Combination of dynamical invariant method and radiation-spin interaction to calculate magnetization damping

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Recently, an effective field method has been proposed, which provides a direct derivation of the Gilbert damping term in the magnetization equation [Phys. Rev. Lett. **92**, 097601 (2004)]. In this approach, the radiation-spin interaction (RSI) was introduced into the spin Hamiltonian. In this paper, it is shown that the damping term of the magnetization equation is also derived from the Lewis-Riesenfeld method when it is combined with the RSI description. This result provides an evidence supporting the RSI description for the magnetization relaxation.

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I. INTRODUCTION

Recently, studies of magnetization dynamics in small magnetic elements have attracted much attention of physics and related research communities. Such attention involves industrial concerns as well as theoretical interests. This is because it is anticipated that the future big progress in the magnetic information technology requires a deep and fundamental understanding of the collective behavior of spins on a short time scale, typically less than a nanosecond. At present, micromagnetic studies using Landau-Lifshitz-Gilbert (LLG) equation provide plausible descriptions to many experimental results, and, therefore, it is considered to be a reasonable route to achieve necessary understanding.

The LLG equation describes the dynamics of the magnetization $\mathbf{M}$ given by

$$\frac{d\mathbf{M}}{dt} = -\gamma\mathbf{M} \times \mathbf{H}_{\mathrm{eff}} + \alpha\mathbf{M} \times \frac{d\mathbf{M}}{dt}, \tag{1}$$

where γ and α are the absolute value of gyromagnetic ratio and the Gilbert damping parameter, respectively.[1] M is the magnitude of the magnetization vector $\mathbf{M}$ and the effective magnetic field $\mathbf{H}_{\mathrm{eff}}$ is defined by the energy variational with magnetization $\mathbf{H}_{\mathrm{eff}} = -\delta F/\delta\mathbf{M}$, where F is the free energy of the system. The physical description for the first term on the right-hand side of Eq. (1) is given by the Zeeman effect, or, in a classical language, magnetization precessional motion with respect to $\mathbf{H}_{\mathrm{eff}}$. It is the second term of Eq. (1), the Gilbert damping term, which raises many interesting issues in the study of the magnetization dynamics.

In general, the Gilbert damping term has not been obtained via a direct method of derivation from first principles, but has been introduced phenomenologically. One of the main difficulties in deriving the damping term is that various kinds of relaxation processes are melded together into a single damping term. Even though many analytical microscopic descriptions for the magnetization relaxation process have been successfully verified, a full version of derivation for the Gilbert damping term would be an indispensable element to achieving a thorough understanding of the magnetization dynamics.

Recently, we have proposed an effective field method to provide a direct derivation for the Gilbert damping term.[2] This proposal is a synthetic scheme in the sense that contributions to the magnetization relaxation are *effectively* represented by the radiation-spin interaction (RSI). In other words, rendering all the degrees of freedom coupled to the magnetization into the effective field $\mathbf{H}_{\mathrm{eff}}$ without considering individual effect to the magnetization dynamics, we have introduced a dynamical effect to *effectively* represent the nonmagnetic degrees of freedom contributing to the magnetization relaxation. In Ref. 2, the dynamical effect was derived from the structural analogy of the radiation damping in the classical electrodynamics, and according to this analogy, the dynamical effect has been termed as the radiation-spin interaction. This approach allows us to concentrate on the phenomenological description of magnetization relaxation processes. Similar concepts for representing the magnetization relaxation can be found in Ref. 3.

Of course, the RSI description can be directly used for studying the effect of the *real* radiation-spin interaction in the magnetization relaxation process.[4,5] In that case, one has to distinguish the damping parameter caused by the radiation-spin interaction α_{RSI} from that contributed by other degrees of freedom α_{other}. Then, the relation between the damping parameter and the radiation parameter λ used in Ref. 2 must be changed by $\alpha_{\mathrm{RSI}} = \lambda/(1-\lambda\alpha_{\mathrm{tot}}M^2)$, where $\alpha_{\mathrm{tot}} \equiv \alpha_{\mathrm{RSI}} + \alpha_{\mathrm{other}}$ and we redefined λJ_λ in Ref. 2 with λ.

