Unified theory for magnetic and electric field coupling in multistacked Josephson junctions

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We develop a theoretical framework for multistacked Josephson junctions. The electric field coupling is newly formulated by a semimiscroscopic treatment for the electric field penetration into the superconducting layer, and a unified equation containing both the magnetic and the electric field coupling is derived. From the equation, we obtain analytical frequency dispersions for collective plasma waves as a function of the in-plane wave number, and we find two unique features in the dispersions, i.e., the electric field coupling hardens the Josephson plasma with a decrease in the superconducting layer thickness and causes the dispersion curves to cross at small in-plane wave number regimes. The latter one means that mode conversions between different plasma waves occur.

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Since the discovery of intrinsic Josephson effects in layered high-T_c superconductors,[1] multistacked Josephson junctions have attracted much interest. Consequently, very rich features in these systems have been clarified,[2] and new ideas toward device applications have been suggested.[3]

In multistacked systems, periodical alternation of material properties strongly affects wave propagations along the stacked direction. In plasma physics, this kind of issue, i.e., the electromagnetic and the electrostatic wave propagation inside spatially varing media, is a topic of broad interest,[4] and the advancement reveals that mode conversions between the electrostatic and the electromagnetic waves occur. This idea is also of importance in stacked Josephson junctions due to the relevance to the electromagnetic wave radiation and absorption. In this paper, in order to examine the possibility of mode conversion in stacked Josephson junctions, we make a unified description for both the electrostatic and the electromagnetic dynamics and clarify the propagation mode profiles.

Several studies on intrinsic Josephson junctions have clarified two novel types of couplings which relate with the penetration of the magnetic field[5] and the electric field[6,7] into each superconducting layer. Here, we note that there is an imbalance between the two theoretical developments. Sakai, Bodin, and Pedersen proposed the magnetic field coupling between neighbor junctions[5] in stacked junctions with any thickness of the superconducting layer, while the authors in Refs. 6–8 formulated the electric field coupling only in the zero thickness limit with a viewpoint that internal field distribution inside the superoconducting layer is negligible. However, in reality, the superconducting layer thickness is comparable to or larger than the electric field penetration depth in intrinsic Josephson junctions, and the internal field distribution makes an important contribution in evaluating the coupling. In this paper, therefore, we seriously consider the electric field penetration at the superconducting layer interface and construct a unified framework for both of the couplings[9] in the treatment, taking into account the finite thickness of the superconducting layer. Such a study, to our knowledge, is the first one. Moreover, we start the theoretical framework in a semimicroscopic way, i.e., the Schrödinger equation for the macroscopic superfluid wave function, and we formulate the electric field coupling through the wave function's dynamics. It is the most straightforward and comprehensive way.

Up to now, several numerical works[10] have been done on the so-called coupled sine-Gordon equation derived by Sakai *et al.*[5] and Bulaevskii *et al.*[11] in order to study Josephson vortex dynamics and related electromagnetic wave radiation.[12] As a result, rich dynamical phases depending on the vortex velocity due to resonances with multiple plasma waves have been revealed,[10] and the presence of the in-phase-like vortex flow suggesting *superradiance* has been predicted.[13,14] However, in these works, influences from the electric field coupling have been neglected except for a few specialized numerical works,[15,16] and the energy transfer between the electrostatic and the electromagnetic components in excited waves has never been examined, although they are quite important themes in the laboratory and the space plasma physics.[4] In this paper, we study the unified model and give a theoretical perspective to the role of the electric field coupling.

