Linear magnetic response of disordered metallic rings: Large contribution from forward-scattering interactions

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We calculate the effect of electron-electron interactions involving vanishing momentum transfer (forward scattering) on the orbital linear magnetic response of disordered metal rings pierced by a magnetic flux ϕ. Using the bulk value of the Landau parameter F_0 for copper, we find that in the experiment by Lévy *et al.* [Phys. Rev. Lett. **64**, 2074 (1990)] the forward-scattering contribution to the *linear* magnetic response is larger than the corresponding contribution from large momentum transfers considered by Ambegaokar and Eckern [Phys. Rev. Lett. **65**, 381 (1990)]. However, outside the regime of the validity of the linear response and to first order in the effective screened interaction the persistent current is dominated by scattering processes involving large momentum transfers.

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I. INTRODUCTION

More than a decade ago the measurement by Lévy *et al.*[1] of persistent currents in mesoscopic normal metal rings pierced by an Aharonov-Bohm flux ϕ triggered a lot of theoretical activity.[2,3] Yet, up until now a truly convincing and generally accepted theoretical explanation of the surprisingly large persistent currents observed in Ref. 1 and in subsequent experiments[4,5] has not been found. It has become clear, however, that this effect cannot be explained within a model of noninteracting electrons. Ambegaokar and Eckern[6] (AE) were the first to examine the effect of electron-electron interactions on mesoscopic persistent currents: they realized that, to first order in the screened Coulomb interaction, the dominant contribution to the disorder-averaged persistent current can be obtained from the two diagrams shown in Fig. 1, representing a special correction $\overline{\Omega}_{\rm AE}(\phi)$ to the disorder-averaged thermodynamic potential which depends strongly on the Aharonov-Bohm flux ϕ. Here the overbar denotes averaging over the disorder. The shaded symbols in Fig. 1 denote Cooperon ladders, defined diagrammatically in Fig. 2. Given the grand canonical potential $\Omega(\phi)$, the corresponding persistent current $I(\phi)$ can be obtained from the thermodynamic relation

$$I(\phi) = -c\frac{\partial\Omega(\phi)}{\partial\phi}. \tag{1}$$

In a bulk metal at high densities the bare Coulomb interaction $V_0(\mathbf{q})=4\pi e^2/\mathbf{q}^2$ is strongly screened. A simple way to take the screening into account diagrammatically is the random-phase approximation (RPA). Following this procedure, AE approximated the effective interaction (in the imaginary frequency formalism) as follows:

$$\overline{V}_{\rm RPA}(\mathbf{q},i\omega) = \frac{V_0(\mathbf{q})}{1+\overline{\Pi}_0(\mathbf{q},i\omega)V_0(\mathbf{q})}. \tag{2}$$

For momentum transfers $|\mathbf{q}|$ small compared with the inverse elastic mean free path ℓ^{-1}, and for frequency transfers $|\omega|$ small compared with the inverse elastic lifetime τ^{-1} the disorder-averaged polarization is given by

$$\overline{\Pi}_0(\mathbf{q},i\omega) \approx 2\nu_0\frac{D_0\mathbf{q}^2}{D_0\mathbf{q}^2+|\omega|}, \tag{3}$$

where D_0 is the diffusion coefficient and ν_0 is the average density of states at the Fermi energy (per spin) in the absence of interactions. Note that $\nu_0=(\Delta_0\mathcal{V})^{-1}$, where $\mathcal{V}$ is the volume of the system and Δ_0 is the average level spacing (per spin) at the Fermi energy. It turns out that both diagrams in Fig. 1

