# Oscillatory ac conductivity and photoconductivity of a two-dimensional electron gas: Quasiclassical transport beyond the Boltzmann equation

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We have analyzed the quasiclassical mechanism of magneto-oscillations in the ac conductivity and photoconductivity, related to non-Markovian dynamics of disorder-induced electron scattering. While the magneto-oscillations in the photoconductivity are found to be weak, the effect manifests itself much more strongly in the ac conductivity, where it may easily dominate over the oscillations due to the Landau quantization. We argue that the damping of the oscillatory photoconductivity provides a reliable method of measuring the homogeneous broadening of Landau levels (single-particle scattering rate) in high-mobility structures.

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## I. INTRODUCTION

An intriguing development in the study of a high-mobility two-dimensional electron gas (2DEG) was the recent observation[1] of magneto-oscillations of the photoconductivity of a sample subjected to microwave radiation, as a function of the ratio $\omega/\omega_c$. Here $\omega$ and $\omega_c$ are the radiation frequency and the cyclotron frequency, respectively. Subsequent experiments,[2–5] working with very-high-mobility samples, yielded yet another dramatic discovery: for sufficiently high radiation power, the minima of the oscillations evolve into "zero-resistance states"; i.e., the dissipative resistance of a sample becomes vanishingly small.

The nature of the oscillations in the photoconductivity $\sigma_{\rm ph}$ has raised a lot of interest. An important step was made in Ref. 6, in which a direct connection between the emergence of the zero-resistance states and the oscillations was emphasized. Specifically, it was recognized that *whatever* the nature of the oscillations, when they become so large that the linear dc response theory yields a negative $\sigma_{\rm ph}$, an instability is developed leading to the formation of domains of counter-flowing currents and thus to the zero measured resistance. Following this approach, a key issue which needs to be settled for understanding the experiments is the microscopic mechanism of the oscillatory photoconductivity (OPC).

Similarly to the conventional Shubnikov–de Haas oscillations, the growing body of theoretical work is focused on the oscillations of the density of states (DOS) induced by the Landau quantization as an essential element of the construction. The mechanism of the OPC identified in Ref. 7 and analyzed in detail in Ref. 8 hinges on the oscillations of the DOS and is related to a radiation-induced change of the electron distribution function in energy space, $f(\varepsilon)$, such that $f(\varepsilon)$ oscillates with varying both $\varepsilon/\omega_c$ and $\omega/\omega_c$. A hallmark of this contribution to $\sigma_{\rm ph}$ is that it yields an amplitude of the OPC which is proportional to the inelastic relaxation time $\tau_{\rm ee}$ due to electron-electron collisions (more effective at low temperatures than electron-phonon scattering). Another mechanism of the OPC, based on the effect of radiation on impurity scattering in the presence of the Landau quantization, was suggested in Ref. 9 (an earlier, closely related variant of this approach was formulated in Ref. 10). A systematical theory of this contribution to $\sigma_{\rm ph}$ was constructed in Ref. 11. Comparing the results of Refs. 7 and 8, and Ref. 11, one sees that the mechanism[7,8] dominates, i.e., leads to much stronger oscillations, if $\tau_{\rm ee} \gg \tau_q$, where $\tau_q$ is the single-particle relaxation time due to impurity scattering. For typical experimental parameters, a characteristic ratio $\tau_{\rm ee}/\tau_q \sim 10^2$.[8] Overall the results of Ref. 8 are in good agreement with the experimental data as regards the behavior of $\sigma_{\rm ph}$ in the range of parameters where the OPC is not too strongly damped, i.e., where the experimental efforts have been focused so far. In particular, Ref. 8 explains the emergence of strong oscillations and, in combination with Ref. 6, the formation of zero-resistance regions.

While the agreement between theory and experiment is very encouraging, the situation is not so clear in the experimental limit of *weak* (strongly damped) oscillations. Central to the identification of the microscopic mechanism of the oscillations is, on top of their period and phase, the behavior of their envelope with decreasing magnetic field $B$. For any mechanism based on the DOS oscillations, the relation between the DOS and OPC damping factors is critically important. The OPC[7,8,11] is damped at low $B$ by a factor $\exp(-\pi/\omega_c\tau_{\rm ph})$, where the ratio $\tau_{\rm ph}/\tau_q = 1/2$ is a distinctive feature not sensitive to microscopic details of either disorder or weak inelastic interactions.[12] However, as emphasized in Ref. 7, the experimentally reported values of $\tau_q$ and $\tau_{\rm ph}$ do not satisfy this relation, with $\tau_{\rm ph}$ noticeably larger than $\tau_q/2$, roughly by a factor of 10 in Ref. 3 and by a factor of 3 in Ref. 2. Taken at face value, the difference would mean that the amplitude of the OPC observed at small $B$ is orders of magnitude higher than given by the mechanism based on the Landau quantization, which might be considered as a hint about a different origin of the OPC at small $B$. Alternatively, the experiments on the damping of Shubnikov–de Haas oscillations might overestimate the single-particle scattering rate $\tau_q^{-1}$, e.g., because of inhomogeneous (due to macroscopic inhomogeneities) broadening of Landau levels. To resolve this dilemma, it is desirable to examine a range

 

of mechanisms of the OPC in the absence of the DOS oscillations.

In this article, we analyze a mechanism of the OPC governed by *quasiclassical* memory effects. These are related to non-Markovian correlations in electron dynamics.[13] We assume that $\pi/\omega_c\tau_q \gg 1$ and completely neglect weak oscillations of the DOS. The OPC induced by the memory effects is not specific to any particular type of disorder; however, below we concentrate on the following two-component model,[14] in which the memory effects are particularly prominent. We assume that there is a smooth random potential of remote donors that are separated by a large spacer $d \gg k_F^{-1}$, where $k_F$ is the Fermi wave vector, from the 2DEG plane and, in addition, there are rare short-range scatterers, e.g., residual impurities located at or near the interface. We consider the case $\tau_S \ll \tau_L$, where $\tau_S$ and $\tau_L$ are the zero-$B$ momentum relaxation times due to the short-range scatterers and the long-range disorder, respectively. From the experimental point of view, this choice is motivated by reports (see, e.g., Ref. 15) that the zero-$B$ mobility in very-high-mobility structures is frequently limited by residual impurities and $\tau_L/\tau_S$ can be as large as 10. Although $\tau_S \ll \tau_L$ in our model, we assume that $\tau_q$ is determined by the smooth disorder; i.e., $\tau_q \simeq \tau_L/(2k_F d)^2 \ll \tau_S$.

The article is organized as follows. First, in Sec. II, we outline the approach to the photoconductivity based on the Boltzmann equation. In Sec. III, we discuss the mechanism of the photoconductivity related to electron-electron interactions. In Sec. IV, we turn to the magneto-oscillations induced by the memory effects. Our central results are presented in Secs. V and VI. Section V deals with the oscillations in the ac conductivity. Finally, in Sec. VI, we compare two mechanisms of the oscillatory photoconductivity, quasiclassical and quantum, related to the memory effects and the Landau quantization, respectively.

## II. PHOTOCONDUCTIVITY: ESSENTIALS

A necessary input to the calculation of the *quasiclassical* OPC is the memory effects, discarded in the Boltzmann equation. However, to set up a systematic formalism, it is instructive to begin with a derivation of $\sigma_{\rm ph}$ within the conventional kinetic theory. The Boltzmann equation for the distribution function $g(p,\phi,t)$ of electrons in momentum space reads

$$Lg(p,\phi,t) = -\mathbf{F}\ \partial_{\mathbf{p}}\ g(p,\phi,t), \tag{1}$$

where $L=\partial_t+\omega_c\ \partial_\phi - I_{\rm el} - I_{\rm in}$, $\mathbf{F}=-e(\vec{\mathcal{E}}_{\rm dc}+\vec{\mathcal{E}}_\omega \cos\omega t)$, $\vec{\mathcal{E}}_{\rm dc}$ is the dc electric field, $\vec{\mathcal{E}}_\omega$ is the ac field, $\phi$ is the angle of the momentum $\mathbf{p}$ with respect to the direction of $\vec{\mathcal{E}}_{\rm dc}$, and $I_{\rm el}$ and $I_{\rm in}$ are the elastic and inelastic collision integrals, respectively.

