Mott insulators, no double occupancy, and non-Abelian superconductivity

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SU(4) dynamical symmetry is shown to imply a no-double-occupancy constraint on the minimal symmetry description of antiferromagnetism and *d*-wave superconductivity. This implies a maximum doping fraction of $\frac{1}{4}$ for cuprates and provides a microscopic critique of the projected SO(5) model. We propose that SU(4) superconductors are representative of a class of compounds that we term *non-Abelian superconductors*. We further suggest that non-Abelian superconductors may exist having SU(4) symmetry and therefore cupratelike dynamics, but without *d*-wave hybridization.

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Suppression of double occupancy on sites in the copper oxide planes is critical in explaining why cuprate systems are antiferromagnetic Mott insulators at half filling become superconductors through hole doping. The symmetric Zhang SO(5) model[1] predicts no charge gap at half filling. To recover Mott insulator phases at half-filling in the Zhang model it is normal to impose a no-double-occupancy rule by Gutzwiller projection. This breaks SO(5) symmetry, but lattice calculations and schematic arguments suggest that many SO(5) features might survive in such a projected SO(5) model.[2–5]

We have proposed a unified description of high temperature superconductivity and antiferromagnetism based on a $U(4) \supset SU(4)$ dynamical symmetry that has analytical solutions in three symmetry limits.[6,7] The SO(4) limit of the SU(4) model corresponds to an antiferromagnetic phase, the SU(2) limit to a *d*-wave superconducting phase, and the SO(5) limit to a critical symmetry interpolating between the antiferromagnetic and superconducting phases.

Although the methodology of the SU(4) model differs substantially from that of the Zhang model, its SO(5) limit represents the Zhang SO(5) algebra subject to constraints implied by embedding SO(5) in the larger algebra SU(4). In this article we address the physical understanding of why SU(4) should play a crucial role in high temperature superconductivity, how no-double-occupancy and Mott insulator properties lie at the basis of this understanding, and provide a microscopic understanding of the projected SO(5) model.

The $U(4) \supset SU(4)$ model has 16 symmetry generators:

$$p_{12}^{\dagger} = \sum_k g(k) c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger}, \quad p_{12} = \sum_k g^*(k) c_{-k\downarrow} c_{k\uparrow},$$

$$q_{ij}^{\dagger} = \sum_k g(k) c_{k+Q,i}^{\dagger} c_{-k,j}^{\dagger}, \quad q_{ij} = (q_{ij}^{\dagger})^{\dagger},$$

$$Q_{ij} = \sum_k c_{k+Q,i}^{\dagger} c_{k,j}, \quad S_{ij} = \sum_k c_{k,i}^{\dagger} c_{k,j} - \frac{1}{2}\Omega \delta_{ij}, \tag{1}$$

where $c_{k,i}^{\dagger}$ creates an electron of momentum k and spin projection $i,j=1$ or $2 \equiv \uparrow$ or $\downarrow$, $Q=(\pi,\pi,\pi)$ is an antiferromagnetic ordering vector, $\Omega/2$ is the electron-pair degeneracy, and $g(k)$ is the *d*-wave form factor

$$g(k) = (\cos k_x - \cos k_y) \approx \mathrm{sgn}(\cos k_x - \cos k_y). \tag{2}$$

By forming new linear combinations, Eq. (1) may be replaced by operators having more direct physical meaning:

$$Q_+ = Q_{11} + Q_{22} = \sum_k (c_{k+Q\uparrow}^{\dagger} c_{k\uparrow} + c_{k+Q\downarrow}^{\dagger} c_{k\downarrow}),$$

$$\vec{S} = \left(\frac{S_{12}+S_{21}}{2}, -i\frac{S_{12}-S_{21}}{2}, \frac{S_{11}-S_{22}}{2} \right),$$

$$\vec{\mathcal{Q}} = \left(\frac{Q_{12}+Q_{21}}{2}, -i\frac{Q_{12}-Q_{21}}{2}, \frac{Q_{11}-Q_{22}}{2} \right),$$

