Possible acoustical probe for the metamagnetic transition

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(Received 26 January 2004; published 15 November 2004)

We propose an acoustical method to investigate the metamagnetic transition in heavy fermion compounds. The discrete Uehling-Uhlenbeck equations together with the model of free orientations (θ_s, the relative direction of scattering of particles w.r.t. to the normal of the propagating plane-wave front) are solved to study the diverse dispersion relations of plane waves in Fermi gases. Our results of ultrasound absorption in Fermi gases resemble qualitatively those measurements reported by Souslov *et al.* and Wolf *et al.* or Yanagisawa *et al.* and Weber *et al.* (for URu_2Si_2, $CeRu_2Si_2$, and UPt_3). The latter observation makes it possible to probe metamagnetic transition as well as the Cooper pairing in atomic gases via the acoustical approach.

DOI: 10.1103/PhysRevB.70.174421 PACS number(s): 75.45.+j, 43.35.+d, 34.10.+x, 36.10.−k

I. INTRODUCTION

The nature of the metamagnetic transition in heavy fermion compounds is still far from clear. Hence it is important to study this transition using various either experimental or theoretical approaches. This difficult task remains among the outstanding problem of the quantum statistical physics, despite the significant success achieved in the field of research during the last 10 years. Meanwhile, recent researches show that this metamagnetic transition point might be a kind of quantum critical points (QCPs, a zero-temperature instability between two phases of matter where quantum fluctuations develop long range correlations both in space and time; these special points exert wide-reaching influence on the finite temperature properties of a material)[1] which are also relevant to the superconducting and/or superfluid transition.

Recently, it has become possible to tune atomic scattering lengths (say, the *s*-wave scattering length: a) to essentially any value, positive or negative, by exploiting Feshbach resonances[2] (also related to a magnetic field). The remaining challenge is to detect the presence of Cooper pairing or superfluid transition (which is crucial for the low-temperature superconductivity mechanism) in atoms within their on-site test section! Pairing in a gas of fermionic atoms and the relevant route to the novel superfluidity state (paired atoms flow without viscosity) still seems to be rather difficult to experimentalists. These lead to many proposals for probing the Cooper pair in atomic gases.[3]

The original idea that the superconductive state should be connected with electron-phonon interaction was conceived in it in the following way: an electron density causes a certain lattice displacement and thereby leads to a decrease in energy. Other electrons will find it favorable to adapt themselves to this displacement and in turn lead to a further increase in it.[4] A situation might thus arise which could be compared with ferromagnetism where a few oriented spins tend to orient others into the same direction which in turn increases the tendency for orientation.

Schafroth[5] has suggested consideration of correlation between electron pairs. He pointed out that an electron pair above, but near the top, of the Fermi surface may form a pseudomolecule, i.e., the mean distance between the two electrons will remain finite, provided an attractive interaction exists. They are bound (in space) although they have positive energy.[6] In this short paper, we propose an acoustical method for observing the metamagnetic transition in heavy Fermi compounds. This is a continuous study of plane-wave propagation in dilute atomic gases.[7,8] In fact, our previous results[8] show that, near the collisionless (or height-frequency) limit, there exists symmetry for bosonic and fermionic systems which might imply the possible regime for the formation of Cooper pairs. Based on the quantum analog of the discrete kinetic model (which can include a free orientation parameter: θ: related to the relative direction of scattering of particles w.r.t. to the normal of the propagating plane-wave front) and the Uehling-Uhlenbeck collision term[9,10] which could describe the collision of atomic gases by tuning a parameter B (via a Pauli *blocking factor* σ), we shall first compare the dispersion relations of ultrasonic waves propagating in dilute atomic gases by the verified approach[7–10] and then identify the specific behavior for the metamagnetic transition (there is no phase transition, at least for $T>0$ away from a possible quantum critical point in this study) as well as the Cooper pairing in Fermi gases. In fact, our preliminary results of dispersion relations[7] qualitatively resemble those reported by by Lea *et al.*[11] or Abrikosov and Khalatnikov[12] or for waves propagating in fermionic systems (say, ^{3}He) and Stamper-Kurn *et al.*[13] for waves propagating in bosonic systems. Our ultrasonic attenuation results of Bose gases[7] qualitatively agree quite well with previous reported ones (cf. Fig. 22 in Ref. 14 by Woods and Cowley, $T \leqslant 1$ K or cf. Ref. 15). The enormous increase of the maximum attenuation we found near the critical point ($\theta=\pi/4$) resemble qualitatively those for the metamagnetic transition (ultrasound propagating in heavy fermion compounds)[16,17] as well as the superfluid transition (possible Cooper pairing).[15] Note that the common understanding of this metamagnetic transition is the change from a more itinerant electron system for small magnetic fields to a more localized electron system for high fields. The theoretical description of this transition was confined so far to the explanation of the susceptibility peak. Therefore, the present calculation can give new insights for this transition.