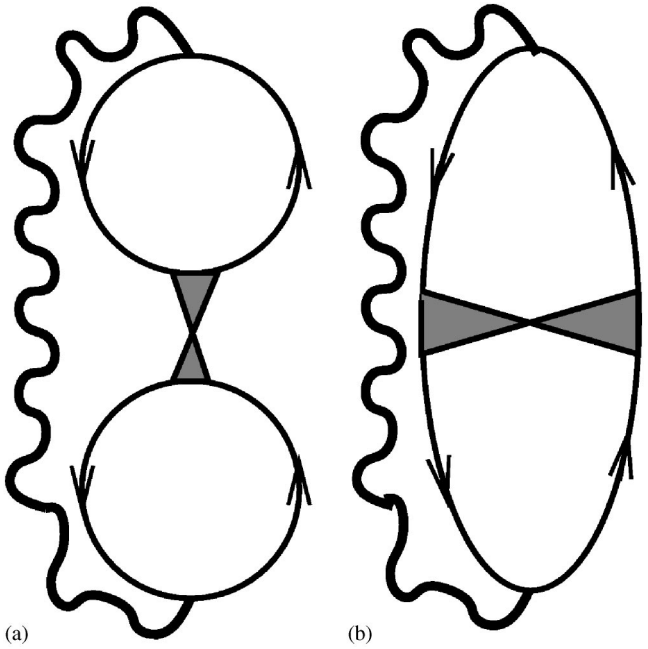

FIG. 1. Feynman diagrams representing the flux-dependent part of the grand canonical potential to first order in the screened interaction. (a) Hartree diagram, (b) Fock diagram. Solid arrows represent noninteracting disorder-averaged Green functions and thick wavy lines represent the effective density-density interaction. The Cooperon (shaded symbol) is defined in Fig. 2.

are dominated by momentum transfers of the order of the Fermi momentum k_F, which for a metallic system is large compared with ℓ^{-1}. Equations (2) and (3) are therefore not suitable for a quantitatively accurate calculation of persistent currents. To make some progress analytically, AE estimated the contribution from the diagrams in Fig. 1 by replacing the effective interaction by a constant:

$$\overline{V}_{\mathrm{RPA}}(\mathbf{k}-\mathbf{k}',i\omega) \to \langle \overline{V}_{\mathrm{RPA}}(\mathbf{k}_F-\mathbf{k}'_F,i0)\rangle \equiv \overline{V}, \tag{4}$$

where $\langle\cdots\rangle$ denotes the Fermi surface average over $\mathbf{k}_F$ and $\mathbf{k}'_F$. For simplicity, it is assumed that the ring is quasi one dimensional, with transverse thickness $L_\perp$ in the range $k_F^{-1} \ll L_\perp \ll \ell \ll L$, where L is the circumference of the ring. Then diffusive motion is only possible along the circumference. At temperature $T=0$ the resulting average persistent current can be written as[6]

$$\overline{I}^{\mathrm{AE}}(\phi) = \sum_{k=1}^{\infty} I_k^{\mathrm{AE}} \sin(4\pi k\phi/\phi_0), \tag{5}$$

where $\phi_0 = hc/e$ is the flux quantum[7] and the Fourier coefficients of the current are

$$I_k^{\mathrm{AE}} = \frac{c}{\phi_0}\frac{16\lambda_c}{k^2} E_c e^{-k\sqrt{\gamma}}[1+k\sqrt{\gamma}]. \tag{6}$$

Here $E_c = \hbar D_0/L^2$ is the Thouless energy and $\gamma = \Gamma/E_c \ll 1$, where at zero temperature $\Gamma = \Delta_0/\pi$ is the cutoff energy that regularizes the singularity in the Cooperon in a finite system;[8] see Eqs. (14) and (20) below. The coupling constant $\lambda_c = \nu_0 \overline{V}$ can be identified with the dimensionless effective interaction in the Cooper channel to first order in perturbation theory. AE estimated $\lambda_c \approx 0.3$, assuming that the validity of the RPA can be extended to momentum transfers of the order of k_F. However, higher-order ladder diagrams in the Cooper channel strongly reduce the effective interaction, so that $\lambda_c \approx 0.06$ is a more realistic estimate[9] for the Cu rings in the experiment.[1]

In real space Eq. (4) amounts to replacing the electron-electron interaction by a local effective density-density interaction:

$$\overline{V}_{\mathrm{eff}}(\mathbf{r}-\mathbf{r}') \to \overline{V}\delta(\mathbf{r}-\mathbf{r}'). \tag{7}$$

More precisely, this replacement means that for distances $|\mathbf{r}-\mathbf{r}'|$ larger than ℓ, the interaction is effectively local. In a recent Letter Schechter *et al.*[10] pointed out that a different type of effective interaction can possibly lead to a much larger persistent current. Specifically, they used the BCS model to calculate the leading interaction correction to the orbital linear magnetic response and found[7,10]

$$\left.\frac{\partial \overline{I}^{\mathrm{BCS}}}{\partial\phi}\right|_{\phi=0} = \frac{c}{\phi_0^2} 32\pi\lambda_{\mathrm{BCS}} E_c \ln\left(\frac{E_{\mathrm{co}}}{\Delta_0}\right), \tag{8}$$

where $\lambda_{\mathrm{BCS}} < 0$ is the attractive dimensionless interaction in the BCS model, and the coherence energy E_{co} is the smaller energy of $\hbar/\tau$ and the Debye energy $\hbar\omega_D$. Equation (8) should be compared with the corresponding result for the local interaction model used by AE, which implies, according to Eqs. (5) and (6),