In this paper, we present another evidence supporting the RSI description for the magnetization relaxation given in Ref. 2. Our newly introducing mechanism is the dynamical invariant method proposed by Lewis and Riesenfeld.[6] We will show that the Lewis-Riesenfeld (LR) method combined with the RSI description produces the damping term of the magnetization equation. In the research on the spin dynamics, the LR dynamical invariant method has been used for the nonadiabatic generalization of the Berry phase.[7,8]

This report is organized as following. In Sec. II, the LR dynamical invariant theory and its application to a simple spin system is briefly reviewed. Based on this review, we propose an ansatz for the dynamical invariant for the spin system, which is represented by ordinary magnetic quantities, i.e., magnetization and magnetic moment operator. In Sec. III, using the proposed ansatz and the LR method combined with the RSI description, we obtain the magnetization equation including the damping term from the Liouville–von Neumann equation. Comments and discussions are presented in Sec. IV.

II. LR DYNAMICAL INVARIANT

Many physically interesting quantum systems exhibit nonequilibrium characteristics, and their Hamiltonians are explicitly time dependent. For solving quantum mechanical problems of time-dependent quantum systems, various methods for nonequilibrium dynamics have been proposed, such as the Schwinger-Keldysh closed-time path method.[9] There is another relatively simple but powerful canonical method suggested by Lewis and Riesenfeld.[6] They constructed explicitly time-dependent dynamical invariants whose eigenstates are used for evaluating the exact quantum states that are solutions of the Schrödinger equation. The dynamical invariant $\mathcal{I}(t)$ is required to satisfy the Liouville–von Neumann (LvN) equation

$$\frac{\partial \mathcal{I}(t)}{\partial t} + \frac{1}{i\hbar}[\mathcal{I}(t),\mathcal{H}(t)] = 0, \tag{2}$$

where $\mathcal{H}(t)$ is the time-dependent Hamiltonian. From this LvN equation, the classical equation governing the dynamics of the spin system is obtained. Especially, the LR method is applied for the nonadiabatic generalization of the Berry phase. In Ref. 8, it has been shown that the LR method can be used even for noncyclic evolutions. In this article, we are interested in the classical equation obtained with LvN method, not the geometrical phase.

Consider the spin Hamiltonian given by

$$\mathcal{H}_0 = g\mu_B \sum_i \hat{\mathcal{S}}_i \cdot \mathbf{H}_{\text{eff}}, \tag{3}$$

where g is the Landé g factor, μ_B is the absolute value of the Bohr magneton, and $\hat{\mathcal{S}}_i$ is the spin operator. The subscript "0" indicates that the corresponding term does not include the RSI effect. The effective field $\mathbf{H}_{\text{eff}}$ includes the exchange field, the anisotropy field, and the demagnetizing field, as well as the external field.

Following Ref. 7, we take the ansatz of the dynamical invariant given by

$$\mathcal{I}_0 = \sum_i \mathbf{R}(t) \cdot \hat{\mathcal{S}}_i, \tag{4}$$

where $\mathbf{R}$ is a parameter vector. Then, the LvN equation becomes

$$\sum_i \hat{\mathcal{S}}_i \cdot \left(\frac{d\mathbf{R}(t)}{dt} + g\mu_B \mathbf{R}(t) \times \mathbf{H}_{\text{eff}} \right) = 0 \tag{5}$$

and we obtain a classical equation for the parameter vector given by

$$\frac{d\mathbf{R}(t)}{dt} = -\gamma \mathbf{R}(t) \times \mathbf{H}_{\text{eff}}, \tag{6}$$

where we have used $\gamma \equiv g\mu_B$ and the commutation relation $[\mathcal{S}_i^a, \mathcal{S}_j^b] = i\hbar \epsilon_{abc} \delta_{ij} \mathcal{S}_i^c$. The equation for the parameter vector given by Eq. (6) explicitly describes the vector precessing with respect to the field $\mathbf{H}_{\text{eff}}$ with the Larmor frequency $\omega = \gamma H_{\text{eff}}$. Thus, without loss of generality, we can identify the parameter vector $\mathbf{R}$ with the magnetization vector $\mathbf{M}_0$ (up to a dimensional constant factor).