We expand the distribution function at energy $\varepsilon$ in a series: $g(p,\phi,t)=\Sigma_{\nu n}\, g_{\nu n}(\varepsilon)\exp(i\nu\phi+in\omega t)$. Elastic collisions lead to relaxation of angular harmonics with $\nu\neq 0$; in particular, $I_{\rm el}g_{1n}=-\tau^{-1}g_{1n}$, where $\tau$ is the momentum relaxation time. Inelastic electron-electron collisions tend to equilibrate electrons among themselves, but are not capable of establishing a steady-state dc photoconductivity. For the quasiclassical OPC (in contrast to that based on the DOS oscillations, cf. Ref. 8), the inelastic transitions due to electron-electron interaction do not play any essential role and will be neglected. To dissipate energy absorbed from the ac field, we introduce coupling to a thermal bath, e.g., to an equilibrium phonon system, characterized by a relaxation time $\tau_{\rm in}$. Under the assumption that both $\tau_{\rm in}$ and the momentum relaxation time due to the coupling to the bath are much longer than $\tau$, the main role of the inelastic scattering is to yield a slow relaxation of the isotropic ($\nu=0$) part of $g$ to the equilibrium Fermi distribution $f_F$ at a bath temperature $T$.

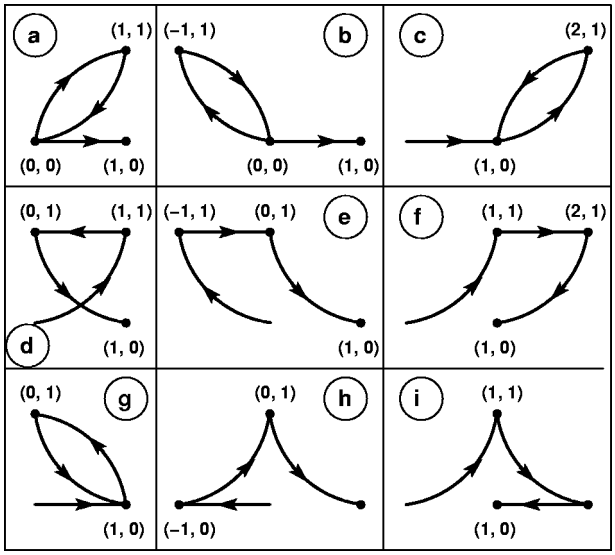


FIG. 1. Graphic representation of various contributions to the photoconductivity [Eqs. (2) and (3)] in the $(\nu,n)$ space at the lowest order $\sigma_{\rm ph}\sim\mathcal{O}(\mathcal{E}_{\rm dc}^0\mathcal{E}_\omega^2)$.

Expanding the nonequilibrium distribution function in powers of the driving force, we have

$$g=\sum_{m=0}(-L^{-1}\mathbf{F}\ \partial_{\mathbf{p}})^m f_F. \tag{2}$$

A useful way of visualizing this solution of Eq. (1) at given order in $\mathcal{E}_{\rm dc}$ and $\mathcal{E}_\omega$ is by counting all possible couplings of harmonics $g_{\nu n}$ represented as points on the $(\nu,n)$ plane (Fig. 1). The dc field $\mathcal{E}_{\rm dc}$ couples nearest-neighbor harmonics along the $\nu$ axis, $g_{\nu n}\leftrightarrow g_{\nu\pm1,n}$. We are interested here in the linear (with respect to $\mathcal{E}_{\rm dc}$) photoconductivity $\sigma_{\rm ph}$, so that only one such link is allowed. The ac field $\mathcal{E}_\omega$ couples harmonics along diagonals, $g_{\nu n}\leftrightarrow g_{\nu\pm1,n\pm1}$ and $g_{\nu\pm1,n\mp1}$. The propagator $L^{-1}$ is a diagonal matrix in $(\nu,n)$ space. The static longitudinal current $j=\sigma_{\rm ph}\mathcal{E}_{\rm dc}$, given by

$$j=-\frac{e}{2\pi\hbar^2}\int d\varepsilon\ p\ {\rm Re}\ g_{10}, \tag{3}$$

is expressed through $g_{10}$; i.e., is given by a sum of all paths starting at (0, 0) and ending at (1, 0). Already at order $\sigma_{\rm ph}\sim\mathcal{O}(\mathcal{E}_{\rm dc}^0\mathcal{E}_\omega^2)$, as many as nine different graphs arise, shown in

Fig. 1, to which one should add their counterparts mirrored in the horizontal axis, which corresponds to the change $\omega \to -\omega$.

Our strategy for finding $g_{10}$ is to select graphs involving couplings whose strength diverges at $\tau_{\rm in}/\tau \to \infty$. These are graphs returning to the point (0, 0) [graphs (a) and (b) in Fig. 1], which are proportional to $(L^{-1})_{00}$. This means that to order $\mathcal{E}_\omega^2$ the path $(0,0)\to(1,1)\to(0,0)\to(1,0)$ (and its counterparts in other quadrants) gives the main contribution to $\sigma_{\rm ph}$ for

$$\tau_{\rm in} \gg \tau. \tag{4}$$

The perturbative expansion in powers of $\mathcal{E}_\omega^2$ proceeds by iterating the loop $(0,0)\to(1,1)\to(0,0)$. In this way we arrive at a simple relation $(\pm i\omega_c+\tau^{-1})g_{\pm1,0}=e\mathcal{E}_{\rm dc}\,\partial_p f/2$, where $f=g_{00}$ satisfies the closed equation

$$\frac{e^2\mathcal{E}_\omega^2}{2m}\partial_\varepsilon[K_\omega\,\partial_\varepsilon f]+I_{\rm in}f=0. \tag{5}$$

The function $K_\omega=K_\omega^+ + K_\omega^-$ which describes the absorption rate at energy $\varepsilon$ is given by

$$2K_\omega^\pm=\frac{\varepsilon\tau}{1+(\omega\pm\omega_c)^2\tau^2}. \tag{6}$$

The photoconductivity at $\tau_{\rm in}\gg\tau$ is thus completely determined by $f$; i.e., in this limit the ac field modifies the dc current through the heating, as

$$\sigma_{\rm ph}=-\frac{e^2}{2\pi\hbar^2}\int d\varepsilon\; K_0\;\partial_\varepsilon f. \tag{7}$$

The function $f(\varepsilon)$ changes abruptly around the Fermi energy $\epsilon_F$ on a scale

$$T_e=\max\{T,\Delta_h\}, \tag{8}$$

where

$$\Delta_h=(e\mathcal{E}_\omega\; l_{\rm in}/2)[K_\omega(\epsilon_F)/K_0(\epsilon_F)]^{1/2} \tag{9}$$

and $l_{\rm in}=v_F[K_0(\epsilon_F)\tau_{\rm in}/\epsilon_F]^{1/2}$ is the inelastic length ($v_F$ is the Fermi velocity). Note that $\tau_{\rm in}$ in the regime of strong heating ($\Delta_h\gg T$) should be found self-consistently with $\Delta_h$ and thus depends on $\mathcal{E}_\omega$.

Turning to the evaluation of $\sigma_{\rm ph}$ under the assumption that $T_e\ll\epsilon_F$, we first notice that a seemingly reasonable approximation that neglects the $\varepsilon$ dependence of $\tau$ in the integrand of Eq. (7) [normally, $\tau(\varepsilon)$ changes on a scale of $\epsilon_F$] yields an identically zero photoresponse. Indeed, in that case $\sigma_{\rm ph}$ is equal to the static Drude conductivity $\sigma_0^{\rm D}$ independently of the detailed shape of $f(\varepsilon)$, since $\int d\varepsilon\;\varepsilon\;\partial_\varepsilon f=-2\pi\hbar^2 n_e/m$ due to particle number conservation ($n_e$ is the electron concentration). It follows that the dependence of $\tau$ on $\varepsilon$ should be taken into account. It is worth mentioning that, contrary to a naive expectation, this does not lead to any additional smallness of $\sigma_{\rm ph}$ since $\tau(\varepsilon)$ enters the result through expressions of the type $\varepsilon\partial_\varepsilon\tau|_{\varepsilon=\varepsilon_F}\sim\tau$.

