Electron tunneling cross-talk: Selective transmission in semiconductor nanowires

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The system presented here avoids, thanks to the quantum tunneling effect, collision between two incoming electrons in a "crossing" made out of monomode wires of one semiconductor. The structure has two input gates, through which two electrons of the same energy and opposite spins can enter the system. These two electrons are resonantly cross-transferred between the two input and the two output gates.

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One of the challenges of modern technology is to control the electronic transport at the nanometer scale. At this length scale, since the electron mean free path becomes large compared to the size of the device, the majority of the observed features is due to the wave nature of electrons, being responsible for a variety of quantum-interference effects (see, for example, Ref. 1).

Selective transmission of electrons from one quantum wire to another one and the associated channel-drop tunneling processes exemplify such quantum-interference phenomena, recently attracting much attention because of their fundamental interest and practical importance.^{2–6} To be more specific, tunable directional transfer from one wave guide to another is particularly important for signal multiplexing in various routing devices and as a potential electronspectroscopy tool. The growing interest in this kind of systems can also be attributed to the advancement in modern semiconductor technology (such as the electron beam lithography) that makes it possible to fabricate the nanostructures with a sufficient degree of control of their geometry and chemical composition.

Accordingly, we propose a resonant-coupling structure, built of semiconductor nanowires, which—under certain conditions—realizes the directional transfer of electrons with a good selectivity. More precisely, this structure lets the electrons of every-but-one energy (from a certain energy window) propagate without perturbation from one input gate to the other, while transmitting one electron of a preselected and well-defined energy to another output gate.

This paper stress that, by linear superposition, two electrons of opposite spins and having the same energy simultaneously sent through two input gates, can be crosstransferred to two other output gates by the quantum tunnel effect. Moreover, the simple system presented here enables one to obtain closed-form expressions for its characteristic wire lengths, and thus, to determine easily all the parameters necessary for its fabrication.

The system under consideration is sketched in Fig. 1. All the wires are quasi-one-dimensional semiconductor guides. For an illustrative example, GaAs quantum wires are taken and assumed to behave as single-mode electron wave guides. For symmetry purposes, the distance d_0 between nodes 1 and 2 is the same as that between nodes 3 and 4, while the rest of the device is built of only two distinct constituents of the same semiconductor, namely, the finite wires of lengths d_1 and d_2 . More specifically, four identical single-stub structures, each consisting of a wire of length d_2 grafted in the middle of another wire of length $2d_1$, are stuck between nodes 1 and 5, 5 and 4, 2 and 6, and 6 and 3, respectively. Such side-branch structures are known to exhibit a gap in the electronic transmission at the eigenenergies of the grafted resonantor.⁷ Between nodes 5 and 6, another double-stub structure of length $3d_1$ is stuck, with two wires of length d_2 branched off at equidistant positions. This supports localized states in the abovementioned energy gap. Owing to simple building blocks of the device, there are only three parameters, namely, wire lengths d_0 , d_1 , and d_2 , fully determining its characteristics.

In general, any incident electronic wave launched onto the coupling structure, e.g., from the input gate 1, generates, as a result of scattering processes, the reflected wave at node 1, along with the three transmitted waves at nodes 2, 3, and 4 (see Fig. 1). The corresponding reflection (r_{11}) and transmission (t_{1n} , n=2,3,4) amplitudes can be expressed in terms of the elements of the Green function g of the system via⁶

 $r_{11} = 2iFg(1,1) - 1$

and

(1a)

$$t_{1n} = 2iFg(1,n) \quad (n = 2,3,4).$$
 (1b)

In Eqs. (1a) and (1b), $i=\sqrt{-1}$ stands for the imaginary unit, while



FIG. 1. Sketch of the considered "electron tunnel cross-talk" structure. Proportions between the distances within the device correspond to the particular system studied in Fig. 2, assuming all the wires are 10 nm wide.

$$F = \frac{\hbar^2}{2m^*} \alpha \tag{2a}$$

is related to the electron wave vector α defined as

$$\alpha = \frac{1}{\hbar} \sqrt{2m^* E}, \qquad (2b)$$

E and m^* being the electron energy and its effective mass, respectively.

The necessary Green-function elements can be readily obtained taking into account the symmetry of the system. Consequently, for electrons incoming through gate 1, the expressions for the reflection and transmission wave function amplitudes can be conveniently written as

$$r_{11} = z_1 + z_2 + z_3 + z_4 - 1, (3a)$$

$$t_{12} = z_1 + z_2 - z_3 - z_4, \tag{3b}$$

$$t_{13} = z_1 - z_2 + z_3 - z_4 \tag{3c}$$

and

$$t_{14} = z_1 - z_2 - z_3 + z_4, \tag{3d}$$

where

$$z_n = \frac{i}{2(i+y_n)} \quad (n = 1, 2, 3, 4), \tag{4}$$

with the y_n 's determined by the particular resonant-coupling structure under consideration.⁶ For the structure of Fig. 1:

$$y_1 = y_2 - \frac{2B_5^2}{2A_5 + A_6 + B_6},\tag{5a}$$

$$y_2 = A_0 + B_0 + A_5, \tag{5b}$$

$$y_3 = A_0 - B_0 + A_5 \tag{5c}$$

and

$$y_4 = y_3 - \frac{2B_5^2}{2A_5 + A_6 - B_6},\tag{5d}$$

where

$$A_5 = A_1 - \frac{B_1^2}{2A_1 + A_2},\tag{6a}$$

$$B_5 = -\frac{B_1^2}{2A_1 + A_2},\tag{6b}$$

$$A_6 = A_1 - \frac{1}{2} \left(\frac{B_1^2}{2A_1 + A_2 - B_1} + \frac{B_1^2}{2A_1 + A_2 + B_1} \right)$$
(6c)

and

$$B_6 = \frac{1}{2} \left(\frac{B_1^2}{2A_1 + A_2 - B_1} - \frac{B_1^2}{2A_1 + A_2 + B_1} \right), \quad (6d)$$

while

$$A_j = -\left[\tan(\alpha d_j)\right]^{-1} \tag{6e}$$

and

$$B_j = [\sin(\alpha d_j)]^{-1}$$
 (j = 0, 1, 2). (6f)

For a cross transfer of one electron from gate 1 to gate 3, one must satisfy the conditions

$$A_5 = A_6 = 0$$
 (7a)

and

$$B_5^2 = B_0 B_6,$$
 (7b)

which, via Eqs. (6a)–(6f), determine the characteristic distances within the coupling structure.

Define the incoming electron probabilities of presence

$$I_j = |\Psi_{I_j}(E)|^2 \quad (j = 1, 2),$$
 (8a)

where $\Psi_{I_j}(E)$ is the wave function of the incoming electrons. In the same manner, we define

$$O_j = |\Psi_{O_j}(E)|^2 \quad (j = 1, 2, 3, 4),$$
 (8b)

the transmission probabilities at the output gates.

Now, for a single input at gate 1, define a quality factor Q characterizing the selectivity of the directional forward transfer from gate 1 to gate 3 by

$$Q = \frac{E_0}{2(E' - E_0)},$$
(9)

such that $O_3(E') = I_1(E')/2$. With a good accuracy, the wire lengths should satisfy

$$\alpha_0 d_0 = (1 + 4n_0)\frac{\pi}{2},\tag{10a}$$

 $\alpha_0 d_1 = (1 + 4n_1 \pm \delta) \frac{\pi}{2},$ (10b)

and

$$\alpha_0 d_2 = (2n_2 \mp \delta) \frac{\pi}{2}, \qquad (10c)$$

where n_i (j=0,1,2) are integers, while

$$\delta = \left[\frac{3(1+4n_1+2n_2)}{2\pi Q}\right]^{1/2} \tag{11}$$

is supposed to be small compared to 1. Note that the positive value of δ in the expression for d_1 corresponds to the negative value of δ in the expression for d_2 and vice versa.

Thus, Eqs. (10a)–(10c) and (11) enable one to estimate the wire lengths for the desired device operation, once the directional-transfer energy E_0 and the corresponding quality factor Q are chosen. Note that several solutions for d_0 , d_1 , and d_2 are feasible, due to the different available values of integers n_0 , n_1 , and n_2 .

The performance of the proposed device can be tested by considering a structure built of GaAs wires. Choosing E_0 = 50 meV and Q=500, the distances within the coupling



FIG. 2. Output-signal intensities $O_1=O_2$ (dashed line) and $O_3 = O_4$ (solid line) as a function of energy for the structure of Fig. 1 with $d_0=90$ nm, $d_1=47$ nm, and $d_2=43$ nm, when two input electrons are simultaneously present.

structure are determined from approximate Eqs. (10a)–(10c) and (11), and rounded up or down to full nanometers, which results in $d_0=90$ nm, $d_1=47$ nm, and $d_2=43$ nm. These wire lengths are next used to compute the reflection and transmission amplitudes as a function of energy through the exact Eqs. (3a)–(3d).

Now, when two electrons are simultaneously transferred, one from gate 1 to gate 3 and the other from gate 2 to gate 4 (see Fig. 1), the output transmission probabilities are

 $O_1 = O_2 = |2(z_1 + z_2) - 1|^2$

$$O_3 = O_4 = |2(z_1 - z_2)|^2.$$
 (12b)

(12a)

In other words, two electrons of particular energy E_0 are cross transferred through the "electron tunnel crossing" structure to gates 3 and 4, respectively. This electron "cross-talk" effect is illustrated in Fig. 2.

A notch in the output signal can also be a property of purely one-dimensional systems with two pairs of propagating states.⁸ The results of the present paper show that this "electron tunnel cross-talk" structure realizes a cross transfer of two electrons, respectively, from gate 1 to gate 3 and from gate 2 to gate 4. Moreover, the derived closed-form expressions for the characteristic wire lengths within the coupling structure prove useful in finding the optimal parameters for the desired device operation, enabling one to engineer it at will for specific applications.

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