II. FORMULATION

The gas is presumed to be composed of identical particles of the same mass. The discrete number density (of particles) is denoted by $N_i(\mathbf{x},t)$ associated with the velocity $\mathbf{v}_i$ at point $\mathbf{x}$ and time t. If only nonlinear binary collisions are considered, and considering the evolution of N_i, we have

$$\frac{\partial N_i}{\partial t} + \mathbf{v}_i \cdot \nabla N_i = F_i \equiv \sum_{j=1}^{p} \sum_{(k,l)} (A_{kl}^{ij} N_k N_l - A_{ij}^{kl} N_i N_j), \quad i = 1, \ldots, p, \tag{1}$$

where (k,l) are admissible sets of collisions,[7–9] $i \in \Lambda = \{1,\ldots,p\}$, and the summation is taken over all $j,k,l \in \Lambda$. Here A_{kl}^{ij} are non-negative constants satisfying[7–9] (i) $A_{kl}^{ji} = A_{kl}^{ij} = A_{lk}^{ij}$, (ii) $A_{kl}^{ij}(\mathbf{v}_i+\mathbf{v}_j-\mathbf{v}_k-\mathbf{v}_l)=0$, (iii) $A_{kl}^{ij}=A_{ij}^{kl}$. The conditions defined for discrete velocities above are valid for elastic binary collisions such that momentum and energy are preserved. With the introducing of the Uehling-Uhlenbeck collision term[9,10] in Eq. (1),

$$F_i = \sum_{j,k,l} A_{kl}^{ij}[N_k N_l (1+\sigma N_i)(1+\sigma N_j) - N_i N_j (1+\sigma N_k)(1+\sigma N_l)], \tag{2}$$

for $\sigma < 0$, we obtain a gas of Fermi-particles [for $\sigma > 0$ we obtain a gas of Bose-particles, and for $\sigma = 0$ we obtain Eq. (1)]. From Eq. (2), the model of quantum (discrete) kinetic equation for dilute atomic gases proposed[9] is a system of $2q(=p)$ semilinear partial differential equations of the hyperbolic type

$$\frac{\partial}{\partial t}N_i + \mathbf{v}_i \cdot \frac{\partial}{\partial \mathbf{x}} N_i = \frac{cS}{q}\sum_{j=1}^{2q} N_j N_{j+q}(1+\sigma N_{j+1})(1+\sigma N_{j+q+1}) - 2cSN_i N_{i+q}(1+\sigma N_{i+1})(1+\sigma N_{i+q+1}), \tag{3}$$

where $N_i = N_{i+2q}$ are unknown functions, and $\mathbf{v}_i = c(\cos[\theta+(i-1)\pi/q], \sin[\theta+(i-1)\pi/q])$, $i=1,\ldots,2q$; c is a reference velocity modulus, S is an effective collision cross-section for the collision system, θ is the orientation starting from the positive x-axis to the v_1 direction. We notice that the right-hand-side of the Eq. (3) is highly nonlinear and complicated for a direct analysis. As passage of the plane wave causes a small departure from an equilibrium resulting in energy loss owing to internal friction and heat conduction, we linearize above equations around a uniform equilibrium state (N_0) by setting $N_i(t,x) = N_0\,(1+P_i(t,x))$, where P_i is a small perturbation, N_0 normally depends on the equilibrium temperature for certain referenced state. After some tedious manipulations,[7,8] with $B=\sigma N_0$, which gives or defines the (proportional) contribution from dilute Fermi gases (if $\sigma<0$, e.g., $\sigma=-1$), or dilute Bose gases (if $\sigma>0$, e.g., $\sigma=1$), we then have