We do not discuss specific microscopic models of the inelastic coupling of electrons to a thermal bath, our purpose here is to use the simplest possible representation of $I_{\rm in}$. In a conserving relaxation-time approximation

$$I_{\rm in}f=-\tau_{\rm in}^{-1}(f-f_F), \tag{10}$$

where the $\varepsilon$-independent $\tau_{\rm in}^{-1}$ is, in general, a functional of $f(\varepsilon)$, we get from Eqs. (5) and (7) for $T_e\ll\epsilon_F$,

$$\sigma_{\rm ph}-\sigma_0^{\rm D}=\sigma_\omega^{\rm D}\frac{e^2\mathcal{E}_\omega^2\tau_{\rm in}}{2m}K_0''=\frac{e^2}{2\pi\hbar^2}\Delta_h^2K_0''. \tag{11}$$

Here $K_0''=\partial_\varepsilon^2K_0|_{\varepsilon=\epsilon_F}$ and $\sigma_\omega^{\rm D}$ is the zero-$T$ dynamic Drude conductivity, $\sigma_\omega^{\rm D}=e^2K_\omega(\epsilon_F)/2\pi\hbar^2$. Note that the only source of nonlinearity of $\sigma_{\rm ph}$ with respect to the ac field power in Eq. (11) is a dependence of $\tau_{\rm in}$ on $\mathcal{E}_\omega$.

Alternatively, assuming the dominant role of soft inelastic scattering with energy transfers much smaller than $T$, we can write $I_{\rm in}$ in the Fokker-Planck (FP) form, as

$$I_{\rm in}f=\partial_\varepsilon\,\{B[\partial_\varepsilon f+T^{-1}f(1-f)]\}, \tag{12}$$

where $B(\varepsilon)=\langle(\delta\varepsilon)^2W(\varepsilon,\delta\varepsilon)\rangle/2$ is the diffusion coefficient in energy space, $W$ is the corresponding rate of inelastic processes, and $\langle\ldots\rangle$ denotes averaging over the energy transfer $\delta\varepsilon$. Equation (5) then becomes first-order in $\partial_\varepsilon$, which gives $f(\varepsilon)$ described by the Fermi distribution with the effective electron temperature

$$T_{\rm eff}=T+\Delta_{\rm FP}, \tag{13}$$

where

$$\Delta_{\rm FP}=e^2\mathcal{E}_\omega^2TK_\omega(\epsilon_F)/2mB(\epsilon_F), \tag{14}$$

and

$$\begin{aligned}\sigma_{\rm ph}-\sigma_0^{\rm D}&=\frac{e^2}{2\pi\hbar^2}\frac{\pi^2}{6}(T_{\rm eff}^2-T^2)K_0''\\&=\frac{e^2}{2\pi\hbar^2}\frac{\pi^2}{6}(2T+\Delta_{\rm FP})\Delta_{\rm FP}K_0''.\end{aligned} \tag{15}$$

The case of typical energy transfers $\sim T$ may be qualitatively described by either model with $B(\epsilon_F)\tau_{\rm in}\sim T_eT$.

The microwave power and temperature dependences of $\sigma_{\rm ph}$ can be found from Eqs. (11) and (15) for a variety of scattering mechanisms. If one assumes that $\tau_{\rm in}$ is determined by scattering on acoustic phonons via the piezoelectric interaction screened by the 2DEG, the characteristic energy transfer is $T_e$ and $\tau_{\rm in}^{-1}\propto T_e^3$. It then follows from Eq. (9) that the heating at $\Delta_h\gg T$ is characterized by $T_e\propto\mathcal{E}_\omega^{2/5}$. By using Eq. (11) we get $\sigma_{\rm ph}-\sigma_0^{\rm D}\propto\mathcal{E}_\omega^2T^{-3}$ for $\Delta_h\ll T$ and $T$-independent $\sigma_{\rm ph}-\sigma_0^{\rm D}\propto\mathcal{E}_\omega^{4/5}$ otherwise.

Having identified the main contribution to $\sigma_{\rm ph}$ in the limit $\tau_{\rm in}/\tau\gg1$ [diagrams (a) and (b) in Fig. 1] it is instructive to compare this contribution with that corresponding to other diagrams [diagrams (c)–(i)]. While the former is related to the heating of electrons by the ac field, the latter can be regarded as an effect of radiation on the impurity scattering and thus represents a classical analog of the quantum effect considered in Refs. 9–11. Following the procedure given by

Eqs. (2) and (3) and making use of the explicit matrix form of the field operator, $[\mathbf{F}\partial_{\mathbf{p}}g]_{\nu n}=F^{\nu\nu'}_{nn'}g_{\nu' n'}$,

$$F^{\nu\nu'}_{nn'} = -\frac{1}{2}\delta_{\nu,\nu'\pm1}(\mathbf{s}_{\nu\nu'}\cdot e\vec{\mathcal{E}}_{nn'})\left[\partial_p + (\nu'-\nu)\frac{\nu'}{p}\right],$$

$$\vec{\mathcal{E}}_{nn'} = \vec{\mathcal{E}}_{\rm dc}\delta_{nn'} + \frac{1}{2}\vec{\mathcal{E}}_{\omega}\delta_{n,n'\pm1},$$

$$\mathbf{s}_{\nu\nu'} = \mathbf{e}_x + i(\nu'-\nu)\mathbf{e}_y, \tag{16}$$

one can readily calculate the photoconductivity at any desirable order in the fields $\vec{\mathcal{E}}_{\rm dc}$ and $\vec{\mathcal{E}}_{\omega}$ ($\mathbf{e}_{x,y}$ are the unit vectors along the $x,y$ axes). At the lowest order the photoconductivity $\sigma_{\rm ph}\sim\mathcal{O}(\mathcal{E}^0_{\rm dc}\mathcal{E}^2_{\omega})$ is given by the diagrams (a)–(i) in Fig. 1 (together with their counterparts in a lower half-plane, $\omega\to-\omega$). In the limit $\tau_{\rm in}^{-1}\ll\tau^{-1}\ll\omega_c\ll\omega$ the result takes a simple form

$$\sigma_{\rm ph}-\sigma_0^{\rm D} = \frac{1}{8}\sigma_0^{\rm D}\left(\frac{eE_{\omega}v_F}{\varepsilon_F\omega}\right)^2 \times\left[2c_1\frac{\tau_{\rm in}}{\tau}+(5c_1+4c_2)+(3c_1+2c_2)\cos 2\phi_E\right], \tag{17}$$

where $c_1=\varepsilon\tau\partial_\varepsilon^2\varepsilon\tau^{-1}|_{\varepsilon=\varepsilon_F}$, $c_2=\varepsilon\tau\partial_\varepsilon\tau^{-1}|_{\varepsilon=\varepsilon_F}$ are numbers (typically of order unity) determined by the type of disorder, and $\phi_E$ is the angle between $\vec{\mathcal{E}}_{\rm dc}$ and $\vec{\mathcal{E}}_{\omega}$. The first term in the square brackets corresponds to the diagrams (a) and (b) in Fig. 1 and reproduces Eq. (11) in the limit of weak heating, $\Delta_h\ll T$. The term $(5c_1+4c_2)$ corresponds to the diagrams (c), (e), and (f). The polarization-dependent part, given by the last term, originates from the diagrams (d) and (i) (in which both diagonal links have the same direction along the $\nu$ axis). Finally, the diagrams (g) and (h) give a contribution which is smaller, compared to the diagrams (c)–(f) and (i), in the parameter $1/\omega_c\tau_{\rm in}\ll1$ and is omitted in Eq. (18). One can clearly see from Eq. (18) that in the limit $\tau_{\rm in}/\tau\gg1$, photoconductivity is dominated by the heating of electrons.

## III. INTERACTION-INDUCED PHOTOCONDUCTIVITY

In the above, we have neglected inelastic electron-electron collisions, whose role is not essential for the quasiclassical OPC, but have also ignored the renormalization of the *elastic* scattering rate by electron-electron interactions. The latter approximation, which fits in with the conventional approach to photoconductivity, in fact misses an important contribution to $\sigma_{\rm ph}$. Recall that the change of the conductivity due to radiation at $\tau_{\rm in}\gg\tau$ comes mainly from the heating. It is most illuminating to focus on the model of Eqs. (12) and (15), within which $\sigma_{\rm ph}-\sigma_0^{\rm D}$ is simply proportional to $T_{\rm eff}^2-T^2$. Clearly, this contribution to $\sigma_{\rm ph}$ is associated with the term in the Drude conductivity that is quadratic in the small parameter $T/\epsilon_F$. Substituting $T_{\rm eff}$ for $T$ in the Drude term yields $\sigma_{\rm ph}$ given by Eq. (15). On the other hand, there are $T$-dependent quantum corrections to the conductivity, neglected above, in which one should similarly change $T\to T_{\rm eff}$. At low $T$, the terms in $\sigma_{\rm ph}$ coming from these quantum corrections may easily become larger than the classical contribution (15), as we now demonstrate.

