Spin depolarization in quantum wires polarized spontaneously in zero magnetic field

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The conditions for a spontaneous spin polarization in a quantum wire positioned in a zero magnetic field are analyzed under weak population of one-dimensional (1D) subbands, which gives rise to the efficient quenching of the kinetic energy by the exchange energy of carriers. The critical linear concentration of carriers above which the quasi-one-dimensional gas undergoes a complete spin depolarization is determined by the Hartree-Fock approximation. The dependence of the critical linear concentration on the carrier's concentration is defined to reveal the interplay of the spin depolarization with the evolution of the "$0.7(2e^2/h)$" feature in the quantum conductance staircase from the e^2/h to $(3/2)(e^2/h)$ values. This dependence is used to study the effect of the hole concentration on the $0.7(2e^2/h)$ feature in the quantum conductance staircase of the quantum wire prepared inside the *p*-type silicon quantum well using the split-gate technique. The 1D channel is demonstrated to be spin-polarized at the linear concentration of holes lower than the critical linear concentration, because the $0.7(2e^2/h)$ feature is close to the value of $0.5(2e^2/h)$ that indicates the spin degeneracy lifting for the first step of the quantum conductance staircase. The $0.7(2e^2/h)$ feature is found to take, however, its normal magnitude when the linear concentration of holes attains the critical value corresponding to the spin depolarization. The variations in the height of the $0.7(2e^2/h)$ feature observed in the hole quantum conductance staircase that is revealed by the p-type silicon quantum wire seem to be related to the evidences of the quantum conductance staircase obtained by varying the concentration of electrons in the 1D channel prepared inside the GaAs-AlGaAs heterojunction [K. S. Pyshkin, C. J. B. Ford, R. H. Harrell, M. Pepper, E. H. Linfield, and D. A. Ritchie, Phys. Rev. B **62**, 15842 (2000)].

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I. INTRODUCTION

Progress in nanotechnology makes it possible to fabricate low-dimensional semiconductor systems with low density of high-mobility charge carriers, which exhibit ballistic behavior under the condition $k_B T\tau/\hbar > 1$, where $\hbar/k_B T$ is the time of electron-electron interaction and $\tau = m^*\mu/e$ is the transport relaxation time.[1–15] In contrast to the diffusion mode $(k_B T\tau/\hbar < 1)$, the role of spin correlations is considerably enhanced in the processes of ballistic transport.[2–6] Among their most dramatic manifestations in the localization and transport processes is the appearance of the "$0.7(2e^2/h)$" feature, which is split off from the first step in the quantum conductance staircase revealed by a one-dimensional (1D) channel.[1,7–9]

The charge transport in such channels that are prepared by the split-gate[10–12] and cleaved edge overgrowth[13] methods is not accompanied with the Joule losses, because their length is less than the mean free path. Therefore, the conductance of the quantum wire that contains a single or several ballistic 1D channels depends only the transmission coefficient T,[14,15] i.e.,

$$G_0 = g_s \frac{e^2}{h} NT, \tag{1}$$

where N denotes the number of the highest occupied 1D subband, which is changed by varying the gate voltage U_g. Furthermore, the dependence $G(U_g)$ represents the quantum conductance staircase, because the conductance of a quantum wire is changed by the value of $g_s e^2/h$ each time when the Fermi level coincides with one of the 1D subbands.[11,12] Spin factor g_s describes the spin degeneration of the wire mode. The value of g_s is equal to 2 for noninteracting fermions if the external magnetic field is absent and becomes unity as a result of Zeeman splitting of a quantum staircase in strong magnetic field. The first step of the quantum conductance staircase has been found, however, to split into two parts even in the absence of external magnetic field.[1,7–9] The height of the substep that is dependent on temperature is usually observed to be about 0.7 of the first step value in a zero magnetic field.

Two experimental observations indicate the importance of the spin component for the behavior of this $0.7(2e^2/h)$ feature. First, the electron g factor was found to increase from 0.4 to 1.3 as the number of occupied 1D subbands decreases.[7] Second, the height of the $0.7(2e^2/h)$ feature attains a value of $0.5(2e^2/h)$ with increasing external magnetic field.[7–9] These results have defined the spontaneous spin polarization of a 1D gas in a zero magnetic field as one of possible mechanisms for the $0.7(2e^2/h)$ feature in spite of the theoretical prediction of a ferromagnetic state instability in ideal 1D systems in the absence of a magnetic field.[16]

The studies of the spontaneous spin polarization in quantum wires that have been carried out recently in frameworks of the Kohn-Sham mean-field approximation with the ultralow linear concentration of charge carriers when the energy of the exchange interaction begins to exceed the kinetic energy in a zero magnetic field allowed to describe qualita-

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tively the quantum conductance staircase of a spin-polarized 1D channel.[17–23] However, the behavior of the $0.7(2e^2/h)$ feature with the onset of spin depolarization that is enhanced by increasing the linear concentration of charge carriers specifically at finite temperatures is still elusive, because the spontaneous spin polarization is analyzed using only numerical calculations, and any analytical expressions of the critical linear concentration corresponding to spin depolarization were not present.[17–19]