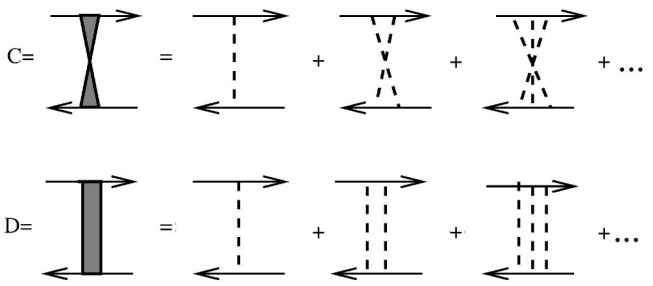

FIG. 2. Diagrammatic definitions of the Cooperon (C) and the diffuson (D). A dashed line represents the covariance of the impurity potential.

$$\left.\frac{\partial \overline{I}^{\mathrm{AE}}}{\partial\phi}\right|_{\phi=0} = \frac{c}{\phi_0^2} 32\pi\lambda_c E_c \ln\left(\frac{E_c}{\Delta_0}\right), \tag{9}$$

where we have used $\Gamma = \Delta_0/\pi$ and retained only the leading logarithmic order. Note that the logarithm is due to the slow decay ($\propto k^{-1}$) of the Fourier coefficients $4\pi k I_k^{\mathrm{AE}}/\phi_0$ of $\partial \overline{I}^{\mathrm{AE}}/\partial\phi$, so that all coefficients with $k \lesssim 1/\sqrt{\gamma}$ contribute to the linear response. For $E_{\mathrm{co}} \gg E_c$ the linear magnetic response in the BCS model is parametrically larger than the linear response in the local interaction model. Whether or not this remains true beyond the linear response has not been clarified. Note also that in the BCS model the linear magnetic response is diamagnetic because the effective interaction is attractive ($\lambda_{\mathrm{BCS}} < 0$), whereas the linear response in the local interaction model is paramagnetic, corresponding to a repulsive effective interaction ($\lambda_c > 0$).

II. MAGNETIC RESPONSE DUE TO FORWARD SCATTERING

An interesting observation made by the authors of Ref. 10 is that an effective interaction different from the local interaction used by AE can lead to a much larger persistent current, at least for sufficiently small flux ϕ, where it is allowed to calculate the current from the linear response. Given the rather crude approximations in the microscopic derivation of the local interaction model, it seems worthwhile to explore the magnetic response for other types of effective interactions. A possibility which so far has not been thoroughly analyzed is an interaction which is dominated by small momentum transfers. Note that the assumption that only forward-scattering processes (corresponding to vanishing momentum transfer) have to be taken into account for a consistent description of the low-energy and long-wavelength properties of normal metals lies at the heart of the Landau's Fermi-liquid theory. The Landau model is in a sense the opposite extreme of the local interaction model, because the effective interaction in the Landau model is proportional to a Kronecker δ in momentum space:

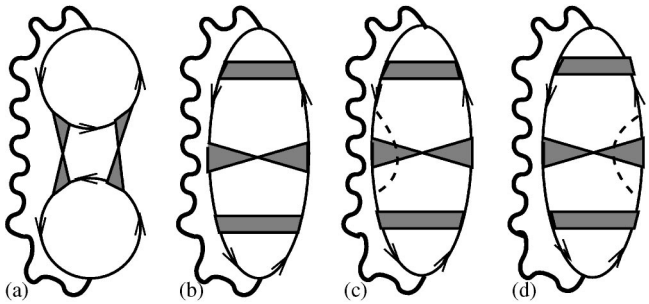

FIG. 3. Feynman diagrams that dominate the flux-dependent part of the grand canonical potential if the effective interaction involves only momentum transfers smaller than the inverse elastic mean free path, $|\mathbf{q}| \lesssim \ell^{-1}$. For vanishing momentum transfer the Hartree diagram (a) dominates the linear magnetic response, whereas outside the regime of validity of the linear response the sum of the three Fock diagrams (b)-(d) has the same order of magnitude as the Hartree diagram (a). Note that the diffuson (shaded box; see Fig. 2) renormalizes only the density vertex in the Fock diagrams; the interaction in the Hartree diagram does not transfer any energy and hence cannot be renormalized by singular diffuson corrections.