For high-mobility samples, we are mostly interested in $\sigma_{\rm ph}$ at not too low temperatures $T\tau/\hbar\gg1$. In this "ballistic" regime, the most important $T$-dependent term in the conductivity at zero $B$, for the limiting case of short-range disorder ($\tau\to\tau_S$), is related to screening of the disorder by Friedel oscillations, which translates into a $T$- and $\varepsilon$-dependent renormalization of the elastic scattering rate. This quantum interaction-induced term is given by $\Delta\sigma_{\rm int}=\alpha(e^2/\pi\hbar^2)T\tau_S$.[16] Here $\alpha$ is the interaction coupling constant, equal to unity for the Coulomb interaction (under the assumption that $k_F^{-1}$ is much smaller than the static screening length). Remarkably, $\Delta\sigma_{\rm int}$ is linear in $T/\epsilon_F$, in contrast to the classical $T$-dependent term, which is quadratic in $T/\epsilon_F$. Substituting $T\to T_{\rm eff}$ in $\Delta\sigma_{\rm int}$ yields an interaction-induced term in the photoconductivity, $\Delta\sigma_{\rm ph}=\alpha(e^2/\pi\hbar^2)(T_{\rm eff}-T)\tau_S$, where $T_{\rm eff}-T=\Delta_{\rm FP}$ is given by Eq. (14). For finite $B$, assuming that $T\gg\hbar\omega_c,\hbar/\tau$, this term in $\sigma_{\rm ph}$ reads

$$\Delta\sigma_{\rm ph} = \alpha\frac{e^2}{\pi\hbar^2}(T_{\rm eff}-T)\tau_S\frac{1-\omega_c^2\tau_S^2}{(1+\omega_c^2\tau_S^2)^2}. \tag{18}$$

This result is obtained by inverting the resistivity tensor for which the leading (for $T\gg\hbar\omega_c,\hbar/\tau$) interaction-induced correction to $\rho_{xx}$ is $B$ independent, while that to $\rho_{xy}$ may be neglected. It follows from the comparison of Eqs. (15) and (18) that this quantum contribution to the photoconductivity is much larger than the classical one provided the effective temperature is low ($T_{\rm eff}\ll\alpha\epsilon_F$), which is satisfied for $\alpha\sim1$ in the whole range of temperatures in a degenerate Fermi system. Thus, sufficiently strong interactions have the effect of greatly enhancing the photoconductivity.

For stronger magnetic fields ($\hbar\omega_c\gg T$), another mechanism of the interaction-induced photoconductivity becomes relevant, related to the interplay[17] of quasiclassical memory effects and electron-electron interactions. For the two-component model of disorder, assuming, as above, that $\tau_S\ll\tau_L$, the $T$-dependent correction to the conductivity is $\Delta\sigma_{\rm int}\sim\alpha(e^2/\hbar)(\tau_L/\tau_S)^{1/2}(T\tau_S/\hbar)^{-1/2}$.[17] With numerical factors included, this yields a contribution to the photoconductivity given by

$$\Delta\sigma_{\rm ph} = -\alpha\frac{e^2}{\pi\hbar}\frac{3\zeta(3/2)}{16\pi^{3/2}}\left(\frac{\tau_L}{\tau_S}\right)^{1/2}\frac{T_{\rm eff}^{-1/2}-T^{-1/2}}{(\tau_S/\hbar)^{1/2}}. \tag{19}$$

Comparing Eqs. (18) and (19), one sees that the latter mechanism gives a larger contribution to $\sigma_{\rm ph}$ in the whole temperature range $T\lesssim\hbar\omega_c$. At $T\sim\hbar\omega_c$, the term (19) is still larger than that given by Eq. (18) by a factor $(\omega_c\tau_L)^{1/2}\gg1$. With increasing $T$, however, the memory-effects-induced correction falls off rapidly, as $\exp(-4\pi^2T/\omega_c)$, so that at $T\sim\hbar\omega_c\ln(\omega_c\tau_L)$ a crossover to Eq. (18) occurs.

## IV. MAGNETO-OSCILLATIONS DUE TO MEMORY EFFECTS

The photoconductivity obtained in Secs. II and III exhibits the cyclotron resonance but shows no oscillations with

varying $\omega/\omega_c$. Let us now incorporate the memory effects. To this end, we have to step back to write the Liouville equation not yet averaged over positions of impurities. More precisely, for the two-component model of disorder (specified in Sec. I), we average only over smooth disorder and represent the Liouville operator as

$$L = L_0 + \delta L - I_{\text{in}}, \tag{20}$$

where $L_0$ includes the effect of scattering on smooth disorder and is given by

$$L_0 = \partial_t + \mathbf{v}\nabla_{\mathbf{r}} + \omega_c\partial_\phi - \tau_L^{-1}\partial_\phi^2, \tag{21}$$

and $\delta L = -\Sigma_i I_{\mathbf{R}_i}(\mathbf{r})$ is a sum of collision operators for short-range impurities located at points $\mathbf{R}_i$. We have to keep in $L_0$ the spatial-gradient term ($\mathbf{v}$ is the velocity).

Averaging the solution of Eq. (1) over $\mathbf{R}_i$ with $L$ given by Eq. (20) can be done systematically along the lines of Ref. 14: a classical diagram technique is formulated by means of the free propagator $L_0^{-1}$ and the disorder correlation function $\langle \delta L(\mathbf{r})\delta L(\mathbf{r}')\rangle$. We proceed by representing the averaged propagator $\langle L^{-1}\rangle = (L_0 + M - I_{\text{in}})^{-1}$ in terms of the self-energy operator $M$. Equations (5)–(7) are then reproduced with $\tau$ in Eq. (6), given by

$$\tau^{-1} = \tau_L^{-1} + \Sigma(\omega), \tag{22}$$

where $\Sigma = \int (d\phi/2\pi)\mathbf{n}M\mathbf{n}$ and $\mathbf{n} = \mathbf{v}/|\mathbf{v}|$. To first order in $\delta L$,

$$\Sigma^{(1)} = -n_S \int d\mathbf{r} \int (d\phi/2\pi)\mathbf{n} I_{\mathbf{R}_i}(\mathbf{r})\mathbf{n}. \tag{23}$$

By definition, $n_S\int d\mathbf{r} I_{\mathbf{R}_i}(\mathbf{r})\mathbf{n} = -\mathbf{n}\tau_S^{-1}$, so that we have $\Sigma^{(1)} = \tau_S^{-1}$, which yields the Drude result for the total scattering rate $\tau^{-1} = \tau_L^{-1} + \tau_S^{-1}$. Expanding now $M$ to second order in $\delta L$, we obtain the leading correction to $\Sigma$ that is due to the memory effects:

$$\Sigma^{(2)}(\omega) = -n_S \int d\mathbf{r} \int d\mathbf{r}' \int \frac{d\phi}{2\pi}\mathbf{n} I_{\mathbf{R}_i}(\mathbf{r}) D_\omega(\mathbf{r}-\mathbf{r}') I_{\mathbf{R}_i}(\mathbf{r}')\mathbf{n}, \tag{24}$$

or more explicitly

$$\Sigma^{(2)}(\omega) = -4\pi n_S \int d\mathbf{r} \int d\mathbf{r}' \int \frac{d\phi}{2\pi} \int \frac{d\tilde{\phi}}{2\pi} \int \frac{d\tilde{\phi}'}{2\pi} \int \frac{d\phi'}{2\pi} \times \cos\phi I_{\mathbf{R}_i}(\mathbf{r},\phi,\tilde{\phi}) D_\omega(\mathbf{r}-\mathbf{r}',\tilde{\phi},\tilde{\phi}') \times I_{\mathbf{R}_i}(\mathbf{r}',\tilde{\phi}',\phi')\cos\phi', \tag{25}$$

where the propagator

$$D = (L_0 + \tau_S^{-1})^{-1} \tag{26}$$

is taken in the $\omega$ representation. Most importantly, the $\omega$ dispersion of $D$ leads to oscillations of $\Sigma^{(2)}(\omega)$ with a period $\omega_c$.