$$\vec{\pi}^{\dagger} = \left(i\frac{q_{11}^{\dagger}-q_{22}^{\dagger}}{2}, \frac{q_{11}^{\dagger}+q_{22}^{\dagger}}{2}, -i\frac{q_{12}^{\dagger}+q_{21}^{\dagger}}{2} \right),$$

$$\vec{\pi} = (\vec{\pi}^{\dagger})^{\dagger}, \quad D^{\dagger} = p_{12}^{\dagger}, \quad D = p_{12}, \quad M = \tfrac{1}{2}(S_{11}+S_{22}), \tag{3}$$

where Q_+ is associated with charge density waves, $\vec{S}$ is the spin operator, $\vec{\mathcal{Q}}$ is the staggered magnetization, $D^{\dagger}(D)$ is the creation (annihilation) operator of spin-singlet *d*-wave pairs, $\vec{\pi}^{\dagger}$ ($\vec{\pi}$) are associated with spin-triplet pairs, and $2M=\hat{n}-\Omega$ is related to the number (charge) operator $\hat{n}$. The representation space of the SU(4) model is built by the coherent D and π pairs:

$$|n_x n_y n_z n_s\rangle = (\pi_x^{\dagger})^{n_x} (\pi_y^{\dagger})^{n_y} (\pi_z^{\dagger})^{n_z} (D^{\dagger})^{n_s} |0\rangle. \tag{4}$$

The operator Q_+ commutes with all generators and will be ignored in this discussion. Thus the most general effective Hamiltonian in the symmetry-dictated truncated space is a linear combination of all scalar products constructed from the remaining 15 SU(4) generators.[6,7] The operator set (3) is closed under $U(4) \supset U(1) \times SU(4)$ symmetry [hereafter termed SU(4)] only if the approximation in Eq. (2) holds. The physics implied in this approximation becomes more transparent if we transform Eqs. (1) to coordinate space [using the exact form of Eq. (2)]:

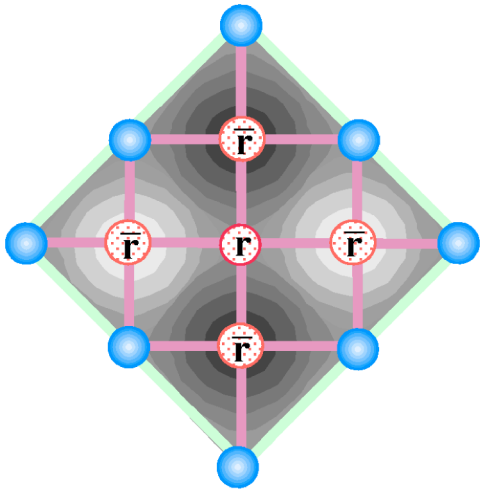

FIG. 1. (Color online) A schematic hole pair. Fuzzy balls are sites where electron holes form a pair: one hole at $\mathbf{r}$, the other with equal probability (1/4) at the four neighboring sites ($\bar{\mathbf{r}}=\mathbf{r}\pm\mathbf{a}$ and $\mathbf{r}\pm\mathbf{b}$). Balls connected by green lines (those on the outer boundary of the figure) are sites where the presence of a hole would imply double occupancy.

$$p_{12}^{\dagger}=\sum_{r} c_{\mathbf{r}\uparrow}^{\dagger}c_{\bar{\mathbf{r}}\downarrow}^{\dagger},\quad p_{12}=\sum_{r} c_{\bar{\mathbf{r}}\downarrow}c_{\mathbf{r}\uparrow},$$

$$q_{ij}^{\dagger}=\sum_{r}(-1)^{r}c_{\mathbf{r},i}^{\dagger}c_{\bar{\mathbf{r}},j}^{\dagger},\quad q_{ij}=\sum_{r}(-1)^{r}c_{\bar{\mathbf{r}},j}c_{\mathbf{r},i},$$

$$Q_{ij}=\sum_{r}(-1)^{r}c_{\mathbf{r},i}^{\dagger}c_{\mathbf{r},j},\quad S_{ij}=\sum_{r}c_{\mathbf{r},i}^{\dagger}c_{\mathbf{r},j}-\frac{1}{2}\Omega\delta_{ij},\qquad(5)$$