$$\overline{V}_{\mathrm{eff}}(\mathbf{q}, i\omega) \to \delta_{\mathbf{q},0} f_0, \tag{10}$$

where the Landau parameter f_0 can be determined from experiments. In fact, the dimensionless Landau parameter[11] $F_0 \equiv 2\nu_0 f_0$ can be written as

$$F_0 = \frac{B}{B_0}\frac{m_*}{m_0} - 1,$$

where B is the bulk modulus, m_* is the effective mass, and B_0 and m_0 are the corresponding quantities in the absence of interactions. Inserting the known bulk values for Cu,[12] $m_*/m \approx 1.3$ and $B/B_0 \approx 2.1$, we find $F_0 \approx 1.7$, which is a factor of 30 larger than the corresponding estimate $\lambda_c \approx 0.06$ in the local interaction model. Note that in real space Eq. (10) corresponds to a constant effective interaction, proportional to the inverse volume of the system:

$$\overline{V}_{\mathrm{eff}}(\mathbf{r} - \mathbf{r}') \to \frac{f_0}{\mathcal{V}}. \tag{11}$$

Given an effective interaction of the form (10), the dominant flux-dependent contributions to the average potential $\overline{\Omega}(\phi)$ to first order in the interaction are shown in Fig. 3. The Fock diagrams (b)–(d) have been discussed previously in Refs. 13 and 14; as first pointed out by Béal-Monod and Montambaux,[13] to leading order in the small parameter $(k_F \ell)^{-1}$, the three Fock diagrams in Figs. 3(b)–3(d) cancel, so that a direct evaluation of the sum of these diagrams is rather difficult. To calculate the leading contribution of these diagrams, we note that the fermion loops in Figs. 3(b)–3(d) can be identified with contributions to the disorder-averaged polarization, which for general frequencies and small wave vectors can be written as[15]

$$\overline{\Pi}_0(\mathbf{q}, i\omega) = 2\nu_0 \frac{D(i\omega)\mathbf{q}^2}{D(i\omega)\mathbf{q}^2 + |\omega|}, \tag{12}$$

where $D(i\omega)$ is a generalized frequency-dependent diffusion coefficient. The crucial observation is now that the sum of the three Fock diagrams in Figs. 3(b)–3(d) corresponds to the usual weak localization correction to the average conductance:[14]

$$D(i\omega) \approx D_0[1 + g_{\mathrm{WL}}(i\omega)], \tag{13}$$

where

$$g_{\mathrm{WL}}(i\omega) = -\frac{2\Delta_0}{\pi}\sum_{\mathbf{q}} \frac{1}{\hbar D_0 \mathbf{q}^2 + |\omega| + \Gamma}. \tag{14}$$

Essentially we have used the equation of continuity to replace the charge vertices in Fig. 3 by current vertices, which cannot be renormalized by singular diffusion corrections. The fact that a gauge transformation replacing charge vertices by current vertices can be used to avoid the explicit calculation of vertex corrections has also been employed in Ref. 16 to calculate the zero-bias anomaly in the tunneling density of states of two-dimensional disordered electrons interacting with Coulomb forces.

The evaluation of the contribution of the three Fock diagrams in Fig. 3 to the persistent current is now straightforward. Note that for a thin ring with $L_\perp \ll \ell \ll L$ the $\mathbf{q}$ summation is one dimensional, with quantized wave vectors $2\pi(n + 2\phi/\phi_0)/L$, $n = 0, \pm 1, \pm 2, \ldots$. Then we obtain for the kth Fourier component of the average current due to the Fock diagrams (b)–(d) in Fig. 3 for the Landau model:[14]

$$I_k^{\mathrm{L,Fock}} \propto k^{-1}\frac{f_0}{\mathcal{V}}. \tag{15}$$

Due to the extra factor of inverse volume, this contribution is, for experimentally relevant parameters,[1] negligible compared with corresponding result in the local interaction model given in Eq. (6).