On the other hand, the magnetization is defined by

$$\mathbf{M}_0 \equiv \frac{1}{V} \mathbf{Tr}\{\rho \hat{\mathcal{M}}_0\}, \tag{7}$$

where ρ is the density operator, V is the volume of the system, and $\hat{\mathcal{M}}_0$ is the magnetic moment operator (MMO) defined by

$$\hat{\mathcal{M}}_0 \equiv \frac{\delta \mathcal{H}_0}{\delta \mathbf{H}_{\text{ext}}}, \tag{8}$$

where $\mathbf{H}_{\text{ext}}$ is the external field. It is to be noted that the density operator is not given by $\exp\{-\beta \mathcal{H}_0\}$ but is given by $\exp\{-\beta \mathcal{I}_0\}$.[10] From Eqs. (3) and (8), we obtain

$$\hat{\mathcal{M}}_0 = g\mu_B \sum_i \hat{\mathcal{S}}_i. \tag{9}$$

Then, according to the above argument, the ansatz of the dynamical invariant given by Eq. (4) can be written as

$$\mathcal{I}_0 = c\mathbf{M}_0 \cdot \hat{\mathcal{M}}_0, \tag{10}$$

where c is a dimensional constant. Note that this expression for the dynamical invariant consists of the magnetization and the MMO, i.e., magnetic quantities. In the next section, we show that the new form of the dynamical invariant given by Eq. (10) leads to a reasonable generalization containing the RSI effect.

III. DYNAMICAL INVARIANT, RSI, AND DAMPING

The effective Hamiltonian including the RSI effect is given by

$$\mathcal{H} = g\mu_B \sum_i \hat{\mathcal{S}}_i [(1 - \lambda \alpha M^2)\mathbf{H}_{\text{eff}} + \lambda \boldsymbol{M} \times \mathbf{H}_{\text{eff}}], \tag{11}$$

where the dynamical parameter λ, which was termed as the radiation parameter, carries the RSI effect.[2] Here, in the Hamiltonian, only the dissipative part of the radiation field has been included, and the remaining part, i.e., nondissipative part, has been assumed to be parallel to the magnetization vector $\mathbf{M}$. Then, the nondissipative part does vanish in the procedure obtaining the magnetization equation. Thus, we omit the nondissipative part in the Hamiltonian. The MMO modified by the dynamical effect is given by

$$\hat{\mathcal{M}} = g\mu_B \sum_i [(1 - \lambda \alpha M^2)\hat{\mathcal{S}}_i + \lambda \hat{\mathcal{S}}_i \times \mathbf{M}] \tag{12}$$

and the magnetization vector becomes

$$\mathbf{M} = \frac{g\mu_B}{V}\sum_i \mathrm{Tr}\{\rho[(1-\lambda\alpha M^2)\hat{\mathcal{S}}_i + \lambda\hat{\mathcal{S}}_i \times \mathbf{M}]\}. \quad (13)$$

After some algebraic calculations, we find that $\mathbf{M}$ is parallel to $\mathbf{M}_0$, and the second term of Eq. (13) vanishes. Thus, the magnitude of magnetization vector satisfies the relation given by

$$M = (1-\lambda\alpha M^2)M_0. \quad (14)$$

The LLG Eq. (1), then, can be obtained by differentiating Eq. (13) with respect to time

$$\frac{d\mathbf{M}}{dt} = +\frac{g\mu_B}{i\hbar V}\sum_i \mathrm{Tr}\{\rho[(1-\lambda\alpha M^2)\hat{\mathcal{S}}_i + \lambda\hat{\mathcal{S}}_i \times \mathbf{M},\mathcal{H}]\} + \frac{\lambda}{1-\lambda\alpha M^2}\mathbf{M}\times\frac{d\mathbf{M}}{dt} = -\gamma\mathbf{M}\times\mathbf{H}_{\mathrm{eff}} + \alpha\mathbf{M}\times\frac{d\mathbf{M}}{dt}, \quad (15)$$

where we use Eq. (14) and the relation

$$\alpha = \frac{\lambda}{1-\lambda\alpha M^2}. \quad (16)$$

Note that the time derivative in Eq. (15) has to be performed before the trace operation.

Now, we address an alternative method to obtain the magnetization equation including the damping term. First of all, we take the ansatz of the dynamical invariant given by Eq. (10)

$$\mathcal{I}(t) = c\mathbf{M}\cdot\hat{\mathcal{M}} = cg\mu_B(1-\lambda\alpha M^2)\mathbf{M}\cdot\sum_i \hat{\mathcal{S}}_i. \quad (17)$$

The dynamical invariant (17) receives the RSI effect as the factor of $(1-\lambda\alpha M^2)^2$ [see Eq. (14)]. According to the LR method, then, we require that the dynamical invariant satisfies the LvN equation

$$\sum_i \hat{\mathcal{S}}_i\cdot\left(\frac{d\mathbf{M}}{dt} + g\mu_B\mathbf{M}\times[(1-\lambda\alpha M^2)\mathbf{H}_{\mathrm{eff}} + \lambda\boldsymbol{M}\times\mathbf{H}_{\mathrm{eff}}]\right) = 0. \quad (18)$$