$$\left\{\frac{\partial^2}{\partial t^2} + c^2\cos^2\left[\theta + \frac{(m-1)\pi}{q}\right]\frac{\partial^2}{\partial x^2} + 4cSN_0(1+B)\frac{\partial}{\partial t}\right\}D_m = \frac{4cSN_0(1+B)}{q}\sum_{k=1}^{q}\frac{\partial}{\partial t}D_k, \tag{4}$$

where $D_m = (P_m + P_{m+q})/2$, $m=1,\ldots,q$, since $D_1 = D_m$ for $1 = m \pmod{2q}$.

We are now to look for the solutions in the form of plane wave $D_m = d_m \exp i(kx-\omega t)$, $(m=1,\ldots,q)$, with $\omega=\omega(k)$. This is related to the dispersion relations of (forced) plane waves propagating in dilute Bose ($B>0$) or Fermi ($B<0$) gases. So we have

$$\left\{1 + ih(1+B) - 2\lambda^2\cos^2\left[\theta + \frac{(m-1)\pi}{q}\right]\right\}d_m - \frac{ih(1+B)}{q}\sum_{k=1}^{q} d_k = 0, \quad m=1,\ldots,q, \tag{5}$$

with

$$\lambda = kc/(\sqrt{2}\omega), \quad h = 4cSN_0/\omega,$$

where λ is complex and $h(\propto 1/\mathrm{Kn})$ is the rarefaction parameter of the gas (Kn is the Knudsen number which is defined as the ratio of the mean free path of Bose or Fermi gases to the wave length of the plane wave).

III. RESULTS AND DISCUSSIONS

We can obtain the complex roots ($\lambda = \lambda_r + i\lambda_i$) from the polynomial equation above. The roots are the values for the nondimensionalized dispersion (positive real part; a relative measure of the phase speed) and the attenuation or absorption (positive imaginary part), respectively.

Both families of curves in Fig. 1 follow the conventional dispersion relations of plane waves propagating in dilute Bose and Fermi gases.[7,8] As for the ultrasound propagating in Bose fluids (say, liquid helium), we can reproduce qualitatively the previous experiments especially for the sound absorption or attenuation (cf. Ref. 14 for Fig. 22 or Ref. 15 for Fig. 7.3 therein). We found attenuation agreement around $T \leqslant 1$ K. (cf. Fig. 22 in Ref. 14 therein). The possible explanation for the peak at about $T=0.8$ K in the ultrasonic attenuation: at very low temperatures the excitations in liquid helium have a lifetime τ which is very long such that $\omega\tau \gg 1$ (ω, the frequency of an ultrasonic wave). Under these conditions the ultrasonic wave propagates in an almost collisionless manner, only occasionally scattering off a thermal excitation. At higher temperatures the excitations become much shorter such that $\omega\tau \ll 1$ the ultrasonic wave then propagates thermodynamically with many collisions of the excitations within each period of the ultrasonic wave. The peak in the attenuation arises as $\omega\tau=1$ and the ultrasonic wave is strongly scattered by the thermodynamically excited excitations. Some researchers might argue that in He^3 (cf. Ref. 11) as well as in He^4 it is a $\omega\tau$ effect, i.e. the attenuation maximum is for $\omega\tau \approx 1$.