Here we use the Hartree-Fock approximation to define analytically the critical linear concentration of carriers above which the spin-polarized quasi-one-dimensional gas undergoes the spin depolarization that seems to be related to the evolution of the height of the $0.7(2e^2/h)$ feature in the quantum conductance staircase. The energies of the completely spin-polarized and unpolarized states of the quasi-one-dimensional gas are analytically calculated and then compared. The completely spin-polarized state is shown to be stable in ultrathin 1D ballistic channels when the linear concentration of carriers is lower than the critical value, above which the quasi-one-dimensional gas undergoes the spin depolarization. This result allows us to explain the variations in the height of the $0.7(2e^2/h)$ feature revealed by the 1D channel prepared inside the GaAs-AlGaAs heterojunction which have been found by varying the concentration of 2D electron gas in leads.[1] These variations are suggested to result from the dependence of the effective mass of carriers on its concentration in two-dimensional semiconductor structures. Finally, the effect of the concentration of the charge carriers on the $0.7(2e^2/h)$ feature is experimentally verified in the studies of the hole quantum conductance staircase revealed by the quantum wire prepared inside the *p*-type silicon quantum well using the split-gate technique.

II. SPONTANEOUS SPIN POLARIZATION IN ONE-DIMENSIONAL SYSTEM IN A ZERO MAGNETIC FIELD

The 1D systems of fermions are described by the Schrödinger equation $H\Psi=E\Psi$ with $H=H_0^{1D}+H_1$, where H_0^{1D} is the Hamiltonian of noninteracting fermions, which depends on the dimensionality of the system under study, and the term H_1 accounts for the interaction of fermions. The form of H_0^{1D} depends on the dimensionality of the system under consideration. Since the motion of charge carriers in 1D systems is quantized in two directions (x,y), the Hamiltonian for noninteracting particles has the form

$$H_0^{1D}=\sum_{j=1}^{N}\left(\frac{p_j^2}{2m}+U(x_j,y_j)\right). \tag{2}$$

The corresponding unperturbed wave functions can be written as

$$\psi_{k,m}(\mathbf{r})=\frac{1}{\sqrt{\Omega_{1D}}}e^{ikz}\varphi_m(\boldsymbol{\rho}), \tag{3}$$

where m is the number of the subband of the quantum confinement in the plane $\mathbf{r}=\mathbf{i}x+\mathbf{j}y$ and Ω_{1D} is the 1D volume that is a quantity with the dimensionality of length.

The interaction operator that is independent of the dimensionality is given by

$$H_1=\frac{1}{2}\sum_{i\neq j}V(|\mathbf{r_i}-\mathbf{r_j}|), \tag{4}$$

where

$$V(|\mathbf{r_i}-\mathbf{r_j}|)=\frac{e^2}{|\mathbf{r_i}-\mathbf{r_j}|}. \tag{5}$$

In the second-quantized representation,

$$H_1=\frac{1}{2}\sum_{i\neq j}\langle KL|V|MQ\rangle c_K^+c_L^+c_Qc_M, \tag{6}$$

where each of the subscripts K, L, M, and Q stand for a particle wave vector, the number of the quantum-confinement subband, and the spin.

If the density of noninteracting carriers is sufficiently low so that only the lowest quantum-confinement subband is occupied, the total energy of the electron gas equals its kinetic energy, and the energy density can easily be calculated both for a 1D gas of charge carriers:

$$\varepsilon_{kin}=\sum_{|k|<k_F}\frac{\hbar^2k^2}{2m}, \tag{7}$$

where k_F is the Fermi wave vector.

Accordingly,

$$\varepsilon_{kin}^{1D}=\sum_{|k|<k_F}g_s\frac{\hbar^2k^2}{2m}=\frac{\hbar^2g_s}{6\pi m}k_F^3=\frac{\pi^2\hbar^2n_{1D}^3}{6mg_s^2}, \tag{8}$$

where the value of k_F is determined from the condition

$$k_F=\frac{\pi}{g_s}n_{1D}. \tag{9}$$

Here n_{1D} is the linear concentration of charge carriers in a 1D gas and g_s is the spin factor, which is equal to the number of electrons per unit cell of phase space. For an unpolarized state, $g_s=2$, whereas for a completely spin-polarized state, $g_s=1$. The values $1<g_s<2$ correspond to the partial spin polarization of charge carriers in a 1D gas.

The spin-polarized state of a 1D gas of noninteracting fermions seems to be unfavorable energetically, because its kinetic energy is always higher than the kinetic energy of an unpolarized state. Therefore the spontaneous spin polarization due to the exchange interaction in quasi-one-dimensional (quasi-1D) systems appears to be unlikely. However, the additional energy term E_1 that can be represented by the following infinite sequence of diagrams

$$\varepsilon_{core}+\varepsilon_{el\text{-}el}= \text{[diagram]}+\text{[diagram]}+\text{[diagram]}+\text{[diagram]}+\text{[diagram]}+\ldots \tag{10}$$

has to be taken into account if a 1D gas consists of interacting particles. The exchange diagrams 3 and 5 are easily seen to be dependent substantially on the spin polarization of the

system. Indeed, the interaction is independent of the spin, which implies spin conservation at the diagram vertices. Thus, only particles with the same spin may be involved in the processes described by the exchange diagrams, the contribution of which is more significant to the spin polarized than to unpolarized systems. Since the contribution from diagram 2 is negative, a spin-polarized state of a 1D gas may be energetically more favorable than an unpolarized state.

