the tunneling current is given by the zero-frequency correlator $\langle\{I_T,I_T\}\rangle$ and similarly to Eq. (41) is generated by the backscattering part of the current (43):

$$\langle\{I_T,I_T\}\rangle = \sum_{i,j=1}^{n} G_{0i}G_{0j}\langle\{I_i^{(0)},I_j^{(0)}\}\rangle. \quad (47)$$

In the lowest nonvanishing order in the backscattering, the correlators on the right-hand side of Eq. (47) are not equal to zero only for $i=j$. Nonzero diagonal correlators are given by the average backscattered currents, as in Eq. (41). In this case, summing the contributions $2\langle I_j^{(0)}\rangle$ to noise from each of the point contacts according to Eq. (47) we get

$$\langle\{I_T,I_T\}\rangle = \frac{q^{2q+1}V}{\pi\Gamma(2q)}(V/D)^{2(q-1)}(1-q)^{(n-1)(2q-1)} \times \sum_j (W_j/v)^2(1-q)^{(n-j)(3-2q)}. \quad (48)$$

Similar to the average backscattered current, the main contribution to the shot noise comes from few contacts with $j \sim n$. If the tunneling amplitudes of these contacts are approximately equal so that we can neglect their variations, $W_n^2 \simeq W_{n-1}^2 \simeq \cdots$, the ratio between the noise and the average backscattered current is

$$\left|\frac{\langle\{I_T,I_T\}\rangle}{2(\langle I_T\rangle + \sigma_0\Delta G_{00}V)}\right| = q\frac{1-(1-q)^{2(1-q)}}{1-(1-q)^{3-2q}}. \quad (49)$$

We see that in a uniform junction with large number n of modes, the splitting of excitations at the point contacts complicates the relation between the backscattered charge and the shot noise. For instance, if $\nu_0 = 1/3$, by a numerical coincidence Eq. (49) gives the noise-to-current ratio also equal to $1/3$, while the backscattered charge is $q=1/2$. For smaller ν_0, the ratio (49) does not correspond directly to either q or ν_0.

VI. CONCLUSION

To summarize, this work provides a description of quasiparticle tunneling in the junction with geometry shown in Fig. 1, in which several point contacts are formed between single-mode FQHL edges with in general different filling factors $\nu_j = 1/odd$ and one common single-mode edge with filling factor ν_0. This description extends to finite reflection in the contacts our previous theory[12] of strong electron tunneling in such a junction. New quasiparticle physics emerging form the multipoint-contact geometry is coherent splitting of quasiparticles at all point contacts downstream the common edge ν_0 from the point contact where the quasiparticles have been created. Quasiparticles are split at each contact in precisely the same way as the incoming edge currents in the strong tunneling limit. This means that the branching ratios of the edge currents determine the quasiparticle charge and statistics. The statistics is given by the permutation relations between the backscattering operators producing quasiparticles and makes the whole process of coherent splitting at successive point contacts causal. The quasiparticle charges determine the magnitude of the shot noise of the outgoing current in each edge. In the non-uniform junctions, in which one point contact dominates the backscattering, the shot noise in any given edge is directly related to the charge split in this edge in the branching process—see Eq. (42). In uniform junctions, the charge-noise relation is less direct [see Eq. (49) and the example discussed next to it] because of averaging over different point contacts.

In the case of a junction between the Fermi-liquid reservoir and the FQHL edge, $\nu_j = 1$ for $j = 1\ldots n$, the quasiparticle backscattering gives corrections to the reservoir-edge equilibration reached with increasing number of reservoir modes, $n\to\infty$, in the absence of backscattering. For temperatures T much larger than the bias voltage V, these corrections are uniformly suppressed by the factor $[(\nu_0-1)/(\nu_0+1)]^n$, exponential in the number n of modes. In the opposite regime, $T \ll V$, the corrections, i.e., the nonlinear junction conductance and current shot noise, do not vanish, however, even for $n\to\infty$.

Description of the charge and statistics of the edge excitations in the multipoint-contact junction, which is developed in this work for simple single-mode edges in the junction geometry with one common edge for all point contacts, should also be important for more complex multimode edges and more general junction geometries.