where $c_{\mathbf{r},i}^{\dagger}$ ($c_{\mathbf{r},i}$) creates (annihilates) an electron of spin i located at $\mathbf{r}$ and $c_{\bar{\mathbf{r}},i}^{\dagger}$ ($c_{\bar{\mathbf{r}},i}$) creates (annihilates) an electron of spin i at the four neighboring sites, $\mathbf{r}\pm\mathbf{a}$ and $\mathbf{r}\pm\mathbf{b}$, with equal probabilities ($\mathbf{a}$ and $\mathbf{b}$ are lattice constants in $\mathbf{x}$ and $\mathbf{y}$ directions, respectively),

$$c_{\bar{\mathbf{r}},i}^{\dagger}=\frac{1}{2}(c_{\mathbf{r}+\mathbf{a},i}^{\dagger}+c_{\mathbf{r}-\mathbf{a},i}^{\dagger}-c_{\mathbf{r}+\mathbf{b},i}^{\dagger}-c_{\mathbf{r}-\mathbf{b},i}^{\dagger}).\qquad(6)$$

The factor $(-1)^r$ in Eq. (5) is $(-1)^{n_x+n_y}$ and (n_x,n_y) are the coordinates of a lattice site on the copper oxide plane, $\mathbf{r}=n_x\mathbf{a}+n_y\mathbf{b}$, which is positive for even sites (n_x+n_y=even) and negative for the odd sites (n_x+n_y=odd). This factor originates from the assumption $e^{i\mathbf{Q}\cdot\mathbf{r}}\approx(-1)^r$ and implies Mott insulator properties: the electrons are localized at lattice sites with small overlap between orbitals of electrons on neighboring lattice sites.

From the coordinate representation (5) we see that spin-singlet and spin-triplet pairs are formed by holes on adjacent sites. Figure 1 illustrates the spatial structure of a hole pair: if one hole is at $\mathbf{r}$, the other hole occupies the four adjacent sites ($\mathbf{r}\pm\mathbf{a}$ and $\mathbf{r}\pm\mathbf{b}$) with equal probability. The summation over $\mathbf{r}$ in the pair creation (annihilation) operators indicates that such pairs are highly coherent. It also can be seen that

$$\hat{n}=\hat{n}^{(e)}+\hat{n}^{(o)},\quad Q_{+}=\hat{n}^{(e)}-\hat{n}^{(o)},\qquad(7)$$

$$S_{ij}=S_{ij}^{(e)}+S_{ij}^{(o)},\quad Q_{ij}=S_{ij}^{(e)}-S_{ij}^{(o)},\qquad(8)$$

with $\hat{n}^{(e)}$ ($\hat{n}^{(o)}$) and $S_{ij}^{(e)}$ ($S_{ij}^{(o)}$) the total electron number and spin operators at even (odd) sites, respectively:

$$\hat{n}^{(e)}=\sum_{i,r=\text{even}}c_{\mathbf{r},i}^{\dagger}c_{\mathbf{r},i},\quad \hat{n}^{(o)}=\sum_{i,r=\text{odd}}c_{\mathbf{r},i}^{\dagger}c_{\mathbf{r},i},\qquad(9)$$

$$S_{ij}^{(e)}=\sum_{r=\text{even}}c_{\mathbf{r},i}^{\dagger}c_{\mathbf{r},j},\quad S_{ij}^{(o)}=\sum_{r=\text{odd}}c_{\mathbf{r},i}^{\dagger}c_{\mathbf{r},j}.\qquad(10)$$

Thus Q_+ and $\vec{Q}$ represent the differences in total charge and spin between even and odd sites, respectively.