We limit our discussion to the first two diagrams, which means that the particle exchange interaction is taken into account within the Hartree-Fock approximation. Thus,

$$E_1 = \frac{1}{2} \sum_{E_K, E_L < E_F} [\langle KL|V|KL\rangle - \langle KL|V|LK\rangle]. \tag{11}$$

Here, the first term is the Hartree correction, whereas the second term is the Fock correction to the exchange energy. The summation is carried out both over spatial and spin variables. The first term diverges in the thermodynamic limit $(N \to \infty, \Omega \to \infty, N/\Omega = n = \text{const})$. This divergence is compensated, however, by the term describing the interaction with a positively charged background. Thus, in the first order, the exchange interaction plays a decisive role. Below, we consider its behavior in a 1D system.

A. Exchange interaction in a quasi-1D system

The matrix element of the exchange interaction for electrons in a quantum wire has the form

$$\begin{aligned}\langle KL|V|LK\rangle &= \frac{e^2}{\Omega_{1D}^2} \int \frac{e^{-ikz'} e^{-ilz''} e^{ilz'} e^{ikz''}}{\sqrt{|\boldsymbol{\rho}' - \boldsymbol{\rho}''|^2 + (z' - z'')^2}} \\ &\quad \times |\varphi(\boldsymbol{\rho}')|^2 |\varphi(\boldsymbol{\rho}'')|^2 d\rho' d\rho'' dz' dz'' \\ &= \frac{e^2}{\Omega_{1D}} \int \frac{e^{-ikz} e^{ilz}}{\sqrt{|\boldsymbol{\rho}' - \boldsymbol{\rho}''|^2 + z^2}} \\ &\quad \times |\varphi(\boldsymbol{\rho}')|^2 |\varphi(\boldsymbol{\rho}'')|^2 d\boldsymbol{\rho}' d\boldsymbol{\rho}'' dz, \end{aligned} \tag{12}$$

where the z coordinate coincides with the wire axis. Here, the transformation of the variables $z = z' - z'', Z = (z' + z'')/2$ was performed with the integral over Z being equal to the sample length Ω_{1D}. Thus, the expression for the exchange interaction energy can be written as

$$\begin{aligned} E_{exc} &= -\frac{1}{2} \sum_{K, L < k_f} \langle KL|V|LK\rangle \\ &= -g_s \frac{e^2 \Omega_{2D}}{2(2\pi)^2} \int_{-k_F}^{k_F} e^{ikz} dk \int_{-k_F}^{k_F} e^{ilz} dl \\ &\quad \times \int \frac{|\varphi(\boldsymbol{\rho}')|^2 |\varphi(\boldsymbol{\rho}'')|^2}{\sqrt{|\boldsymbol{\rho}' - \boldsymbol{\rho}''|^2 + z^2}} d\boldsymbol{\rho}' d\boldsymbol{\rho}'' dz \\ &= -g_s \frac{e^2 \Omega_{1D}}{2\pi^2} \int \frac{\sin^2(k_F z)}{z^2 \sqrt{|\boldsymbol{\rho}' - \boldsymbol{\rho}''|^2 + z^2}} dz \\ &\quad \times \int |\varphi(\boldsymbol{\rho}')|^2 |\varphi(\boldsymbol{\rho}'')|^2 d\boldsymbol{\rho}' d\boldsymbol{\rho}''. \end{aligned} \tag{13}$$

Subsequent substitution, $u = z/|\boldsymbol{\rho}' - \boldsymbol{\rho}''|$ and $\alpha = k_F|\boldsymbol{\rho}' - \boldsymbol{\rho}''|$, in the integral over z results in the following expression for the density of the exchange interaction energy in a quasi-1D system:

$$\varepsilon_{exc} = E_{exc}/\Omega_{1D} = -g_s \frac{e^2}{2\pi^2} \int \frac{|\varphi(\boldsymbol{\rho}')|^2 |\varphi(\boldsymbol{\rho}'')|^2}{|\boldsymbol{\rho}' - \boldsymbol{\rho}''|^2} I(\alpha) d\boldsymbol{\rho}' d\boldsymbol{\rho}''. \tag{14}$$

Here,

$$I(\alpha) = \int_{-\infty}^{+\infty} \frac{\sin^2(\alpha u)}{u^2 \sqrt{1 + u^2}} du.$$

In the limit of the low linear concentration of the charge carriers, when

$$k_F R \ll 1$$

and

$$\alpha = k_F |\boldsymbol{\rho}' - \boldsymbol{\rho}''| \ll 1, \tag{15}$$

where R is the width of the quantum wire, this integral is estimated as

$$I(\alpha) \approx \alpha^2 \left(-\frac{1}{2} \ln \alpha + \frac{3}{4} - \frac{C}{2} \right). \tag{16}$$