ACKNOWLEDGMENTS

The authors would like to acknowledge useful discussions with V. J. Goldman and J. K. Jain. This work was supported by the NSA and ARDA under ARO contract No. DAAD19-03-1-0126.

APPENDIX: ONE POINT CONTACT

In this appendix, we review the known results[8,18] for strong tunneling in one point contact between two edges with different filling factors using the approach similar to that employed in the main text. For consistency of notations, we number the two edges 0 and j, with the filling factors ν_0 and ν_j. The Hamiltonian of one point contact can be written as in the model described in Secs. II and III

$$\mathcal{H} = \int dx\left(\frac{v}{4\pi}[(\partial_x\phi_0)^2 + (\partial_x\phi_j)^2] + [V_0\rho_0 + V_j\rho_j]\right) - u_j \cos\left(\frac{\phi_0}{\sqrt{\nu_0}} - \frac{\phi_j}{\sqrt{\nu_j}}\right)\Bigg|_{x=0}. \quad (A1)$$

The two differences with Secs. II and III are that now we explicitly include the bias voltages V_0 and V_j applied to the edges, and omit the Klein factors in the tunnel part of the Hamiltonian, since they are irrelevant in the case of one point contact. (The Klein factors $\xi_0\xi_j$ and $\xi_j\xi_0$ in the forward and backward tunneling terms commute with each other and are constants of motion. This means that they can at most produce an irrelevant constant shift of the fields ϕ.)

The tunneling field φ_j is introduced according to Eqs. (29) which can be viewed as the rotation in the space of the fields ϕ:

$$\hat{O} = \begin{pmatrix}\cos\vartheta_j, & -\sin\vartheta_j\\ \sin\vartheta_j, & \cos\vartheta_j\end{pmatrix}$$

by the angle ϑ_j defined through the relations:

$$\cos\vartheta_j = \sqrt{\frac{\nu_j}{\nu_0+\nu_j}}, \quad \sin\vartheta_j = \sqrt{\frac{\nu_0}{\nu_0+\nu_j}}.$$

Performing this rotation in the Hamiltonian (A1), in particular rotating the "vector" of bias voltages $\sqrt{\nu_0}V_0$, $\sqrt{\nu_j}V_j$, we see that the Hamiltonian $\mathcal{H}_T$ of the tunneling mode φ_j can be separated from other terms:

$$\mathcal{H}_T = \int \frac{dx}{2\pi}\left[\frac{v}{2}(\partial_x \varphi_j)^2 + \frac{V}{\lambda_j}\partial_x\varphi_j\right] - u_j \cos[\lambda_j \varphi_j(x=0)],$$

$$V \equiv V_0 - V_j. \quad \text{(A2)}$$

From now on we omit the index j of all variables. Heisenberg equation of motion for φ obtained from the Hamiltonian (A2) and the commutation relations (5) is

$$(\partial_t + v\partial_x)\varphi(x,t) = -\frac{V(t)}{\lambda} + u\pi\lambda \sin[\lambda\varphi(0,t)]\mathrm{sgn}(x). \quad \text{(A3)}$$

Separating the bias voltage:

$$\varphi(x,t) = \varphi_0(x,t) - \frac{1}{\lambda}\int^t dt' V(t'), \quad \text{(A4)}$$

we can write the solution of Eq. (A3) as

$$\varphi_0(x,t) = \frac{2\pi m}{\lambda} + \begin{cases} \theta(t - x/v) + \pi\lambda Q(t), & x < 0, \\ \chi(t - x/v) - \pi\lambda Q(t), & x > 0, \end{cases} \quad \text{(A5)}$$

where an arbitrary constant $2\pi m/\lambda$ is written in this form for later convenience, and $Q(t)$ is the operator of charge transferred through the point contact:

$$\dot{Q}(t) = -u \sin[\lambda\varphi(0,t)]. \quad \text{(A6)}$$

The function $\theta(t)$ in Eq. (A5) is an arbitrary operator which has the meaning of fluctuations of the field $\varphi(x,t)$ incident on the point contact, and χ will be determined from the final solution of Eq. (A3).