Then, using the relation (16), we obtain the magnetization equation given by

$$\frac{d\mathbf{M}}{dt} = -\frac{\gamma}{1+\alpha^2M^2}\mathbf{M}\times\mathbf{H}_{\mathrm{eff}} - \frac{\gamma\alpha}{1+\alpha^2M^2}\mathbf{M}\times(\mathbf{M}\times\mathbf{H}_{\mathrm{eff}}). \quad (19)$$

It is well known that Eq. (1) is obtained from Eq. (19), or vice versa, using $\mathbf{M}\cdot d\mathbf{M}/dt=0$. Note that the second term of MMO Eq. (12), which played the central role in the derivation of the Gilbert damping term in Eq. (15), simply vanishes in the expression for the dynamical invariant given by Eq. (17). Instead, the damping term in Eq. (19) is derived from the RSI terms of the spin Hamiltonian as shown in Eq. (18). The factor $(1-\lambda\alpha M^2)$ also plays a critical role in the derivation.

IV. DISCUSSIONS

It has been shown that the LR dynamical invariant method including the RSI effect produces the damping term in the magnetization equation. This observation supports the RSI proposal for describing the magnetization relaxation. Especially, since the LR method was originally designed for the nonequilibrium evolution of time-dependent quantum systems, and the RSI description introduces nonlinearity into the spin Hamiltonian, their combination seems to be appropriate.

The relation of $\mathbf{M}\cdot d\mathbf{M}/dt=0$, or equivalently $\mathbf{M}\perp d\mathbf{M}/dt$, which is obtained from Eq. (1) or (19), has the same range of validity that is given by the approximation that the magnitude of magnetization vector is a constant and does not vary in time. This is just an alternative argument about the fact that Eqs. (1) and (19) describe the coherent magnetization dynamics, and we need to generalize the magnetization equations for the incoherent magnetization dynamics. In fact, the ansatz given by Eq. (17) is prepared for the coherent magnetization dynamics. It appears that a generalized expression for the dynamical invariant of the spin system would be required to describe the incoherent magnetization dynamics.

The RSI description proposed in Ref. 2 is not designed for a fundamental quantum analysis of the magnetization relaxation process, but for providing an effective picture of the magnetization relaxation. So, in order to get a fundamental understanding of the physical parameters of the magnetization dynamics, one has to return to the microscopic studies of that. However, for the purpose of phenomenological description of the magnetization dynamics, the RSI description provides a compact tool eliminating complicated elements in the magnetization dynamics so that one can concentrate on the phenomenological magnetization dynamics. Such an advantage of the RSI description lies with the fact that the RSI effect is not included at the equation of motion level, but at the Hamiltonian level, would lead to an economical way to study generalization of the LLG equation for highly nonlinear magnetization dynamics.

As mentioned in the Introduction, the RSI description can be also used for the *real* radiation-spin interaction after slight modification from the original proposition given in Ref. 2. However, in general, α_{RSI} is too small to make a reasonable detection in the experiment. As an example, in the case of a ferromagnetic film with thickness of $d=4\times10^{-7}$cm, it is of the order of 10^{-6}.[5] It is well known that the magnetization damping can be remarkably enhanced by applying a spin-polarized current to a spin valve system.[11] So, it would be interesting to study the radiation-spin interaction effect to the magnetization dynamics in the situation. Moreover, in that case, the task to calculate the damping constant α_{RSI} from the microscopic study[4,12] would be also desirable.

Related to the above issue, an important observation has been reported in Ref. 5; in order to obtain the Gilbert form of the magnetization damping, a nonzero conduction current $\mathbf{J}(t)=\sigma\mathbf{E}(t)$ is essential, where σ is the conductivity and $\mathbf{E}(t)$ is the electric field. This argument also makes sense even in the case that the RSI is used as an effective description for the magnetization relaxation discussed in Ref. 2. This is because the effective description follows the structural analogy of the radiation damping in the classical electrodynamics, and the radiation field can be derived from the Maxwell equation as shown in Ref. 5. Thus, it seems to be quite interesting to study the phenomenological prescription corresponding to the Ohm's law in the real radiation-spin interaction procedure in the magnetization dynamics.

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