Here, C is the Euler constant $(C \approx 0.5772)$. Thus, we obtain the following expression for the density of the exchange interaction energy as a function of the concentration of the charge carriers in a 1D system:

$$\begin{aligned} \varepsilon_{exc} &\approx -g_s \frac{e^2 k_F^2}{2\pi^2} \int |\varphi(\boldsymbol{\rho}')|^2 |\varphi(\boldsymbol{\rho}'')|^2 \\ &\quad \times \left[-\frac{1}{2} \ln(k_F |\boldsymbol{\rho}' - \boldsymbol{\rho}''|) + \frac{3}{4} - \frac{C}{2} \right] d\boldsymbol{\rho}' d\boldsymbol{\rho}'' \\ &\approx -\frac{\beta_{1D}}{g_s} n_{1D}^2 + \frac{\gamma_{1D}}{g_s} n_{1D}^2 \ln\left(\frac{n_{1D} R}{\pi g_s} \right). \end{aligned} \tag{17}$$

Here,

$$\beta_{1D} = e^2 \left(\frac{3}{8} - \frac{C}{4} \right) \approx 0.28 e^2,$$

$$\gamma_{1D} = \frac{e^2}{4}.$$

It should be noted that the expression obtained is an approximation and independent of the model of the quantum wire used.

Since this expression is obtained in the limit of the low linear concentration of charge carriers, when $k_F R \ll 1$, the logarithmic factor in the second term should be noted to be negative that results in the corresponding negative exchange energy correction.

The opposite limiting case when

$$k_F R \sim \alpha \gg 1 \tag{18}$$

is also followed to be analyzed. The integral $I(\alpha)$ is estimated in frameworks of this limit as

$$I(\alpha) = \int_{-\infty}^{+\infty} \frac{\sin^2(\alpha u)}{u^2\sqrt{1+u^2}} du \approx A\alpha,$$

$$A = \int_{-\infty}^{+\infty} \frac{\sin^2(t)}{t^2} dt \approx 3.1375. \tag{19}$$

Then, the linear density of the exchange energy

$$\varepsilon_{exc} = E_{exc}/\Omega_{1D} = -\frac{g_s e^2}{2\pi^2}\int \frac{|\varphi(\boldsymbol{\rho}')|^2|\varphi(\boldsymbol{\rho}'')|^2}{|\boldsymbol{\rho}'-\boldsymbol{\rho}''|^2} I(\alpha) d\boldsymbol{\rho}' d\boldsymbol{\rho}''$$
$$\approx \frac{Ae^2}{2\pi^2} k_F = \chi_{1D} n_{1D},$$

$$\chi_{1D} \approx \frac{e^2}{2\pi}\int \frac{|\varphi(\boldsymbol{\rho}')|^2|\varphi(\boldsymbol{\rho}'')|^2}{|\boldsymbol{\rho}'-\boldsymbol{\rho}''|^2} d\boldsymbol{\rho}' d\boldsymbol{\rho}'', \tag{20}$$

is independent of the spin factor.

B. Spontaneous spin polarization due to the exchange interaction energy exceeding the kinetic energy

To answer the question of whether the exchange interaction may result in the appearance of a spontaneous spin polarization, we have to compare the total energies of spin-polarized and unpolarized states of a quasi-1D gas of charge carriers. In the limit of the low charge carrier concentration [see Eqs. (8) and (17)], the energy density of a quasi-1D gas equals

$$\varepsilon^{1D} = \varepsilon_{kin} + \varepsilon_{exc},$$

$$\varepsilon^{1D} = \frac{\pi^2\hbar^2 n_{1D}^3}{6mg_s^2} - \frac{n_{1D}^2}{g_s}\left[\beta_{1D} - \gamma_{1D}\ln\left(\frac{n_{1D}R}{\pi g_s}\right)\right]. \tag{21}$$

Here, the first and the second terms correspond to the kinetic and the exchange interaction energies, respectively. Since the approach of the infinitely long quantum wire is used, the Hartree term is by itself infinite [see Eq. (11)], but is completely canceled by positive background according to the general theorem.

If the linear concentration n_{1D} is less than a critical value n_0 that results from the equality of the energies of the spin-polarized (g_s=1) and unpolarized (g_s=2) states,

$$\varepsilon|_{g_s=1} = \varepsilon|_{g_s=2},$$

$$\frac{3\pi^2\hbar^2 n_0}{12m} = \beta_{1D} - \gamma_{1D}\ln\left(\frac{2n_0R}{\pi}\right), \tag{22}$$

the exchange interaction energy exceeds the kinetic energy and, thus, the spin-polarized state is energetically more favorable than the unpolarized state. At the same time, if the linear concentration of the charge carriers exceeds the critical value n_0 and the kinetic energy is dominant, the unpolarized state is energetically more favorable. The value of n_0 should be noted to be dependent only on the width of the quantum wire and the effective mass that appears to be a function of the concentration of electrons[24,25] and holes[26–32] in low-dimensional systems.