Equation (A3) implies that $\varphi(x,t)$ is continuous as a function of x at $x=0$, i.e.,

$$\varphi_0(0,t) - \frac{2\pi m}{\lambda} = \theta(t) + \pi\lambda Q(t) = \chi(t) - \pi\lambda Q(t). \quad \text{(A7)}$$

In the strong tunneling limit $u \to \infty$, we assume that the value of $\varphi(0,t)$ is very close to one of the minima $2\pi m/\lambda$, $m = 0, \pm 1, \ldots$ of the tunneling term in the Hamiltonian (A2) and we can linearize the sine in Eq. (A6). As we will see in the end of the calculation, this assumption is indeed correct. Applying it to the first of the Eqs. (A7), we get the following simple equation for $\varphi_0(0,t)$:

$$\dot{\varphi}(0,t) = -u\pi\lambda\left[\lambda\varphi_0(0,t) - \int^t dt' V(t')\right] + \dot{\theta}. \quad \text{(A8)}$$

Solution of this equation under the condition $\dot{\theta}, V \ll u$ appropriate for the strong-tunneling limit gives

$$\varphi(0,t) = \frac{2\pi m}{\lambda} + \frac{1}{u\pi\lambda^2}\left[\dot{\theta}(t) - \frac{V(t)}{\lambda}\right]. \quad \text{(A9)}$$

We see that up to small terms of the order of $1/u$, $\varphi(0,t)$ is indeed pinned to one of the minima $2\pi m/\lambda$ of the tunneling energy. Inserting Eq. (A9) into Eq. (A6) we see that the tunnel current in the point contact is

$$\dot{Q}(t) = \sigma_0 \frac{2}{\lambda^2} V(t) - \frac{\dot{\theta}(t)}{\pi\lambda}. \quad \text{(A10)}$$

The first term on the right-hand side of this equation gives the strong-tunneling conductance of the point contact, while the second term represents the tunnel current induced by the incident field fluctuations. Finally, combining Eqs. (A9) and (A10) with the continuity condition (A7) we can determine the transmitted field $\chi(t)$ in Eq. (A5):

$$\chi(t) = \frac{2}{\lambda}\int^t dt' V(t') - \theta(t). \quad \text{(A11)}$$

[Equations (A10) and (A11) include only the terms that do not vanish in the limit $u \to \infty$.] Comparison of Eq. (A11) with Eqs. (A4) and (A5) shows explicitly that both the voltage contribution to $\varphi(x,t)$ and incident fluctuations $\theta(t)$ change sign at the point contact in the strong-tunneling limit. This means that the field $\theta_j(x)$ defined by Eq. (28) of the main text indeed propagates freely in the strong tunneling limit.

The last remark is that Eq. (A5) enables us to obtain directly the charge of backscattered quasiparticles. Indeed, instanton tunneling that describes the backscattering changes the number m of the minimum the field $\varphi(x,t)$ is pinned to, affecting in the process the charge transfer through the point contact. Equation (A5) shows that the transition from one tunneling minimum to another changing m by 1 happens if the transferred charge changes by the amount

$$\delta Q = \frac{2}{\lambda^2}, \quad \text{(A12)}$$

which coincides with the junction tunnel conductance in units of σ_0. One can also see from Eqs. (A5) and (A7) that the large-scale distribution of the field $\varphi(x,t)$ created by such a process of instanton tunneling occurring at time t_i coincides essentially, as it should, with the retarded Green's function discussed in Sec. II:

$$\varphi(x,t) = -\frac{2\pi}{\lambda}\Theta(t - t_i)\mathrm{sgn}[x - v(t - t_i)]. \quad \text{(A13)}$$

---

*On leave from St. Petersburg State Polytechnical University, Center for Advanced Studies, St. Petersburg 195251, Russia.

[1] B.I. Halperin, Phys. Rev. B **25**, 2185 (1982); P. Streda, J. Kucera, and A.H. MacDonald, Phys. Rev. Lett. **59**, 1973 (1987); M. Buttiker, Phys. Rev. B **38**, 9375 (1988).