To find $\Sigma^{(2)}(\omega)$, we first note that, since $I_{\mathbf{R}_i}(\mathbf{r})$ as a function of $\mathbf{r}$ falls off fast beyond a small vicinity of $\mathbf{R}_i$, one can put $\mathbf{r} = \mathbf{r}'$ in the argument of $D_\omega$ in Eqs. (24) and (25). $\Sigma^{(2)}(\omega)$ is then given by

$$\Sigma^{(2)}(\omega) = -\frac{2}{n_S\tau_S^2}\int \frac{d\phi}{2\pi}\int d\phi' \cos\phi D_\omega(0;\phi,\phi')\cos\phi', \tag{27}$$

where $D_\omega(0;\phi,\phi')$ is the Fourier transform in $t$ of the probability density to return with a direction of $\mathbf{v}$ specified by $\phi'$ if one starts at an angle $\phi$. Let us now focus on the case $\omega_c\tau_S \gg 1$. In this limit, $D_\omega(0;\phi,\phi')$ is sharply peaked at $\phi = \phi'$ and, introducing the total probability of return $P_\omega = \int d\phi' D_\omega(0;\phi,\phi')$, we finally get

$$\Sigma^{(2)}(\omega) = -P_\omega/n_S\tau_S^2. \tag{28}$$

A return-induced correction to the effective scattering rate, which comes according to Eq. (28) from Re $P_\omega$, yields, away from the cyclotron resonance, oscillations of the absorption rate through a correction to the function $K_\omega(\epsilon_F)$ [cf. Eq. (6)],

$$\Delta K_\omega^{\pm}(\epsilon_F) = -(\epsilon_F/2n_S\tau_S^2)\text{Re } P_\omega/(\omega \pm \omega_c)^2. \tag{29}$$

The oscillatory part of $K_\omega(\epsilon_F)$ leads to *classical* oscillations of the linear ac conductivity,[19]

$$\Delta\sigma_\omega^{(c)} = -\sigma_\omega^{\text{D}}\text{Re } P_\omega/n_S\tau_S, \tag{30}$$

and being substituted in Eqs. (5)–(7), to those of $\sigma_{\text{ph}}$. To first order in $\mathcal{E}_\omega^2$, the classical oscillatory correction to $\sigma_{\text{ph}}$ reads

$$\frac{\Delta\sigma_{\text{ph}}^{(c)}}{\sigma_{\text{ph}} - \sigma_0^{\text{D}}} = \frac{\Delta\sigma_\omega^{(c)}}{\sigma_\omega^{\text{D}}} = -\frac{\text{Re } P_\omega}{n_S\tau_S}. \tag{31}$$

It is worth noting once more that both the smooth correction $\sigma_{\text{ph}} - \sigma_0^{\text{D}}$ and the oscillatory contribution $\Delta\sigma_{\text{ph}}^{(c)}$ are proportional to the inelastic time $\tau_{\text{in}}$.

In the above, we have analyzed the oscillatory correction to the self-energy in terms of the return probability $P_\omega$. In fact, there are other contributions to the OPC that are not reduced to the self-energy corrections and cannot be represented through $P_\omega$. To illustrate this point, it is convenient to switch to a more conventional (dual) representation of the diagrams in Fig. 1, now with lines corresponding to the propagators and vertices representing the field operators (16), as shown in Fig. 2. The diagram (a) in Fig. 2 reproduces the graph (a) in Fig. 1. The diagram Fig. 2(b) represents the oscillatory correction to $\sigma_{\text{ph}}$ of the self-energy type [Eq. (31)]. Both diagrams (a) and (b) in Fig. 2 contain the inelastic propagator $(L^{-1})_{00} = \tau_{\text{in}}$ at zero momentum $q$, which is much larger than all other propagators, $(L^{-1})_{\nu n}$ with at least one of the indices $\nu, n \neq 0$. By contrast, the diagram (c), which exemplifies an oscillatory vertex correction to $\sigma_{\text{ph}}$, is not proportional to $\tau_{\text{in}}$, because of large $q$ running along the internal propagators $D(q,\omega)$ [defined in Eq. (26)]. The vertex type corrections, which are of the same order in all of the diagrams (a)–(i), are thus by a factor $\tau_{\text{in}}/\tau_S$ smaller than the self-energy contribution (31).

The function $P_\omega$ for $\omega_c\tau_L \gg 1$ is most directly evaluated by using Eq. (21), which represents the time evolution of $\phi$ as a diffusion process with a white noise spectrum of $\partial_t\phi$. This approach is justified for not too strong $B$, namely, for $\delta \gg d$, where (see Appendix)

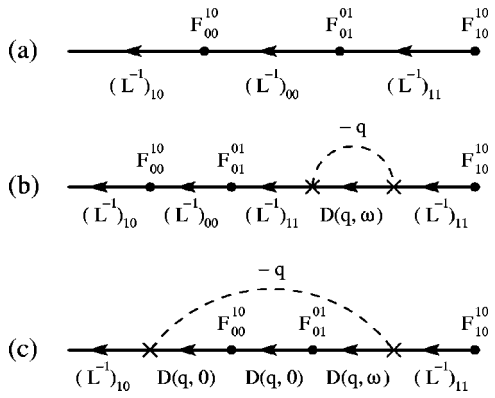


FIG. 2. Diagrams describing the memory effects in the photoconductivity $\sigma_{\rm ph}$: oscillatory self-energy (b) and vertex (c) corrections to the smooth part (a) of $\sigma_{\rm ph}$.

$$\delta = 2\pi^{1/2} v_F \tau_L/(\omega_c \tau_L)^{3/2} \tag{32}$$

is a mean-square fluctuation of the guiding center of a cyclotron orbit after one cyclotron revolution (otherwise adiabatic drift dynamics is developed). The probability density $p_n(x_\perp, x_\parallel)$ for particles on the Fermi surface to be scattered from the starting point on a cyclotron orbit by a distance $x_\perp$ across the orbit and a distance $x_\parallel$ along it in time $2\pi n/\omega_c$ is then given by the anisotropic Gaussian distribution with averages $\langle x_\perp^2\rangle = \langle x_\parallel^2\rangle/3 = n\delta^2/2$ (see Appendix). Summing over multiple cyclotron revolutions, we thus express $P_\omega$ as

$$P_\omega = \sum_{n=1} \int_{-\infty}^{\infty} dt\, e^{-i(\omega - i/\tau_S)t} p_n[0, v_F(t - 2\pi n/\omega_c)]. \tag{33}$$

Note that once the particle hits a short-range impurity, its guiding center is shifted by a distance of the order of the cyclotron radius. As a result the contribution of such trajectories to the return probability can be neglected and only noncolliding orbits should be taken into account, which is expressed by the exponential factor $\exp(-t/\tau_S)$. Equation (33) gives oscillations of $P_\omega$ as $\omega/\omega_c$ is varied, as

$$P_\omega = \frac{1}{\sqrt{\pi} v_F \delta} \sum_{n=1} \frac{1}{\sqrt{n}} \exp\left[ -\frac{2\pi n}{\omega_c}(i\omega + \Gamma) \right], \tag{34}$$

whose damping with decreasing $B$ is characterized by

$$\Gamma = \frac{3}{2\tau_L}\left(\frac{\omega}{\omega_c}\right)^2 + \frac{1}{\tau_S}. \tag{35}$$

In the limit of weak damping, $\pi\Gamma \ll \omega_c$, we perform the summation in Eq. (34) by means of Poisson's formula to represent Re $P_\omega$ as a series of sharp peaks centered at $\omega = N\omega_c$. A peak at $\omega \simeq N\omega_c$ is of the form[20]

$$\text{Re } P_\omega = \frac{\omega_c^3 \tau_L}{2\sqrt{3}\pi v_F^2 \omega} \mathcal{F}\left( \frac{\omega - N\omega_c}{\Gamma} \right), \tag{36}$$

$$\mathcal{F}(x) = \left[ \frac{1 + (1 + x^2)^{1/2}}{2(1 + x^2)} \right]^{1/2}, \quad \mathcal{F}(0) = 1. \tag{37}$$

Note that the amplitude of the peaks in Eq. (36) falls off with decreasing $\omega_c$ or increasing $\omega$ as a power law, namely, as $\omega_c^3/\omega$. The power-law suppression of the oscillations crosses over into the exponential damping only for very large $\pi(\omega/\omega_c)^2 \gg \omega_c \tau_L$, when one can neglect all terms in Eq. (34) but the first one, which gives

$$\text{Re } P_\omega = \frac{(\omega_c \tau_L)^{3/2}}{2\pi v_F^2 \tau_L} \cos\frac{2\pi\omega}{\omega_c} \exp\left( -\frac{2\pi\Gamma}{\omega_c} \right). \tag{38}$$

It is worth noting that, because of the condition $\omega_c \tau_S \gg 1$, the term $\tau_S^{-1}$ in Eq. (35) may be neglected in the damping factor of Eq. (38), so that the exponential damping is determined by the momentum relaxation time for scattering off the long-range disorder.