Two features of the discussed mechanism of a spontaneous spin polarization in low-dimensional semiconductor systems have to be taken into account. First, the conditions for the appearance of ferromagnetic ordering in quasi-1D systems in the limit of low linear concentration of charge carriers are actually reduced to those obtained for strictly 1D systems, in frameworks of the inclusion of correlation corrections that does not destroy the stability of the ferromagnetic state due to the exchange interaction.[17–19] Second, the correlation energy is also taken into account to determine the linear concentration value corresponding to the onset of the Wigner crystallization that competes strongly with the transition to a spontaneously spin-polarized state with extended wave functions.[20,33–36] However, the Wigner crystallization should take place for $r_s \geqslant 39$,[35] while a spontaneous spin polarization appears at r_s=3.3,[20] where r_s is the ratio of the potential to the kinetic energy. Thus, the transition to the crystalline state in quasi-1D systems occurs at the concentration of carriers that is two to four orders of magnitude lower than the value corresponding to the transition to a spontaneously spin-polarized state with extended wave functions. The spontaneous spin-polarized state with extended wave functions seems to be expected in a quantum wire at higher values of n_{2D} than in a 2D gas of charge carriers because of an additional partial decay of the kinetic energy with a reduction in the system dimensionality.[20]

We stress once more that this consideration corresponds to the limit of low linear concentration of charge carriers, $k_FR \ll 1$, for quasi-1D systems. This circumstance imposes serious restrictions on the width of quantum wires. If this condition is not satisfied, the system should be considered in the limiting case of high charge-carrier concentration, $k_FR \gg 1$, for quasi-1D systems when an unpolarized state is always energetically more favorable than a spin-polarized state, because the exchange energy becomes independent of g_s. Besides, all of the preceding is valid only when the wave function corresponding to the motion along the quantum wire can be present in the context of a plane wave that imposes the following condition: $L \gg R$, where L is the length of the quantum wire. To put it in another way, the mechanisms of the $0.7(2e^2/h)$ feature that are caused by the Kondo effect in the process of the transport through the quantum point contact are outside the province of this consideration.[37]

III. SPIN DEPOLARIZATION AND QUENCHING OF THE $0.7(2e^2/h)$ FEATURE IN THE QUANTUM CONDUCTANCE STAIRCASE OF A QUANTUM WIRE

A. Spin depolarization of electrons in the GaAs based quantum wires

The spin-polarized state in low-density quasi-1D gas appears to be revealed in the studies of the quantum conductance staircase. Since the exchange interaction gives rise to

the energy gap between states with the spin parallel and antiparallel to the magnetization of the quantum wire, the carriers of the first type pass ballistically through the quantum wire, while the potential barrier is formed for the carriers with opposite spin direction. The height of this energy barrier seems to be estimated using the relationship for the energy density of a quasi-1D gas (21):

$$E_{\uparrow} - E_{\downarrow} = 2n_{1D}\left[\left(\beta_{1D} - \frac{\gamma_{1D}}{2}\right) - \gamma_{1D}\ \ln\left(\frac{n_{1D}R}{\pi}\right)\right] - \frac{\pi^2\hbar^2 n_{1D}^2}{2m}. \tag{23}$$

Thus, the probability of passing through the quantum wire is sufficiently suppressed for the carriers with the spin antiparallel to the magnetization thereby forming a spin island inside ballistic channel. As the carriers with only one spin projection cause the transport processes, the value of the first step in the quantum conductance staircase is less than $2e^2/h$, being e^2/h in the case of full polarization of the 1D gas in the quantum wire. However, the variations in the split-gate voltage appear to increase the linear concentration of carriers above critical value corresponding to the spin depolarization of the 1D gas that results in the suppression of the potential barrier revealed by the standard value of the quantum conductance step, $2e^2/h$. We would like to stress that the consideration present above was analytically performed, whereas previous data obtained in the studies of the spin properties of the quantum point contacts were based on the numerical calculations.[17–19,38] Nevertheless, the results of both approaches agree with each other rather well.

The critical linear density n_0 corresponding to a complete spin depolarization in a quantum wire connecting two 2D reservoirs, which is given by Eq. (23), depends upon both the width of the quantum wire and the effective mass that increases as the value of n_{2D} decreases.[24–26] Such behavior of the effective mass for electrons specifically in the GaAs based quantum wires seems to be caused by its energy dependence that was calculated in the case when the kinetic energy and the quantum-confinement energy are dominant in low-dimensional semiconductor systems:[39,40]

$$m = m_0(1 + 1.447E + 0.245E^2). \tag{24}$$

Here, E is the sum of the kinetic and quantum-confinement energies and the coefficients of E account for the band parameters of GaAs.

The electron effective mass value has been found to increase by a factor of 1.1–1.2 that is due to a rise in quantum-confinement energy as the width of the quantum well decreases below 10 nm.[39–41] The electron effective mass is of importance to rise as the value of n_{2D} increases in the GaAs based quantum wells, even though the exchange energy that compensates the kinetic energy is taken into account in Eq. (24). However, the exchange interaction may significantly affect the effective mass of charge carriers in quantum wires, because the spin-polarized states with extended wave functions in a quasi-1D system are spontaneously formed at higher values of n_{2D} or p_{2D} than in a 2D gas.[20] Therefore, the kinetic energy is effectively quenched in the middle part of a