Our results show that as $|B|$ increases, the dispersion (λ_r) will reach the hydrodynamical limit ($h \to \infty$) rather slowly

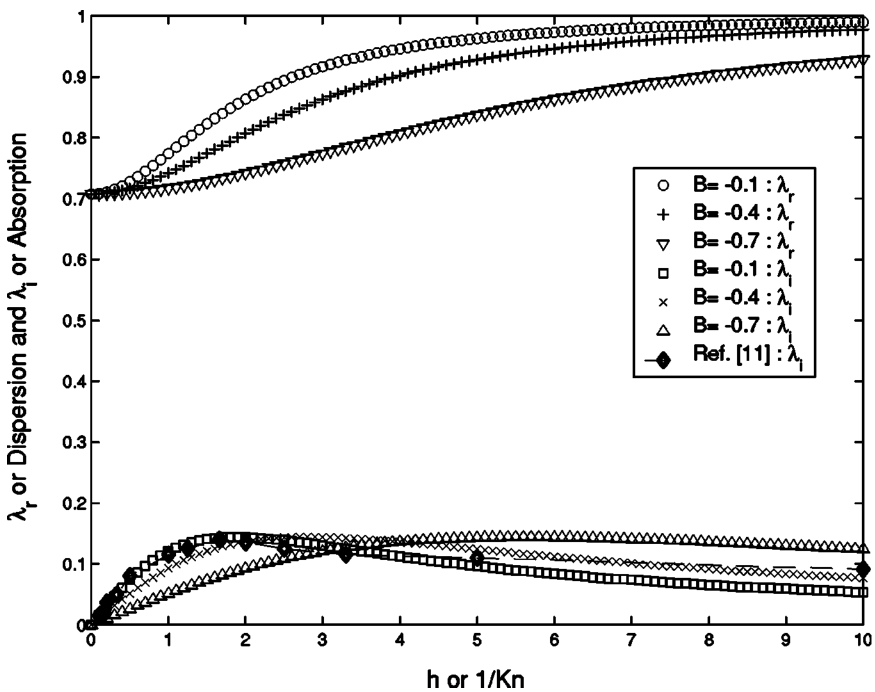

FIG. 1. Variations of λ_r and λ_i w.r.t. h for different Bs (away from $B=-1$) $B=\sigma N_0$, σ corresponds to -1; λ_r denotes the dispersion and λ_i denotes the attenuation. $h=4cSN_0/\omega$, S is the effective collision cross section.

for Fermi gases. Meanwhile, the maximum absorption (or attenuation) for all the rarefaction parameters h keeps the same for all B as observed in the lower part of Fig. 1. There are only shifts of the maximum absorption state (defined as $h_{\max}$) w.r.t. the rarefaction parameter h when $|B|$ increases.

Note that, curves of the attenuation or absorption and dispersion for Fermi gases resemble qualitatively those behavior reported before[11,12] (e.g., cf. Ref. 12 for the low- and high-frequency theory approximations; and cf. Fig. 3 of Ref. 11, waves propagating in ^{3}He which could be thought of a mixture of Fermi and other atomic gases).

We shall examine the detailed differences of the dispersion relation when ultrasonic waves propagating in dilute Bose and Fermi gases. As for the former, once there are Cooper pairings in Fermi gases, we can treat them (atomic pairs) as bosonic particles although the number density of them might be one-half of the original.