## V. OSCILLATORY AC CONDUCTIVITY: QUASICLASSICAL VERSUS QUANTUM

Now we compare the classical oscillatory ac conductivity $\sigma_\omega^{(c)}$, given by Eqs. (30), (36), and (38), with the quantum contribution $\sigma_\omega^{(q)}$ calculated in Ref. 7. Let us represent $\sigma_\omega^{(c)}$ for weak damping at $\omega = N\omega_c$ as

$$\sigma_\omega^{(c)}|_{\omega = N\omega_c} = \sigma_\omega^{\rm D}\left[ 1 - \frac{a}{\sqrt{3\pi} N\delta} (\omega_c \tau_L)^{1/2} \right], \tag{39}$$

where $\delta$ is given by Eq. (32) and we have introduced a characteristic size of the short-range impurities $a = (n_S v_F \tau_S)^{-1}$. It follows that, apart from the harmonics number $N$, the amplitude of the oscillations is given by the product of a small factor $a/\delta$ and a large factor $(\omega_c \tau_L)^{1/2}$. In the exponential damping regime, $\sigma_\omega^{(c)}$ is re-written as

$$\frac{\sigma_\omega^{(c)}}{\sigma_\omega^{\rm D}} = 1 - \frac{a}{\sqrt{\pi}\,\delta} \cos\frac{2\pi\omega}{\omega_c} \exp\left[ -\left(\frac{\omega}{\omega_c}\right)^2 \frac{3\pi}{\omega_c \tau_L} \right], \tag{40}$$

so that the pre-exponential factor is simply given by $a/\delta$. An important point to notice is that the damping in Eq. (40) is characterized solely by the long transport time for scattering off the smooth disorder. On the other hand, the envelope of the quantum oscillations of the ac conductivity is determined by the single-particle time $\tau_q$, as

$$\frac{\sigma_\omega^{(q)}}{\sigma_\omega^{\rm D}} = 1 + 2\, \cos\frac{2\pi\omega}{\omega_c} \exp\left( -\frac{2\pi}{\omega_c \tau_q} \right) \tag{41}$$

(this equation is valid for $2\pi T \gg \hbar/\tau_q$, for smaller $T$ see Ref. 7).

Note the difference in the sign of the oscillatory terms: there is a $\pi$ shift of the quantum and classical oscillations with respect to each other. Another difference is that the damping of the classical oscillations is $\omega$ dependent, in contrast to the quantum case. One sees that, despite the small factor $a/\delta$, the classical oscillations may be stronger than the quantum ones since in high-mobility structures $\tau_q \ll \tau_L$ and the quantum oscillations are damped much more strongly.

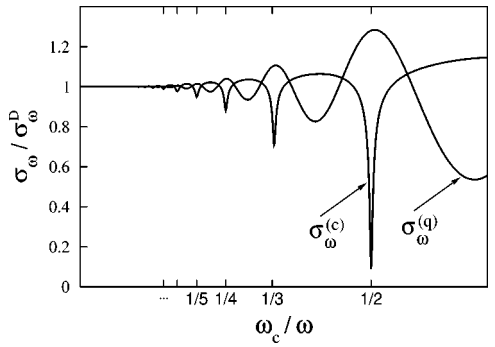


FIG. 3. Quasiclassical [$\sigma_\omega^{(c)}$, Eq. (40)] and quantum [$\sigma_\omega^{(q)}$, Eq. (41)] oscillatory ac conductivity (normalized to the Drude conductivity $\sigma_\omega^{\mathrm{D}}$) vs $\omega_c/\omega$ for $\omega/2\pi=100$ GHz, $\tau=0.6$ ns, $\tau/\tau_q=50$, $\tau_S/\tau_L=0.1$, $a/\delta=0.25$ at $\omega_c/\omega=1/2$.

The behavior of the two contributions to the oscillatory ac conductivity is illustrated in Fig. 3.

## VI. MECHANISMS OF THE OSCILLATORY PHOTOCONDUCTIVITY: QUASICLASSICAL VERSUS QUANTUM

Having found the classical contribution $\Delta\sigma_{\mathrm{ph}}^{(c)}$ to the OPC [Eqs. (31), (36), and (38)], let us compare it with the quantum oscillatory contribution $\Delta\sigma_{\mathrm{ph}}^{(q)}$,[7,8] related to the oscillations of the DOS. Using Eqs. (31), (35), and (38) and omitting numerical factors, we write down the essential factors in $\Delta\sigma_{\mathrm{ph}}^{(c)}$ for $\pi\Gamma \gtrsim \omega_c$ for the case of noninteracting electrons:

$$\Delta\sigma_{\mathrm{ph}}^{(c)} \sim \sigma_0^{\mathrm{D}} \frac{\tau_{\mathrm{in}}}{\tau}\left(\frac{e\mathcal{E}_\omega v_F}{\epsilon_F \omega}\right)^2 \frac{a}{\delta}\cos\frac{2\pi\omega}{\omega_c}\exp\left[-\left(\frac{\omega}{\omega_c}\right)^2 \frac{3\pi}{\omega_c\tau_L}\right]. \tag{42}$$

The sign of $\Delta\sigma_{\mathrm{ph}}^{(c)}$ in Eq. (42) depends on that of $K_0''$ [see Eq. (11)].

As shown in Sec. III, unless the electron-electron interaction is extremely weak, the largest contribution to the smooth part of $\sigma_{\mathrm{ph}}$ comes from the interaction correction to the conductivity. For $T \gtrsim \hbar\omega_c \ln^{1/2}(\omega_c\tau_L)$, the main interaction-induced term in the smooth part of $\sigma_{\mathrm{ph}}$ is given by Eq. (18) and, according to Eq. (31), this yields in turn the main term in the oscillating part $\Delta\sigma_{\mathrm{ph}}^{(c)}$. Setting $\alpha \sim 1$ (long-range Coulomb interaction), we have

$$\Delta\sigma_{\mathrm{ph}}^{(c)} \sim \sigma_0^{\mathrm{D}} \frac{\tau_{\mathrm{in}}}{\tau}\frac{(e\mathcal{E}_\omega v_F/\omega)^2}{\epsilon_F T}\frac{a}{\delta}\cos\frac{2\pi\omega}{\omega_c}\exp\left[-\left(\frac{\omega}{\omega_c}\right)^2 \frac{3\pi}{\omega_c\tau_L}\right], \tag{43}$$

which is larger than the noninteracting part [Eq. (42)] by a factor $\epsilon_F/T$.

For $T \ll \hbar\omega_c$, the main contribution to $\sigma_{\mathrm{ph}}$ is related to the interaction correction given by Eq. (19), which yields the oscillatory part $\Delta\sigma_{\mathrm{ph}}^{(c)}$ similar (in terms of the phase of the oscillations and their damping factor) to that in Eq. (43) but multiplied by a large factor $(\omega_c\tau_L)^2/(T\tau_L/\hbar)^{3/2}$. In the intermediate range of temperature, $\hbar\omega_c \ll T \ll \hbar\omega_c \ln(\omega_c\tau_L)$, there is an exponentially fast crossover between the two regimes. The regime most relevant to the experiments[1–5] is that of high temperature, $T \gtrsim \hbar\omega_c$. It is worth noting that Eqs. (42) and (43) remain valid in the regime of strong heating as well, provided the effective electron temperature $T_e$ [Eq. (9)] is substituted for $T$.

For convenience, we also reproduce here $\Delta\sigma_{\mathrm{ph}}^{(q)}$ in the case of overlapping Landau levels; specifically, for $|\omega \pm \omega_c| \gtrsim \omega_c$ in the regime linear with respect to the microwave power [see Eq. (17) of Ref. 7 or Eq. (8) of Ref. 8; here we omit numerical factors]:

$$\Delta\sigma_{\mathrm{ph}}^{(q)} \sim -\sigma_0^{\mathrm{D}} \frac{\tau_{\mathrm{ee}}}{\tau}\left(\frac{e\mathcal{E}_\omega v_F}{\hbar\omega^2}\right)^2 \frac{\omega}{\omega_c}\sin\frac{2\pi\omega}{\omega_c}\exp\left(-\frac{2\pi}{\omega_c\tau_q}\right). \tag{44}$$

The electron-electron scattering time[8] $\tau_{\mathrm{ee}} \propto T_e^{-2}$ (up to a logarithmic factor) depends on the effective electron temperature $T_e$ [Eq. (9)]. Although both contributions [Eqs. (43) and (44)] have the same period in $\omega/\omega_c$, crucial distinctions are clear.