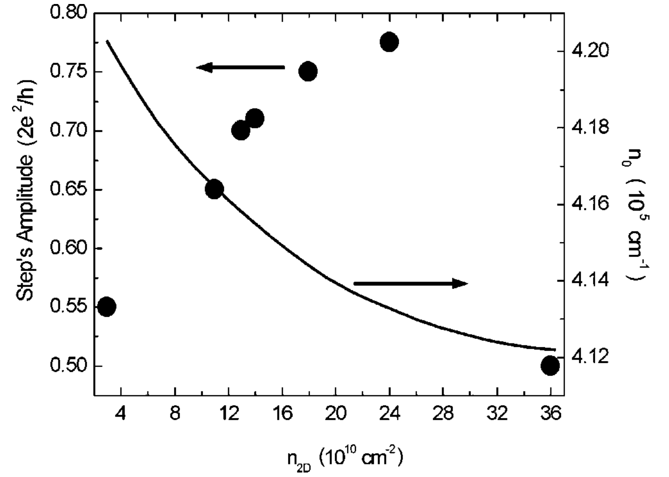

FIG. 1. Dependence of the critical linear concentration corresponding to a complete spin depolarization of the quasi-1D electron gas in a quantum wire connecting 2D reservoirs in a GaAs/GaAlAs QW on the concentration of electrons; R=100 nm, d=20 nm. Circles indicate the height of the $0.7(2e^2/h)$ feature determined in the studies of the 1D channels prepared by split-gate method in GaAs/GaAlAs QWs (Refs. 1 and 7–9).

quantum wire that connects two 2D reservoirs, because a competition with the exchange energy is available, which may favor a reduction in the effective mass with increasing the values of n_{2D} or p_{2D} [see Eqs. (22) and (24)]. Such a gain in the exchange interaction may account for a rise in the effective mass of electrons as their concentration decreases in quantum wells,[25] because the low density 2D gas is able to decay in the system of two-dimensional lakes connected by quantum wires or quantum point contacts, which result from specifically the presence of disorder.[42]

The dependences of the electron effective mass in the GaAs based quantum wells on the value of n_{2D} that were calculated when the exchange energy has been taken into account in the relationship (24) allowed to determine the values of the critical linear concentration n_0 [see Eq. (22)], which corresponds to a complete spin depolarization of electrons in the quantum wire connecting two 2D GaAs reservoirs (Fig. 1). Here, these dependences of n_0 on the value of n_{2D} are used in analysis of the $0.7(2e^2/h)$ feature in the quantum conductance staircase as a function of the electron concentration in the quantum wire prepared in the GaAs based quantum well by the split-gate method, with the electron sheet density tuned controllably over one order of magnitude by biasing an overall top gate[1] (Fig. 1).

The $0.7(2e^2/h)$ feature is seen to attain almost the value of $0.5(2e^2/h)$ at sufficiently small values of n_{2D}. Thus, spin degeneracy of the substep in the quantum conductance staircase is lifted, when the 1D channel is completely spin polarized. However, if the electron concentration in the 2D reservoir attains the value corresponding to the critical linear concentration in the 1D channel, n_0, the $0.7(2e^2/h)$ feature evolves towards its normal value because of spin depolarization. Besides, the apparent level-off of the $0.7(2e^2/h)$ feature near the value of $0.75(2e^2/h)$ appears to be due to its temperature dependence, which results from partial spin depolar-

ization of the electron gas near the bottom of the 1D subband at finite temperatures.[23]

A most interesting result is the unexpected transformation of the $0.7(2e^2/h)$ feature to the value of $0.5(2e^2/h)$ with a subsequent increase in the electron sheet density (see Fig. 1). This recreation of the $0.5(2e^2/h)$ value seems to be caused by also the spin polarization in the quantum wire, which originates probably from the lowest 1D subband that is magnetically ordered by the indirect exchange via electrons excited to the upper subband at a finite temperature. Indirect-exchange mechanisms that cause such nonequilibrium spin polarization in a 1D channel are most probably related to the processes of spin-correlated transport within a narrow band[43,44] and spin polarization due to the formation of spin polarons.[35] Finally, the $0.5(2e^2/h)$ substep is of importance to be observable readily in the quantum wires with a higher level of disorder,[45] which is the evidence of the indirect exchange in lifting the spin degeneracy of the $0.7(2e^2/h)$ feature in the quantum conductance staircase at large values of n_{2D}.

B. Spin depolarization of holes in the Si based quantum wires

Studies of the quantum conductance staircase revealed by ballistic channels have shown that the $0.7(2e^2/h)$ feature is observed not only in various types of the electron GaAs based quantum wires,[7–13,46–48] but also in the hole Si based quantum wires.[49–51] The latter findings were made possible by the developments of the diffusion nanotechnology that allows us to fabricate the ultranarrow silicon quantum wells of the p-type on the n-type Si(100) surface, which are located between the δ barriers heavily doped with boron.[49,52,53]