[2] X.G. Wen, Phys. Rev. Lett. **64**, 2206 (1990); J. Frölich and T. Kerler, Nucl. Phys. B: Field Theory Stat. Syst. **354 [FS]**, 369 (1991); M. Stone and M.P. A. Fisher, Int. J. Mod. Phys. B **8**, 2539 (1994); N. Nagaosa and M. Kohmoto, Phys. Rev. Lett. **75**, 4294 (1995).

[3] C.L. Kane and M.P.A. Fisher, Phys. Rev. Lett. **72**, 724 (1994).
[4] V.J. Goldman and B. Su, Science **267**, 1010 (1995); V.J. Goldman, I. Karakurt, J. Liu, and A. Zaslavsky, Phys. Rev. B **64**, 085319 (2001).
[5] L. Saminadayar, D.C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. **79**, 2526 (1997); R. de-Picciotto, M. Reznikov, M. Heiblum, V. Umansky, G. Bunin, and D. Mahalu, Nature (London) **389**, 162 (1997); M. Reznikov, R. de Picciotto, T.G. Griffiths, M. Heiblum, and V. Umansky, *ibid.* **399**, 238 (1999).
[6] I. Safi, P. Devillard, and T. Martin, Phys. Rev. Lett. **86**, 4628 (2001); C.L. Kane, *ibid.* **90**, 226802 (2003); S. Vishveshwara, *ibid.* **91**, 196803 (2003).
[7] D.V. Averin and V.J. Goldman, Solid State Commun. **121**, 25 (2002).
[8] N.P. Sandler, C. de C. Chamon, and E. Fradkin, Phys. Rev. B **57**, 12324 (1998); **59**, 12521 (1999).
[9] C.L. Kane and M.P.A. Fisher, Phys. Rev. B **52**, 17393 (1995).
[10] C. de C. Chamon and E. Fradkin, Phys. Rev. B **56**, 2012 (1997).
[11] A.M. Chang, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **77**, 2538 (1996); M. Grayson, D.C. Tsui, L.N. Pfeiffer, K.W. West, and A.M. Chang, *ibid.* **80**, 1062 (1998); A.M. Chang, M.K. Wu, C.C. Chi, L.N. Pfeiffer, and K.W. West, *ibid.* **86**, 143 (2001); M. Hilke, D.C. Tsui, M. Grayson, L.N. Pfeiffer, and K.W. West, *ibid.* **87**, 186806 (2001); A.M. Chang and J.E. Cunningham, Surf. Sci. **229**, 216 (1990).
[12] V.V. Ponomarenko and D.V. Averin, JETP Lett. **74**, 87 (2001); Phys. Rev. B **67**, 035314 (2003).
[13] E. Comforti, Y.C. Chung, M. Heiblum, and V. Umansky, Phys. Rev. Lett. **89**, 066803 (2002); E. Comforti, Y.C. Chung, M. Heiblum, V. Umansky, and D. Mahalu, Nature (London) **416**, 515 (2002).
[14] J.K. Jain, Phys. Rev. Lett. **63**, 199 (1989).
[15] R. Guyon, P. Devillard, T. Martin, and I. Safi, Phys. Rev. B **65**, 153304 (2002).
[16] Note that Eq. (11) defines only the commutator of the fields η_j. Because of the vanishing energy of these fields, however, the uncertainty in their definition affects only the zero-frequency component of the imaginary-time correlator (13) which is not obtained by the analytical continuation from real time, and would not change Eq. (16) on which the subsequent calculations are based.
[17] A. Schmid, Phys. Rev. Lett. **51**, 1506 (1983).
[18] Ch. Nayak, M.P.A. Fisher, A.W.W. Ludwig, and H.H. Lin, Phys. Rev. B **59**, 15694 (1999).
[19] C.L. Kane and M.P.A. Fisher, Phys. Rev. B **46**, 15233 (1992).
[20] V.V. Ponomarenko and N. Nagaosa, Phys. Rev. B **60**, 16865 (1999); Solid State Commun. **110**, 321 (1999).