Firstly, their phases are shifted by $\pi/2$. Secondly, despite both contributions being proportional to a certain inelastic relaxation time, they are different in that the amplitude of $\Delta\sigma_{\mathrm{ph}}^{(q)}$ is limited by $\tau_{\mathrm{ee}}$ (which at low $T$ is much shorter than the electron-phonon scattering time), whereas the classical term is not sensitive to the inelastic electron-electron scattering in any essential way[18] and is proportional to the energy relaxation time [$\tau_{\mathrm{in}}$ in Eqs. (10) and (43)], limited by coupling to the external bath (phonons). It follows that in the limit of small $T$ the ratio of the amplitudes of the OPC, classical-to-quantum, contains a large-$T$-dependent factor $\tau_{\mathrm{in}}/\tau_{\mathrm{ee}}$, which may be easily as large as $10^2$. The sensitivity of $\Delta\sigma_{\mathrm{ph}}^{(q)}$ to electron-electron collisions stems from the fact that the quantum contribution is due to a radiation-induced change of the distribution function $f(\varepsilon)$ that oscillates with *both* $\varepsilon$ and $\omega$. By contrast, the classical contribution $\Delta\sigma_{\mathrm{ph}}^{(c)}$ is associated with an oscillatory term in the characteristic electron temperature; i.e., with a smooth part of $f(\varepsilon)$, which oscillates with $\omega$ only.[21]

Thirdly, the dependences of the envelope of the OPC on $\omega$, $\omega_c$, and the degree of disorder are quite different. The most important point is that although there is a small factor $\propto \epsilon_F^{-1}$ in Eq. (43), in addition to another small factor $a/\delta$, the *damping* of the classical term is much weaker than that of $\Delta\sigma_{\mathrm{ph}}^{(q)} \propto \exp(-2\pi/\omega_c\tau_q)$. Indeed, the exponential damping of $\Delta\sigma_{\mathrm{ph}}^{(c)}$ is governed by $\tau_L$ [Eq. (43)], which is far larger than $\tau_q$ in high-mobility samples. It is only that in the limit of very low $B$ that the $\omega_c^{-3}$ factor in the exponent of Eq. (43) suppresses the classical OPC more effectively than the linear in $\omega_c^{-1}$ Dingle factor in the quantum case.

It is important to stress that the amplitude of the classical OPC in units of the dark conductivity is not large under the conditions of the experiments on the zero-resistance states. Indeed, the pre-exponential factor of Eq. (43) may be written as $\sigma_0^{\mathrm{D}}(\Delta_h^2/\epsilon_F T)(a/\delta)$ for the regime linear with respect to the microwave power. Now, the crossover to the regime of

strong heating occurs when the classical OPC is still small; namely, the ratio $\Delta\sigma_{\rm ph}^{(c)}/\sigma_0^{\rm D}$ is of order $(T/\epsilon_F)(a/\delta)\lesssim 10^{-2}$. This should be contrasted with the quantum OPC, which may become large (and thus lead to the zero-resistance states) when the heating may be still negligible. For the regime of strong heating, when the effective electron temperature $T_e \gtrsim T$, the amplitude of the classical OPC shows a sublinear growth with increasing microwave power and may be estimated as $\sigma_0^{\rm D}(T_e/\epsilon_F)(a/\delta)$. In particular, for the piezoelectric mechanism of the energy relaxation due to electron-phonon coupling, the classical OPC grows as $\mathcal{E}_\omega^{2/5}$ [see the discussion below Eq. (15)]. We conclude that, because of the slow growth with increasing microwave power, the characteristic ratio $\Delta\sigma_{\rm ph}^{(c)}/\sigma_0^{\rm D}$ can hardly exceed the level of a few percent in the current experiments. That is to say, the zero-resistance states are related to the *quantum* OPC. The most favorable conditions for the observation of the classical OPC should be realized if the quantum contribution to the OPC is strongly damped, which means sufficiently large $2\pi/\omega\tau_q \gtrsim 7$. For a high-mobility sample with $\tau_q \sim 3$ ps, this would require $\omega/2\pi \lesssim 50$ GHz.

The above analysis shows that the classical OPC cannot possibly explain the experimentally reported strong deviations of the ratio $\tau_{\rm ph}/\tau_q$ from $1/2$, the value predicted by the theory of the quantum OPC, as discussed in Sec. I. We thus argue that the experiments on the damping of Shubnikov–de Haas oscillations might strongly overestimate the single-particle scattering rate $\tau_q^{-1}$. One of the reasons could be the presence of macroscopic inhomogeneities leading to an inhomogeneous broadening of Landau levels, which might be by far larger than the homogeneous broadening given by $\tau_q^{-1}$ and measured in the photoconductivity experiments (such a possibility was mentioned in Ref. 1). We suggest that measuring the damping of the OPC provides a reliable means of extracting $\tau_q^{-1}$ from the magneto-oscillations, free from the effect of the additional inhomogeneous damping characteristic to the Shubnikov–de Haas measurements. The method based on the OPC is particularly useful in high-mobility samples, where $\tau_q^{-1}$ is small, so that in the conventional Shubnikov–de Haas measurements one has to go to fairly low temperatures to separate the impurity-induced damping from that related to the thermal smearing of the Fermi surface.

## VII. CONCLUSIONS

In summary, we have analyzed the quasiclassical mechanism of magneto-oscillations in the ac conductivity and photoconductivity, related to non-Markovian dynamics of disorder-induced scattering of electrons in high-mobility structures. We have calculated the leading contribution associated with a radiation-induced change of the electron distribution function, which is proportional to the inelastic (electron-phonon) relaxation time. We have found that the quasiclassical oscillations in the photoconductivity are weak under the conditions of current experiments. Therefore, the zero-resistance states and the strong oscillations that have been observed in the experiments are likely due to the quantum mechanism of Refs. 7 and 8. We argue that the damping of the oscillatory photoconductivity provides a reliable method of measuring the homogeneous broadening of Landau levels (single-particle scattering rate $\tau_q^{-1}$) in high-mobility structures (which also resolves the dilemma posed in Sec. I: the analysis of the damping of Shubnikov–de Haas oscillations apparently gives overestimated values of $\tau_q^{-1}$ due to an inhomogeneous broadening).

On the other hand, we have identified a range of parameters within which the quasiclassical mechanism yields oscillations of the photoconductivity that may dominate at *small B* over those based on the Landau quantization. In addition to the different low-$B$ damping factor, the quasiclassical oscillations are shifted in phase by $\pi/2$ with respect to the quantum oscillations [see Eqs. (43) and (44)]. We have also shown that the quasiclassical magneto-oscillations in the *ac conductivity* are much stronger than in the photoconductivity and may easily compete with the quantum oscillations.[7]

## ACKNOWLEDGMENTS

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## APPENDIX: RETURN PROBABILITY IN A MAGNETIC FIELD

The return probability $P_\omega$ [Eq. (28)] can be directly evaluated by using the quasiclassical propagator $D=(L_0+\tau_S^{-1})^{-1}$ [Eq. (26)]. In this appendix, we present a different, more illustrative way to derive $P_\omega$. We recall that the Liouville operator $L_0$ [Eq. (21)] represents the time evolution of the direction of the electron velocity $\mathbf{n}=\mathbf{v}/|\mathbf{v}|=(-\sin\phi,\cos\phi)$ as a combination of the cyclotron motion and the angle diffusion due to scattering off smooth disorder. The random part $\chi$ of the angle $\phi$ is characterized by a white noise spectrum of $\partial_t\chi$:

$$\phi(t) = \phi_0 + \omega_c t + \chi(t),$$

$$\langle \partial_t\chi(t)\partial_{t'}\chi(t')\rangle = \frac{2}{\tau_L}\delta(t-t'). \tag{A1}$$