The angular dependencies of the cyclotron resonance spectra and the conductivity have demonstrated that the self-assembled silicon quantum well (SQW) prepared by short-time diffusion of boron contain the high-mobility 2D hole gas with long transport relaxation time of heavy and light holes at 3.8 K, $\tau \geqslant (5\times10^{-10})c$.[52–56] Thus, the transport relaxation time of holes in SQW appeared to be longer than in the best metal oxide-semiconductor structures,[2] contrary to what might be expected from strong scattering by the δ barriers. This passive role of the δ barriers between which the SQW is formed was quite surprising, when one takes into account the level of their boron doping, $\approx 10^{21}$ cm^{-3}.[49–53] To eliminate this contradiction, the temperature dependencies of the conductivity and the Seebeck coefficient as well as the electron paramagnetic resonance (EPR) spectra and the local tunneling current-voltage characteristics have been studied.[49,52–57] The δ barriers heavily doped with boron appeared to consist of the trigonal dipole centers that seem to result from the negative-U reaction, $2B^0 \rightarrow B^- + B^+$, which define their ferroelectric properties responsible for the suppression of backscattering in the SQW.[52–57] Therefore even with small drain-source voltage the electrostatic ordering within the ferroelectric δ barriers is able to stabilize the formation of the one-dimensional subbands, when the quantum wires are created inside SQW using the split-gate technique. Thus, the preparation of the narrow p-type SQW confined by the δ barriers with ferroelectric properties that quench even short-range scattering potential made it possible to use the split-gate constriction to study the quantum conductance staircase of holes at the temperature of 77 K.[49–51,58]

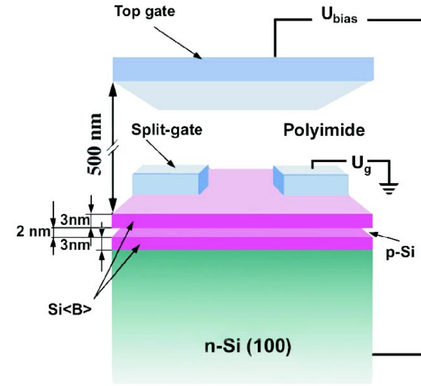

FIG. 2. Schematic diagram of the device that demonstrates a perspective view of the p-type silicon quantum well located between the δ barriers heavily doped with boron on the n-type Si(100) substrate as well as the top gate and the depletion regions created by split-gate method, which indicate a D channel connecting two 2D reservoirs.

The device applied here to analyze the dependence of the $0.7(2e^2/h)$ feature on the concentration of holes in the Si based quantum wires has been first advanced in Ref. 1 (see Fig. 2). The basis of this sample is the self-assembled SQW of the p-type that was formed between δ barriers by the short-time diffusion of boron from the gas phase into the n-type Si(100) surface. The parameters of the SQW that contains the high-mobility 2D hole gas were defined by the, scanning tunnel microscope cyclotron resonance, and EPR methods. The sheet density of holes was found to tune controllably over one order of magnitude, from 5×10^{12} m^{-2} to 9×10^{13} m^{-2}, by biasing the top gate above a layer of insulator, which fulfills the application of the p^+-n bias voltage. The variations in the mobility measured at 3.8 K that correspond to this range of the values of p_{2D} appeared to occur between 80 and 420 m^2/V s. Thus, the mobility of holes remains high even at low densities. Besides, the high value of mobility appeared to decrease no more than two times in range of temperatures from 3.8 K to 77 K that seems to be caused by both the ferroelectric properties for the δ barriers and the electric field of the p^+-n junction.[49,52,56,57] These parameters of 2D hole gas allowed us to study the quantum staircase revealed by the heavy holes at 77 K. The experiments were provided by the effective 1D channel length, 0.2 μm, and the QW cross section, 2×2 nm^2, which is determined by the SQW width and the lateral confinement due to ferroelectric properties for the δ barriers. The number of the highest occupied mode of the short quantum wire inserted in the right side arm was controlled by varying the split-gate voltage (U_g) (see Fig. 2).

Figure 3 shows the quantum staircase revealed by the heavy holes in the 1D channel defined by the split-gate voltage inside the SQW provided that the p^+-n bias voltage is kept to be zero, which appeared to result in the value of p_{2D} equal to 4×10^{13} m^{-2}. Under these conditions, the $0.7(2e^2/h)$

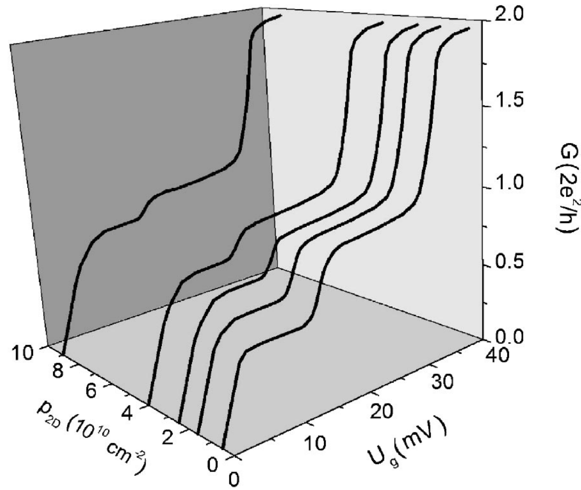

FIG. 3. The quantum conductance staircase of the silicon quantum wire as a function of the sheet density of holes that was tuned controllably by biasing the top gate, which fulfills the application of the p^{+}-n bias voltage to the p-type silicon quantum well on the n-type Si(100) surface. The p^{+}-n bias voltage is varied from the forward branch to the reverse branch between +110 mV and −120 mV, which establishes the range of the magnitude of p_{2D} from 5×10^{12} m^{-2} to 9×10^{13} m^{-2} provided that zero p^{+}-n bias voltage results in the value of p_{2D} equal to 4×10^{13} m^{-2}.