We start to include the orientation effects ($\theta \neq 0$, cf. the first cited paper in Ref. 7, where $\theta=0$). Most of the variations of ultrasonic absorptions w.r.t. h in Fig. 2 follow the previous zero-orientation ones ($B<0$; cf. also the first paper in Ref. 8). Our results show that as θ increases, the maximum absorption (λ_i) will decreases continuously except at $\theta=\pi/4$ where there is a sudden jump (maximum)! Once we consider the enormous increase of the ultrasound attenuation: $\lambda_i=0$ (maximum) at $\theta=\pi/4$, this resembles qualitatively those reported in Ref. 16 for the ultrasonic absorption in heavy fermion compound URu_2Si_2 near the metamagnetic transition point or those reported in Ref. 17 for the ultrasonic attenuation in heavy fermion compound $CeRu_2Si_2$ near the metamagnetic transition point (cf. Fig. 2 therein, the ultrasonic echo disappeared completely in the vicinity of the metamagnetic transition point due to the enormous increase of ultrasonic attenuation which might be caused by the critical slowing-down of the fluctuating rate for the order parameter of the metamagnetic transition point; as for UPt_3, cf. Thalmeier and Lüthi in Ref. 17). Thus, the orientation parameter (θ) here plays the role of the (magnetic) field-tuned order parameter. In fact, experimentalists[17] found the scaling relation for the (isothermal) elastic constant $c(B_m)$ and differential magnetic susceptibility $\chi_m(B_m): \Delta c \propto B_m^2 \chi_m$ (B_m is the magnetic field; cf. p. 322 in Thalmeier and Lüthi[17]). As Δc therein is related to the sound attenuation of our results (w.r.t. the corresponding sound velocity or speed at specific fixed h state), it is possible to obtain the relation between the sound velocity, sound attenuation calculated here and the magnetic susceptibility after some detailed comparison in-between since our results are mainly dimensionless and the

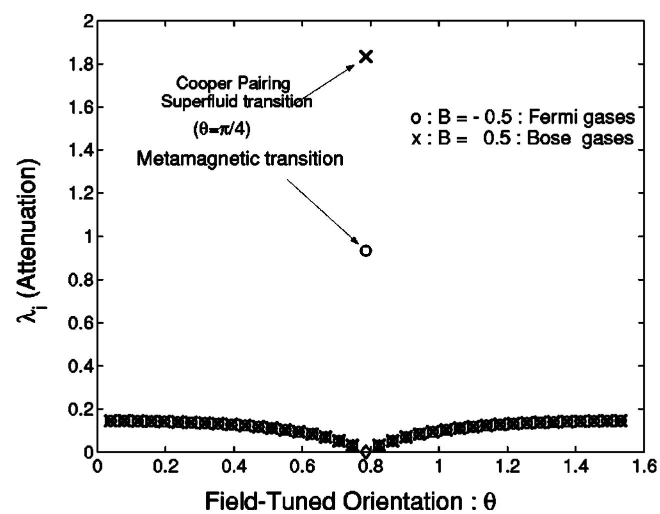

FIG. 2. Variations of λ_i w.r.t. free orientation: θ for $B=-0.5$ (fermion particles). The sudden jump at $\theta=\pi/4$ implies the metamagnetic transition (cf. Fig. 2 in Ref. 16) in Fermi liquids or T_λ transition in Bose liquids (cf. Fig. 20 in Ref. 14, it may also occur for Fermi fluids: in bound pairs). λ_i for $B=0.5$ (Bose gases) is almost twice of that value for $B=-0.5$ (Fermi gases) at $\theta=\pi/4$.

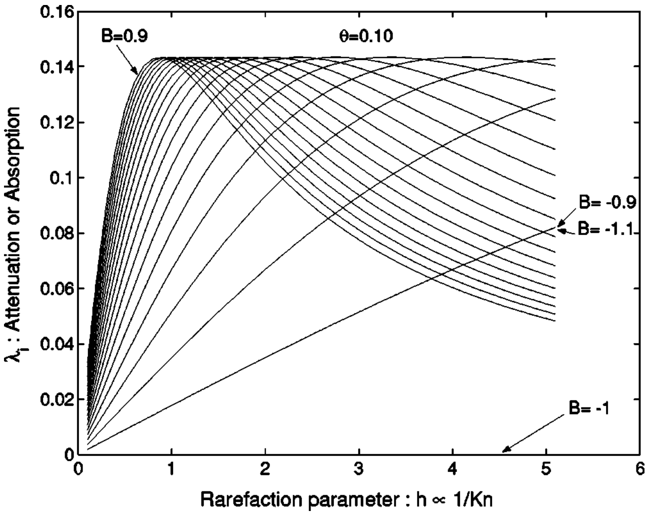

FIG. 3. Variations of λ_i w.r.t. h for different Bs (θ=0.1) $B=\sigma N_0$, σ corresponds to -1; λ_i denotes the attenuation.