In what follows we calculate the mean-square fluctuation of the guiding center of cyclotron motion $\delta$, Eq. (32), and mean-square shifts of an electron along and across the cyclotron orbit after $n$ cyclotron periods at $t=nT_c=2\pi\, n/\omega_c$. For definiteness, let the guiding center be initially placed at the origin, $\mathbf{R}(t=0)=(0,0)$, and the electron coordinate and velocity be $\mathbf{r}(t=0)=(R_c,0)$, $\mathbf{v}(t=0)=(0,v_F)$, respectively ($R_c=v_F/\omega_c$ is the cyclotron radius). Using Eq. (A1) we get the mean-square shifts of the position of guiding center $\mathbf{R}$ and the fluctuating angle $\chi$ in time $t=nT_c$:

$$\langle R_x^2\rangle = \left\langle \left( \int_0^{nT_c} \mathrm{d}t R_c \cos\,\phi(t)\partial_t\chi(t) \right)^2 \right\rangle = \frac{2R_c^2}{\tau_L}\int_0^{nT_c} \mathrm{d}t\,\cos^2\phi(t) = R_c^2\frac{nT_c}{\tau_L},$$

$$\langle R_y^2\rangle = \langle R_x^2\rangle,$$

$$\langle \chi^2\rangle = \left\langle \left( \int_0^{nT_c} \mathrm{d}t\ \partial_t\chi(t) \right)^2 \right\rangle = \frac{2\ n\ T_c}{\tau_L}. \qquad \text{(A2)}$$

The diffusion approximation is valid as long as the rms shift of the guiding center after one cyclotron revolution ($n=1$) exceeds the characteristic length scale of the random potential,

$$\delta = [\langle R_x^2\rangle + \langle R_y^2\rangle]^{1/2} = R_c\left(\frac{4\pi}{\omega_c\tau_L}\right)^{1/2} \gg d. \qquad \text{(A3)}$$

In the same manner, we calculate the mean square of electron shifts along and across the cyclotron orbit, $x_\parallel \equiv y(t=n\ T_c) = \int_0^{nT_c}\mathrm{d}t v_F \cos\,\phi(t)$ and $x_\perp \equiv x(t=nT_c) - R_c = -\int_0^{nT_c}\mathrm{d}t\ v_F \times \sin\,\phi(t)$, respectively, as

$$\langle x_\perp^2\rangle = \left\langle \left( \int_0^{nT_c} \mathrm{d}t\ v_F \cos(\omega_c t)\int_0^t \mathrm{d}t'\,\partial_{t'}\chi(t') \right)^2 \right\rangle = \langle R_x^2\rangle,$$

$$\langle x_\parallel^2\rangle = \langle (R_y + R_c\ \chi)^2\rangle = \langle R_y^2\rangle + R_c^2\langle\chi^2\rangle. \qquad \text{(A4)}$$

It follows that fluctuations along the cyclotron orbit are enhanced with respect to those across the orbit, $\langle x_\parallel^2\rangle = 3\langle x_\perp^2\rangle = 3n\delta^2/2$, and we arrive at the anisotropic electron distribution after $n$ cyclotron revolutions, given by

$$p_n(x_\perp, x_\parallel) = \frac{1}{\sqrt{\pi}n\delta}\exp\left(-\frac{x_\perp^2}{n\ \delta^2}\right)\frac{1}{\sqrt{3\pi}\,n\ \delta}\exp\left(-\frac{x_\parallel^2}{3\,n\ \delta^2}\right), \qquad \text{(A5)}$$

which enters Eq. (33) for the return probability.

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[1] M. A. Zudov, R. R. Du, J. A. Simmons, and J. L. Reno, Phys. Rev. B **64**, 201311(R) (2001); P. D. Ye, L. W. Engel, D. C. Tsui, J. A. Simmons, J. R. Wendt, G. A. Vawter, and J. L. Reno, Appl. Phys. Lett. **79**, 2193 (2001).

[2] R. G. Mani, J. H. Smet, K. von Klitzing, V. Narayanamurti, W. B. Johnson, and V. Umansky, Nature (London) **420**, 646 (2002); Phys. Rev. B **69**, 193304 (2004); cond-mat/0306388.

[3] M. A. Zudov, R. R. Du, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **90**, 046807 (2003); C. L. Yang, M. A. Zudov, T. A. Knuuttila, R. R. Du, L. N. Pfeiffer, and K. W. West, *ibid.* **91**, 096803 (2003).

[4] S. I. Dorozhkin, JETP Lett. **77**, 577 (2003).

[5] R. L. Willett, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **93**, 026804 (2004).

[6] A. V. Andreev, I. L. Aleiner, and A. J. Millis, Phys. Rev. Lett. **91**, 056803 (2003).

[7] I. A. Dmitriev, A. D. Mirlin, and D. G. Polyakov, Phys. Rev. Lett. **91**, 226802 (2003).

[8] I. A. Dmitriev, M. G. Vavilov, I. L. Aleiner, A. D. Mirlin, and D. G. Polyakov, cond-mat/0310668.

[9] A. C. Durst, S. Sachdev, N. Read, and S. M. Girvin, Phys. Rev. Lett. **91**, 086803 (2003).

[10] V. I. Ryzhii, Sov. Phys. Solid State **11**, 2078 (1970); V. I. Ryzhii, R. A. Suris, and B. S. Shchamkhalova, Sov. Phys. Semicond. **20**, 1299 (1986).

[11] M. G. Vavilov and I. L. Aleiner, Phys. Rev. B **69**, 035303 (2004).

[12] To avoid confusion, it is worthwhile to note that the Shubnikov–de Haas oscillations of the dark resistivity were damped in Refs. 1–5 mostly by temperature rather than by disorder.

[13] For smooth disorder, manifestations of the memory effects in the magnetoresistance and ac transport were studied in A. D. Mirlin, J. Wilke, F. Evers, D. G. Polyakov, and P. Wölfle, Phys. Rev. Lett. **83**, 2801 (1999); J. Wilke, A. D. Mirlin, D. G. Polyakov, F. Evers, and P. Wölfle, Phys. Rev. B **61**, 13774 (2000). Recently, the role of the memory effects in the photoconductivity, in the case of smooth disorder, was discussed in terms of the influence of microwave radiation on the collision integral in Ref. 11 and in I. L. Aleiner, B. L. Altshuler, and A. V. Andreev (unpublished). However, the memory effects manifest themselves in the photoconductivity much more strongly through a radiation-induced change of the electron distribution function.

[14] A. D. Mirlin, D. G. Polyakov, F. Evers, and P. Wölfle, Phys. Rev. Lett. **87**, 126805 (2001).

[15] V. Umansky, R. de Picciotto, and M. Heiblum, Appl. Phys. Lett. **71**, 683 (1997).

[16] G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **64**, 214204 (2001).

[17] I. V. Gornyi and A. D. Mirlin, Phys. Rev. B **69**, 045313 (2004).

[18] More precisely, the effect of inelastic electron-electron scattering on the OPC is twofold: firstly, it suppresses oscillations of $f(\varepsilon)$ induced by those of the DOS; secondly, it affects the exact shape of the smooth part of $f(\varepsilon)$ by thermalizing electrons among themselves. The former effect leads to the damping of the quantum term $\Delta\sigma_{\mathrm{ph}}^{(q)}$, as discussed in Ref. 8. The latter can only yield a numerical coefficient of order unity in the classical term $\Delta\sigma_{\mathrm{ph}}^{(c)}$, depending on what other mechanisms of inelastic scattering are. Moreover, this numerical factor is 1, so that electron-electron inelastic scattering does not manifest itself in $\Delta\sigma_{\mathrm{ph}}^{(c)}$ at all, if the inelastic coupling to the thermal bath in the presence of a driving force yields $f(\varepsilon)$ whose shape is given by the Fermi distribution (but with an effective temperature different from that of the bath). This is the case, e.g., for the Fokker-Planck mechanism of Eqs. (12) and (15).

[19] Similar oscillations of the ac conductivity were studied for the case of a random antidot array, where the memory effects are still stronger, in D. G. Polyakov, F. Evers, and I. V. Gornyi, Phys. Rev. B **65**, 125326 (2002). The antidot-array model also describes correctly rare strong short-ranged scatterers in the absence of background smooth disorder.

[20] Equation (36) has the same structure as an interaction-induced correction to the tunneling DOS, studied in A. M. Rudin, I. L. Aleiner, and L. I. Glazman, Phys. Rev. Lett. **78**, 709 (1997), which is also proportional to the return probability $P_\omega$.

[21] It is worth noting that there also exists a subleading quantum contribution to the OPC that is due to the oscillatory heating; i.e., due to a smooth part of $f(\varepsilon)$ which oscillates with $\omega$ (because of the oscillatory absorption rate coming in turn from the Landau quantization). It has the same damping factor as in Eq. (44), but the phase of the oscillations as in Eq. (43). This quantum term is, however, much smaller than given by Eq. (44); specifically, by a factor $(\hbar^2\omega\omega_c/\epsilon_F^2)\tau_{\text{in}}/\tau_{\text{ee}} \ll 1$.