The Hartree diagram in the Landau model is more interesting. The fact that the diagram with two Cooperons shown in Fig. 3(a) dominates the persistent current due to electron-electron interactions with momentum transfers $|\mathbf{q}| \lesssim \ell^{-1}$ has already been pointed out in Ref. 17. A similar diagram with two Cooperons (but without interaction line) dominates the fluctuations of the number of energy levels in a fixed energy window centered at the Fermi energy.[18] Using the approximate relation

$$I_N(\phi) = -\frac{c}{2}\Delta_0 \frac{\partial(\delta N)^2}{\partial \phi} \tag{16}$$

between the persistent current $I_N(\phi)$ at constant particle number and the fluctuation $(\delta N)^2$ of the particle number at constant chemical potential μ, several authors have realized[19–21] that without interactions the two-Cooperon diagram determines the average persistent current in a canonical ensemble. Note that the Hartree diagram in Fig. 3(a) does not contain any vertex corrections analogous to the diffusion corrections of the vertices in the Fock diagams (b)–(d). This is

due to the fact that the interaction line in the Hartree process does not transfer any energy. Hence, the two Green functions attached to the vertex of a Hartree interaction are either both retarded or both advanced, so that it is impossible to attach a singular diffuson to the vertex.

For the Landau model the Hartree diagram in Fig. 3(a) yields at finite temperature T the following correction to the disorder-averaged grand canonical potential:

$$\overline{\Omega}^{\mathrm{L,Hartree}}(\phi)=\frac{f_0}{2\mathcal{V}}4\sum_{\mathbf{q}}T^2\sum_{\tilde{\omega}_n,\tilde{\omega}_{n'}}\theta(-\tilde{\omega}_n\tilde{\omega}_{n'})\times\left(\frac{\Delta_0}{2\pi}\frac{\hbar}{\tau}\right)^2\left[\frac{\hbar/\tau}{\hbar D_0\mathbf{q}^2+|\tilde{\omega}_n-\tilde{\omega}_{n'}|+\Gamma}\right]^2\times\sum_{\mathbf{k}}[\overline{G}_0(\mathbf{k},i\tilde{\omega}_n)]^2\overline{G}_0(-\mathbf{k}+\mathbf{q},i\tilde{\omega}_{n'})\times\sum_{\mathbf{k}'}[\overline{G}_0(\mathbf{k}',i\tilde{\omega}_{n'})]^2\overline{G}_0(-\mathbf{k}'+\mathbf{q},i\tilde{\omega}_n). \tag{17}$$

Here $\tau=\ell/v_F$ is the elastic lifetime, $\tilde{\omega}_n=2\pi(n+\frac{1}{2})T$ are fermionic Matsubara frequencies, and

$$\overline{G}_0(\mathbf{k},i\tilde{\omega}_n)=\frac{1}{i\tilde{\omega}_n-\frac{\hbar^2\mathbf{k}^2}{2m}+\mu+i\frac{\hbar}{2\tau}\,\mathrm{sgn}\,\tilde{\omega}_n} \tag{18}$$

is the disorder-averaged noninteracting Matsubara Green function. Since the Cooperons (i.e., the second line) in Eq. (17) are only singular for $|\mathbf{q}|\lesssim\ell^{-1}$ and because the $\mathbf{k}$ and $\mathbf{k}'$ sums are dominated by momenta of the order of the Fermi momentum, we may approximate $\overline{G}_0(-\mathbf{k}+\mathbf{q},i\tilde{\omega}_{n'})\approx\overline{G}_0(-\mathbf{k},i\tilde{\omega}_{n'})$ and $\overline{G}_0(-\mathbf{k}'+\mathbf{q},i\tilde{\omega}_n)\approx\overline{G}_0(-\mathbf{k}',i\tilde{\omega}_n)$ in Eq. (17). The product of the last two lines of Eq. (17) gives then rise to a factor of $[(2\pi/\Delta_0)(\tau/\hbar)^2]^2$, so that we obtain

$$\overline{\Omega}^{\mathrm{L,Hartree}}(\phi)=\frac{f_0}{2\mathcal{V}}P(\phi), \tag{19}$$

with the dimensionless coefficient

$$P(\phi)=\frac{4T}{\pi}\sum_{0<\omega_m<\hbar/\tau}\sum_{\mathbf{q}}\frac{\omega_m}{[\hbar D_0\mathbf{q}^2+\omega_m+\Gamma]^2}, \tag{20}$$

where $\omega_m=2\pi mT$ are bosonic Matsubara frequencies. Assuming again a thin ring with $L_\perp\ll\ell\ll L$, we find in the limit $T\to0$ for the Fourier components of the persistent current,

$$I_k^{\mathrm{L,Hartree}}=\frac{16}{\pi}\frac{c}{\phi_0}\frac{f_0}{2\mathcal{V}}e^{-k\sqrt{\gamma}}=\frac{c}{\phi_0}8F_0\frac{\Delta_0}{\pi}e^{-k\sqrt{\gamma}}. \tag{21}$$