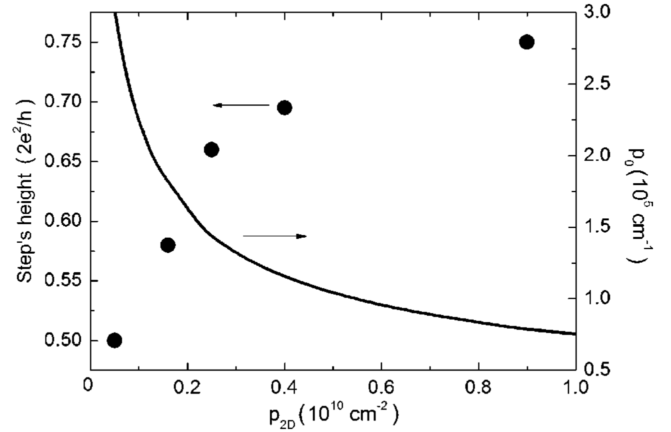

FIG. 4. Dependence of the critical linear concentration corresponding to a complete spin depolarization of the quasi-1D hole gas in a quantum wire connecting 2D reservoirs in a silicon quantum well on the concentration of holes; $R=2$ nm, $d=2$ nm. Circles indicate the values of the height of the $0.7(2e^2/h)$ feature that are shown in Fig. 1.

feature is seen to be coincident practically with its normal value. Tuning the value of p_{2D} by biasing the top gate causes, however, the variations in the height of this feature (Fig. 3). The value of $0.5(2e^2/h)$ is found under forward p^{+}-n bias voltage, whereas at large values of p_{2D} induced by reverse p^{+}-n bias voltage the $0.7(2e^2/h)$ feature attains the value of $0.75(2e^2/h)$. The height of the $0.7(2e^2/h)$ feature studied as a function of the hole concentration in the p-type silicon SQW is worthwhile to be related to the behavior of the critical linear concentration, p_0 [see Eq. (22)], which was calculated by extrapolation from the known dependence of the hole effective mass in the p-type silicon quantum wells on the value of p_{2D},[27–32] with the exchange energy that compensates the kinetic energy (Fig. 4). The model of a square quantum wire with hard walls connecting 2D reservoirs in a silicon quantum well of the p-type, $R=2$ nm, $d=2$ nm, was a basis of these calculations. Thus, the variations in the height of the $0.7(2e^2/h)$ feature that result from controllable tuning the value of p_{2D} can be perceived as a result of partial spin depolarization of holes, which is enhanced as the critical linear concentration in the 1D channel, p_0, is approached.

Finally, the behavior of the $0.7(2e^2/h)$ feature in the quantum conductance staircase shares a common trait related to the critical linear concentration of holes and electrons that corresponds to their complete spin depolarization in the 1D channels prepared, respectively, inside the p-type Si based quantum well studied in this work and inside the n-type GaAs based quantum well discussed above.[1]

IV. CONCLUSIONS

Analysis of the conditions for the appearance of a spontaneous spin polarization in one-dimensional systems placed in a zero magnetic field, which has been carried out within the Hartree-Fock approximation, enabled us to determine the critical concentration of carriers that defines a complete spin depolarization of a quasi-1D gas. The range of the linear concentration of carriers that imposes the restrictions to use the plane waves in the studies of the ferromagnetic ordering in one-dimensional systems has been evaluated in the case of dominance of the exchange energy over the kinetic energy.

The transition of a quasi-1D gas to the crystalline state has been demonstrated to occur at the concentration of carriers that is two to four orders of magnitude lower than those corresponding to the transition to a spontaneously spin-polarized state with extended wave functions. The spontaneous spin-polarized state with extended wave functions seems to be expected in a quantum wire at higher values of n_{2D} than in a 2D gas of charge carriers because of an additional partial decay of the kinetic energy with a reduction in the system dimensionality.

The dependence of the critical linear concentration that defines a complete spin depolarization in a 1D channel connecting two 2D reservoirs on the carrier's concentration has been derived to analyze the corresponding evolution of the $0.7(2e^2/h)$ feature from the e^2/h to $3/2(e^2/h)$ values in the quantum conductance staircase of the quantum wire prepared inside the p-type silicon quantum well using the split-gate technique. The 1D channel studied seems to be spin-polarized at the linear concentration of holes lower than the critical linear concentration, because the $0.7(2e^2/h)$ feature is close to the value $0.5(2e^2/h)$ that indicates the spin degeneracy lifting for the first step of the quantum conductance staircase. The $0.7(2e^2/h)$ feature has been found, however, to tend to the value of $0.75(2e^2/h)$ when the linear concentration of holes attains the critical value corresponding to the spin depolarization.

The variations in the height of the $0.7(2e^2/h)$ feature observed in the hole quantum conductance staircase of the *p*-type silicon quantum wire seem to be related to the evidences of the quantum conductance staircase obtained by varying the concentration of electrons in the 1D channel prepared inside the GaAs-AlGaAs heterojunction.[1]

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