However, Eqs. (5) do not close under commutation unless

$$\{c_{\bar{\mathbf{r}}',i},c_{\bar{\mathbf{r}},j}^{\dagger}\}=\delta_{\mathbf{r}'\mathbf{r}}\delta_{ij},\quad \{c_{\bar{\mathbf{r}}',i},c_{\bar{\mathbf{r}},j}\}=0\qquad(11)$$

[that is, $c_{\bar{\mathbf{r}},i}^{\dagger}$ ($c_{\bar{\mathbf{r}},i}$) is a basis for particles occupying sites adjacent to $\mathbf{r}$]. This separates lattice sites into categories A and B: if r=even are A sites [with operators $c_{\mathbf{r},i}^{\dagger}$ ($c_{\mathbf{r},i}$)], r=odd are the B sites [operators $c_{\bar{\mathbf{r}},i}^{\dagger}$ ($c_{\bar{\mathbf{r}},i}$)], or vice versa. Then Eqs. (11) permits Eqs. (5) to be written as

$$p_{12}^{\dagger}=\sum_{\mathbf{r}\in A}(c_{\mathbf{r}\uparrow}^{\dagger}c_{\bar{\mathbf{r}}\downarrow}^{\dagger}-c_{\mathbf{r}\downarrow}^{\dagger}c_{\bar{\mathbf{r}}\uparrow}^{\dagger}),$$

$$q_{ij}^{\dagger}=\pm\sum_{\mathbf{r}\in A}(c_{\mathbf{r},i}^{\dagger}c_{\bar{\mathbf{r}},j}^{\dagger}+c_{\mathbf{r},j}^{\dagger}c_{\bar{\mathbf{r}},i}^{\dagger}),$$

$$S_{ij}=\sum_{\mathbf{r}\in A}(c_{\mathbf{r},i}^{\dagger}c_{\mathbf{r},j}-c_{\bar{\mathbf{r}},j}c_{\bar{\mathbf{r}},i}^{\dagger}),$$

$$\tilde{Q}_{ij}=\pm\sum_{\mathbf{r}\in A}(c_{\mathbf{r},i}^{\dagger}c_{\mathbf{r},j}+c_{\bar{\mathbf{r}},j}c_{\bar{\mathbf{r}},i}^{\dagger}),$$

$$p_{12}=(p_{12}^{\dagger})^{\dagger},\quad q_{ij}=(q_{ij}^{\dagger})^{\dagger},\qquad(12)$$

with $\tilde{Q}_{ij}\equiv Q_{ij}+(\Omega/2)\delta_{ij}$, where the $\pm$ sign is + (−) if A is chosen to be even (odd) sites. (Whether A sites are taken to be even or odd is a labeling choice and does not influence the physics.) Then by explicit commutation the operators (12) close an SU(4) algebra. But by Eq. (6),

$$\{c_{\bar{\mathbf{r}}',i},c_{\bar{\mathbf{r}},j}^{\dagger}\}=\delta_{\mathbf{r}'\mathbf{r}}\delta_{ij}+\delta_{ij}\frac{1}{4}\sum_{t}g(t)\delta_{\mathbf{r}',\mathbf{r}+\mathbf{t}},\qquad(13)$$

with

$$g(\mathbf{t})=\begin{cases}+1 & \text{for } \mathbf{t}=\pm\mathbf{2a},\pm\mathbf{2b},\\ -1 & \text{for } \mathbf{t}=+\mathbf{a}\pm\mathbf{b},-\mathbf{a}\pm\mathbf{b},\end{cases}\qquad(14)$$

and Eq. (11) is generally *not* satisfied unless the second term on the right side of Eq. (13) vanishes. This term vanishes if a constraint is imposed that whenever there is a hole pair $c_{\mathbf{r}i}^{\dagger}c_{\bar{\mathbf{r}}j}^{\dagger}$ at $\mathbf{r}$ (see Fig. 1), no pair is permitted at $\mathbf{r}'=\mathbf{r}+\mathbf{t}$, leaving nothing to be annihilated by $c_{\bar{\mathbf{r}}'i}$. This is exactly a no-double-occupancy constraint because without it there is a finite amplitude for double site occupancy. For instance, if one pair is at $\mathbf{r}'=\mathbf{r}+2\mathbf{a}$ and a second pair at $\mathbf{r}$, the probability is $\frac{1}{16}$ for two holes to be located at $\mathbf{r}+\mathbf{a}$ (see Fig. 1). We conclude that *closure of the SU(4) algebra is a direct consequence of no*

double occupancy in the copper oxide conducting plane.