Comparing this expression with the corresponding result (6) of the local interaction model, we see that in the Landau model the Fourier components $I_k^{\mathrm{L,Hartree}}$ are independent of k as long as $k\lesssim1/\sqrt{\gamma}$. Therefore the linear magnetic response is determined by all Fourier components up to $k\lesssim\sqrt{E_c/\Gamma}$,

$$\left.\frac{\partial\overline{I}^{\mathrm{L,Hartree}}}{\partial\phi}\right|_{\phi=0}=\frac{4\pi}{\phi_0}\sum_{k=1}^{\infty}kI_k^{\mathrm{L,Hartree}}\approx\frac{c}{\phi_0^2}16\pi F_0E_c, \tag{22}$$

where we have assumed that $\gamma=\Gamma/E_c\ll1$, so that

$$\sum_{k=1}^{\infty}ke^{-k\sqrt{\gamma}}=\frac{e^{\sqrt{\gamma}}}{[1-e^{-\sqrt{\gamma}}]^2}\approx\frac{1}{\gamma}\approx\frac{\pi E_c}{\Delta_0}. \tag{23}$$

Note that the small energy scale Δ_0 of Eq. (21) has disappeared on the right-hand side of Eq. (22) and is replaced by the much larger Thouless energy E_c. Due to the faster decay of the Fourier components (15) of the Fock contribution in the Landau model, the linear response due to the Fock diagrams shown in Figs. 3(b)–3(d) is a factor of $\sqrt{\Gamma/E_c}$ smaller than the corresponding Hartree contribution. Interestingly, the anomalously large linear magnetic response in the BCS model given in Eq. (8) is also dominated by the Hartree process.[10] Thus, the importance of Hartree interactions for persistent currents is to some extent independent of a specific model for the interaction. For the Cu rings used in the experiment[1] we estimate $F_0\approx1.7$, $\lambda_c\approx0.06$, and $E_c/\Delta_0\approx25$; with these values the linear magnetic response due to forward scattering is more than 4 times larger than the linear response in the local interaction model considered by AE.[6] To take both contributions into account one should parametrize the total effective interaction as

$$\overline{V}_{\mathrm{eff}}(\mathbf{r}-\mathbf{r}')=\overline{V}\delta(\mathbf{r}-\mathbf{r}')+\frac{f_0-\overline{V}}{\mathcal{V}}, \tag{24}$$

which in momentum space amounts to

$$\overline{V}_{\mathrm{eff}}(\mathbf{q})=\begin{cases}\overline{V} & \text{for } \mathbf{q}=0,\\ f_0 & \text{for } \mathbf{q}\neq0.\end{cases} \tag{25}$$

Using the estimate for bulk Cu given above, $F_0=2\nu_0f_0\approx1.7$ and $\lambda_c=\nu_0\overline{V}\approx0.06$, we find $f_0/\overline{V}\approx14$, which supports our assumption that there is indeed a strong enhancement of the effective interaction in the forward-scattering channel.

III. CONCLUSIONS

In summary, we have shown that a forward-scattering excess interaction, involving a vanishing momentum transfer, yields the dominant contribution to the *linear* magnetic response of mesoscopic metal rings for experimentally relevant parameters.[1] On the other hand, outside the linear response regime the persistent current is dominated by the term $\overline{V}$ involving large momentum transfers, at least if we use the bulk estimates for $\overline{V}$ and F_0 for Cu. However, for a mesoscopic-disordered Cu ring it is not obvious that the bulk estimates are reliable. Note also that in the bulk the normal Fermi liquid is stable as long as $F_0>-1$, so that for $0>F_0>-1$ the linear magnetic response in the normal state can be diamagnetic, in spite of the fact that the effective coupling λ_c in the Cooper channel is positive. Moreover, in the vicinity of an *s*-wave Pomeranchuk instability,[22,23] where

$F_0<0$ and $|1+F_0|\ll 1$, one should replace F_0 by $F_0/(1+F_0)$. In this case we predict a strongly enhanced diamagnetic linear response. In fact, the forward-scattering channel might then dominate the persistent current even beyond the linear order in the flux ϕ. To clarify this point, a better microscopic theory of the effective electron-electron interaction in mesoscopic-disordered metals is necessary. In particular, a microscopic theory should properly treat the problem of screening in a finite system and incorporate the breakdown of Fermi-liquid theory in quasi-one-dimensional disordered metals at sufficiently low temperatures.[24]

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