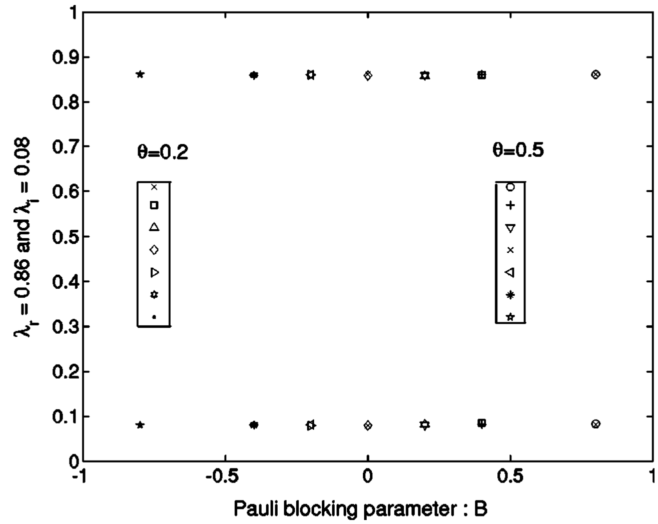

FIG. 4. Demonstration of obtaining the same dispersion relations by adjusting the Pauli parameters (Bs) [or the rarefaction parameters (hs)] and the free orientations (θ=0.2 and 0.5). The baseline is (λ_r=0.86, λ_i=0.08). Data points are obtained via an inverse procedure (instead of the direct formula). The crucial point is that there exist possibilities of Cooper pairings (bound from Fermi particles: $B<0$) which scatter ultrasonic waves like Bose particles ($B>0$).

proportional factor in above expression is, however, closely linked to the Grüneisen parameters in different materials which are dimensional in essence. We also noticed that the experiments[16,17] for these metamagnetic transitions are in the limit $\omega\tau \ll 1$.

Meanwhile, this unusual absorption (or attenuation) peak is similar to those observations found in the T_λ transition (temperature) for liquid helium (Bose liquids) (cf. Figs. 20 and 21 in Ref. 14). Pippard[18] suggested that the attenuation above the transition (T_λ) point arises because there are small inclusion of He II in the He I. These inclusions become larger and more numerous as the T_λ point is approached; not that, the absorption value obtained for B=0.5 (Bose gases) is almost twice of that value for $B=-0.5$ (Fermi gases) at $\theta=\pi/4$. One more interesting observation here, is that this transition at specific scattering orientation $\theta=\pi/4$ is quite similar to that report by Greiner *et al.*[19] for the Mott transition in an optical lattice.

To examine effects due to different-sign Pauli blocking parameters (Bs), we fix θ to be 0.1 and plot the absorption curves in Fig. 3 by changing B from negative to positive. There is sudden (symmetric) jump at $B=-1$. Similar to that in Fig. 1, the maximum absorption (or attenuation) for all hs keeps the same for all Bs (except $B=-1$) as observed in Fig. 3. The general characteristics of our dispersion relationship (especially the attenuation for higher h), in fact, agrees with (qualitatively) that reported in Ref. 20 by Gurarie and Chalker for the generic aspects of the density of small-frequency (ω) excitations in fermionic as well as bosonic systems (symmetry for small h). This symmetry is broken when $|B|$ is large and when h is $\sim O(1)$. Does it imply that the probability of forming Cooper pairs depends on the selection of B and h? Note that, at low temperatures, the Pauli exclusion principle forces Fermi-gas particles to be farther apart than the range of the collisional interaction, and they therefore cannot collide and rethermalize. The much more spreading characteristics of dispersion relations for dilute Fermi gases ($B<0$) obtained and illustrated in Figs. 1 and 3 seems to confirm above theoretical reasoning. The deviations for different B (positive vs negative) in curves of dispersion and absorption shown in Figs. 1 and 3 also highlight their dissimilar quantum statistical nature.