Additional insight follows from observing that the validity of Eqs. (11) actually follows from the more general requirement that no pairs overlap (a consistency condition ensuring that the pair space and the pairing correlations be well defined), is sufficient to satisfy Eqs. (11). The no-pair-overlap constraint implies naturally that if a pair is centered at $\mathbf{r}$, no pair may be located at $\mathbf{r}'=\mathbf{r}+\mathbf{t}$ with $\mathbf{t}$ given in Eq. (14), and thus Eq. (11) holds.

For an N-dimensional basis the minimum closed algebra is SO($2N$) if all bilinear particle-hole and pair operators are taken as generators. The simplest basis for cuprates may be regarded as four-dimensional since electrons can exist only in four basic states, on A sites or B sites, with spin up or down. Thus, absent further constraints, the minimum Lie algebra for the set of generators that can describe high-T_c superconductivity and antiferromagnetism simultaneously in a cuprate system is SO(8) and not SU(4). The 28 generators of SO(8) are the 16 operators in Eq. (12) plus the 12 operators

$$\overrightarrow{p}_{12}^{\dagger} = \sum_{\mathbf{r}\in A} (c_{\mathbf{r}\uparrow}^{\dagger} c_{\mathbf{r}\downarrow}^{\dagger} - c_{\overline{\mathbf{r}}\downarrow}^{\dagger} c_{\overline{\mathbf{r}}\uparrow}^{\dagger}),$$

$$\overrightarrow{q}_{12}^{\dagger} = \pm \sum_{\mathbf{r}\in A} (c_{\mathbf{r}\uparrow}^{\dagger} c_{\mathbf{r}\downarrow}^{\dagger} + c_{\overline{\mathbf{r}}\downarrow}^{\dagger} c_{\overline{\mathbf{r}}\uparrow}^{\dagger}),$$

$$\overline{S}_{ij} = \sum_{\mathbf{r}\in A} (c_{\mathbf{r},i}^{\dagger} c_{\overline{\mathbf{r}},j} - c_{\mathbf{r},j} c_{\overline{\mathbf{r}},i}^{\dagger}),$$

$$\overline{Q}_{ij} = \pm \sum_{\mathbf{r}\in A} (c_{\mathbf{r},i}^{\dagger} c_{\overline{\mathbf{r}},j} + c_{\mathbf{r},j} c_{\overline{\mathbf{r}},i}^{\dagger}),$$

$$\overline{p}_{12} = (\overrightarrow{p}_{12}^{\dagger})^{\dagger}, \quad \overline{q}_{12} = (\overrightarrow{q}_{12}^{\dagger})^{\dagger} \tag{15}$$

[± depends on the even-odd choice for A sites; see Eqs. (12)].

Equations (15) contain two kinds of new (spin-singlet) pairs created by $\overrightarrow{p}_{12}^{\dagger}$ and $\overrightarrow{q}_{12}^{\dagger}$, which may be termed S and S^* pairs, respectively. In both the two electrons (holes) occupy the same site, with equal probability to appear anywhere in the lattice coherently. The S^* pairs differ from S pairs in their phases. The operators $\overline{S}_{ij}$ are the hopping operators with and without spin flip, and $\overline{Q}_{ij}$ is the staggering of the hopping. These operators change D and π pairs into S and S^*, or vice versa.

The SO(8) algebra reduces to SU(4) if the S and S^* pairs may be neglected, which occurs if we assume on-site Coulomb repulsion pushing the S and S^* pairs to sufficiently high energy. Thus, restriction to no double occupancy effectively allows the operators in Eqs. (15) to be ignored and reduces SO(8) to the subalgebra SU(4).

Therefore, the minimal Lie algebra that can describe antiferromagnetism and d-wave superconductivity in a cuprate system is in general SO(8), but under the constraint of no double occupancy the symmetry effectively reduces to SU(4). The assumption of an SU(4) symmetry in a cuprate system automatically implies the imposition of a no-double-occupancy constraint on the general SO(8) symmetry in the copper-oxygen planes.

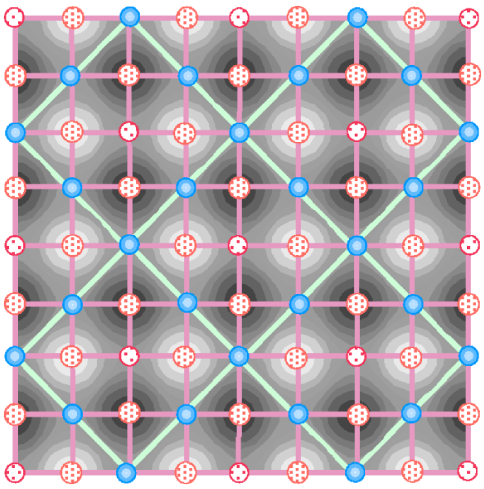

FIG. 2. (Color online) Schematic spatial distribution for maximal hole pair occupation. Fuzzy balls are lattice sites where the electron holes form pairs. Balls connected by green lines (diagonal or antidiagonal lines) indicate sites where the presence of a hole would lead to double occupancy.

It is likely that the hopping operator $\overline{S}_{ij}$ in Eq. (15) is the source of the most important SU(4) symmetry breaking terms. It breaks SU(4) but is a generator of SO(8), so this perturbation may be taken into account by an extension from SU(4) to SO(8). However, we may expect the no-double-occupancy rule and thus the SU(4) symmetry to be a good initial approximation.

The implicit SU(4) occupancy constraint dictates an upper limit for the doping fraction in SU(4)-conserving states. Figure 2 illustrates the spatial distribution when the number of hole pairs is maximal. By counting, the maximum number of holes is $\Omega=\Omega_e/4$, where Ω_e is the total number of lattice sites. Thus the largest doping fraction preserving SU(4) symmetry is $P_f=\Omega/\Omega_e=\frac{1}{4}$. The empirical maximum doping fraction (0.23–0.27) for cuprate superconductivity may then be interpreted as a direct consequence of SU(4) symmetry.

The preceding discussion implies the following: (1) The physical origin of cuprate SU(4) symmetry is proximity of antiferromagnetism and d-wave pairing, coupled with suppression of double occupancy by on-site Coulomb repulsion. (2) Two important facts in cuprates, that normal states are Mott insulators and that superconductivity exists only in a narrow doping range ($P<0.27$), are direct consequences of an SU(4) dynamical symmetry.

Superconductivity in cuprates is a specific example of what we shall term *non-Abelian superconductivity*, which differs from conventional superconductivity in the richness of pair structure for condensed states and in the appearance of competing sources of long-range order. The key issues for SU(4) non-Abelian superconductivity are that coherent pairs are formed by holes on adjacent sites so that both singlet and triplet states contribute, and that alternative long-range order (antiferromagnetism) enters on an equal footing with superconductivity.

In contrast to BCS superconductivity, which is described by a single dynamical symmetry chain having only Abelian subgroups [SU(2) ⊃ U(1)], the minimal symmetry consistent

with cuprate data is SU(4), which has a much richer structure (three dynamical symmetries having non-Abelian subgroups and differing fundamentally in their properties). We propose that the differences in observational characteristics for these two types of superconductivity originate in this difference in dynamical symmetry structure and in non-Abelian superconductivity resulting from electron-electron interactions instead of electron-phonon interactions.