To further present our last results, which are to demonstrate the possibility of Cooper pairings (bosoniclike particles forming in fermionic gases) in fermionic gases as well as superconductors by using the ultrasound diagnostics, we shall utilize the diverse dispersion relations we could obtain (w.r.t. different Pauli parameters, Bs; different orientations, hs; and different rarefaction parameters, θs). Note that, $h=4cSN_0/\omega$, there are free parameters: effective collision cross-section (S), number density (N_0), and frequency (ω) for fixing h. Although, ultrasound propagating in different B-sign (positive and negative) gases show specific dispersion relations as illustrated in Figs. 1–3. However, we can reproduce the unique or specific one (say, for a prescribed dispersion or attenuation) from different B-sign and θ parameters. This means that there might be no differences for plane waves propagating in Fermi and Bose gases considering some combination of physical parameters (as shown here, different θ, h, and B, even the inverse problem or reconstruction is rather difficult and tedious). Here, as for the Bose case, we should remind that they are fermionic particles in essence even though they are forming a (bound) pair[4–6] or pseudomolecule (bosoniclike particles).

To show the test case, we select two θs (0.2 and 0.5 for all B cases) and, by using the diverse dispersion relations (λ_r, λ_i) we can find out the different combinations of h, θ, and B (either positive or negative) that give the same value, say, λ_r=0.86, λ_i=0.08. For instance, ultrasonic absorptions of Fermi gases ($B<0$) with θ=0.2 match with those of Bose gases ($B>0$) with θ=0.5! They are shown in Fig. 4. The solving procedure is related to an inverse problem. We did not calculate these by using Eq. (5) directly! Thus, this concept is useful to the experimentalists. Note that, as argued before by other researchers, ultrasonic waves would not discriminate between various order parameters such as metamagnetism and Cooper pairing, except perhaps the symmetry of the order parameter. For example, if the fluctuations were more pronounced in certain directions, the ultrasound attenuation would also reflect that symmetry. Here, we would like to remind the readers by referring to an analog with the recent successful experimental procedure[21] for tuning the threshold below and above the critical or transition point. The results will be different, even though, the differences might be too small in terms of certain defined parameters. As the (thermodynamic) process which is a little bit away from the critical (phase) transition point is essentially nonequilibrium and perhaps nonlinear, thus different routes, say, experimental procedures, to approach the critical point will result in different measurements which, however, might have only rather small differences in-between. But, that is enough for us. It means our proposed probe can be useful for studying magnetic transitions. One possible way is to approach the critical θ by either increasing the θ or decreasing the θ abruptly. This latter sequential procedure will enhance the relative differences near the critical point. The field-dependence effect could be calibrated with the tuned (increasing or decreasing θs) orientations which might be linked to the (relative) spin orientations or the induced changes of the confined and stressed texture of the heavy fermion compounds.

To conclude in brief, the diverse dispersion relations (in Figs. 1–3) we obtained for plane waves propagating in both Fermi and Bose gases not only resemble qualitatively those reported before[11–17] (e.g., especially for those attenuation peaks observed[16,17] as a function of field in URu_2Si_2, $CeRu_2Si_2$, and UPt_3) but also can give researchers the understanding of the symmetry[20] between bosonic and fermionic systems (for smaller h here) and the possible probing of the metamagnetic transition in heavy fermion compounds as well as Cooper pairings (relevant to the superfluid state formation) in Fermi gases by using the ultrasound diagnostics. We shall investigate other issues (say, the coupling between the sound wave and the electronic spin system which might be studied via the tuning of σ together with specific relative orientations in our approach, please see Eqs. (2) and (3), $\sigma < 0$ for fermions and $\sigma > 0$ for bosons[7] which have different spin characteristics; some systems which show a phase transition and therefore, the symmetry breaking and order parameter description should be involved) in the future.[1,17,22]

ACKNOWLEDGMENTS

The author is partially supported by the China Post-Dr. Science Foundation (Grant No. 1999-17) and NSFC (under Grant No.: 10274061).

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