The primal role of SU(4) symmetry in non-Abelian cuprate superconductivity suggests that any pairing structure leading to the SU(4) algebra entails dynamics similar to that of cuprates. Therefore, *d*-wave symmetry of the pairs need not be critical to non-Abelian superconductivity in general and SU(4) superconductivity in particular. Pairs with any internal symmetry (extended *s*-wave, *p*-wave, mixed symmetry, etc.) could exhibit SU(4) superconductivity if the no-double-occupancy constraint is valid and correlations can form adjacent-site pairs. Generally, $c^{\dagger}_{\overline{\mathbf{r}},i}$ may be defined as

$$c^{\dagger}_{\overline{\mathbf{r}},i} = \sum_{\mathbf{t}} f(\mathbf{t}) c^{\dagger}_{\mathbf{r}+\mathbf{t},i}, \quad \sum_{\mathbf{t}} |f(\mathbf{t})|^2 = 1, \tag{16}$$

where $\mathbf{t}$ is a few finite lattice displacements of $\mathbf{r}$ and $f(\mathbf{t})$ is the form factor. Different forms of $f(\mathbf{t})$ reflect different internal symmetries of the pairs, but they all satisfy the condition (11) under no double occupancy and thus preserve the SU(4) algebra and the general Hamiltonians implied by its dynamical symmetry chains. The structure (6) of the *d*-wave pairs is only a special case of (16) with $\mathbf{t}=\pm a, \pm b$ and $f(\pm\mathbf{a})=\frac{1}{2}$ and $f(\pm\mathbf{b})=-\frac{1}{2}$.

In summary, we have shown that SU(4) is the minimal symmetry accommodating superconductivity, antiferromagnetism, and a no-double-occupancy constraint in cuprate systems, and that SU(4) symmetry implies a maximum doping fraction of $\frac{1}{4}$ in the cuprates, by symmetry alone. Because the Zhang SO(5) algebra is a subalgebra of SU(4), these results indicate that *closure of the* SO(5) *algebra also implies no double occupancy if that algebra is embedded in* SU(4). Why then does the Zhang SO(5) model require Gutzwiller projection? The work presented here suggests that the projection requirement is not a consequence of the SO(5) symmetry itself but rather arises from approximations and assumptions in the Zhang effective Lagrangian formulation, where five of the SU(4) generators $(D^{\dagger}, D, \vec{Q})$ are treated as order parameters forming a superspin vector. Thus one cannot apply algebraic constraints to them through the commutators. This is most easily seen if not only the SO(5) generators but also the elements of the Zhang superspin vector are treated as operators rather than order parameters, thereby enlarging the algebra to SU(4). Within the SU(4) framework, there is no need for projection since the symmetry itself enforces a no-double-occupancy condition. As demonstrated in Refs. 6 and 7, within the parent SU(4) group antiferromagnetism and superconductivity are described by *different* dynamical symmetries [SO(4) and SU(2), respectively], and SO(5) is a critical dynamical symmetry that interpolates between SO(4) and SU(2). In the SU(4) model it is the SO(4) symmetry, not the SO(5) symmetry, that naturally describes undoped states, and the spectrum for *unbroken* SO(4) dynamical symmetry is intrinsically antiferromagnetic with gapped charge excitations. In a forthcoming paper,[8] we shall show that this SU(4) structure leads to pseudogaps, and to a quantum critical doping point that requires explicit breaking of SO(5) symmetry.

Our results imply some important consequences of attributing cuprate superconductor behavior to an SU(4) algebra that follow directly from symmetry, independent of details: (1) Normal states are antiferromagnetic Mott insulators. (2) Hole doping of normal compounds leads first to SO(5) fluctuations in both antiferromagnetic and superconducting order (implying phases that may exhibit spin glass or stripe character), and then to SU(4) non-Abelian superconductivity. (3) SU(4) superconductivity is strongly suppressed for doping fractions exceeding $\frac{1}{4}$. (4) Symmetry breaking resulting from violation of the no-double-occupancy constraint may be described by a parent SO(8) algebra where terms that break SU(4) may still respect SO(8) symmetry. (5) SU(4) symmetry, not *d*-wave pairing per se, is the ultimate cause of cuprate behavior, implying that systems could exist having non-*d* pairing but cupratelike dynamics. The first three consequences are postdictions in strong accord with existing data. The fourth is a prediction that may be tested through detailed applications of the SU(4) model to data. The final prediction may be tested by searching experimentally for compounds having pairing structures other than $d_{x^2-y^2}$ that satisfy the SU(4) algebra.

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