First, let us study the electric field penetration into superconducting electrodes in a single Josephson junction. A schematic view for scalar, chemical, and electrochemical potential distribution is displayed in Fig. 1. The dynamics of the macroscopic wave function in this system are well described by

$$\left(\frac{\partial}{\partial t}+\frac{2i\psi_i}{\hbar}\right)\Psi_i=0,\ i=1,2, \tag{1}$$

where Ψ_i and ψ_i are the macroscopic wave function and the electrochemical potential of ith superconductors, respectively. ψ_i is decomposed into a sum of the scalar potential φ_i

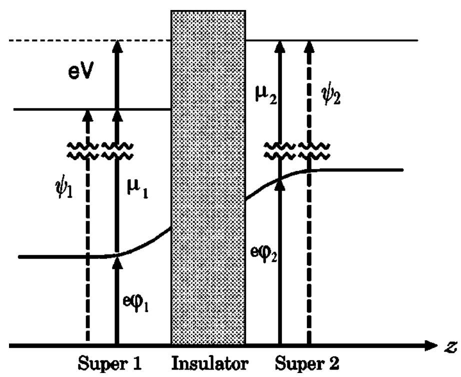

FIG. 1. A schematic figure for a distribution of the electrochemical potential ψ_i $(i=1,2)$, the chemical potential μ_i, and the scalar potential φ_i under a gauge $dA_z/dt=0$ in the single Josephson junction system, in which the energy difference eV exists between two superconducting electrodes.

and the chemical potential μ_i as $\psi_i = e\varphi_i + \mu_i$. From Eq. (1), the time development of the phase γ_i of $\Psi_i(\equiv|\Psi_0|e^{i\gamma_i})$ is given as

$$\frac{\partial \gamma_i}{\partial t} = -\frac{2\psi_i}{\hbar}. \quad (2)$$

Here, we note that the electrochemical potential can be treated as a constant value everywhere inside each superconductor electrode as shown in Fig. 1. The reason is explained as follows. ψ_i must satisfy

$$\frac{1}{e}\frac{\partial \psi_i}{\partial z} = -\frac{m}{2e^2 n_{s,i}}\frac{\partial j^z_{s,i}}{\partial t} - \frac{1}{c}\frac{\partial A_{z,i}}{\partial t}.$$

The first term on the right-hand side is negligibly small on both the time and the current scales of the Josephson effect. Thus, spatially constant ψ_i is automatically assured under a gauge condition $\partial A_z/\partial t=0$. On the other hand, the chemical and the scalar potential can vary at the interface region and the electric field consequently penetrates into the superconducting electrode. Let us formulate how the electric field penetrates with variations of φ and μ. The phase difference between two arbitrary points (z_1 and z_2) in the two superconductors written as $\phi_{2,1} \equiv \gamma_2 - \gamma_1 - (2\pi/\phi_0)\int_{z_1}^{z_2} A_z dz$ (Ref. 17) follows the equation

$$\frac{\partial \phi_{2,1}}{\partial t} = \frac{2}{\hbar}\left(\delta\mu_1(z_1) - \delta\mu_2(z_2) + e\int_{z_1}^{z_2} E_z dz\right) = \frac{2e}{\hbar}V, \quad (3)$$

where we define $\delta\mu_2(z) \equiv \mu_2(z) - \mu_2(\infty)$ and $\delta\mu_1(z) \equiv \mu_1(z) - \mu_1(-\infty)$ and, for simplicity, assume the same material electrode, i.e., $\mu_1(-\infty) = \mu_2(\infty)$. The charge density induced by the chemical potential shift $\delta\mu_i$ is given as

$$\rho_i(z) = 2eN_0\delta\mu_i(z), \quad (4)$$

where N_0 is the density of states at Fermi energy. By combining Eqs. (3) and (4), the Josephson relation is found to be the following one:

$$\frac{\partial \phi_{2,1}}{\partial t} = \frac{2}{\hbar}\left(\rho_1(z_1)/2eN_0 - \rho_2(z_2)/2eN_0 + e\int_{z_1}^{z_2} E_z dz\right). \quad (5)$$

This is an exact Josephson relation which holds for any Josephson junction systems. In addition, we note that the relation is more general, e.g., it is applicable for the relation between the voltage and the time variation of the superconducting phase induced by the flux motion. Now, the charge density ρ_i that emerged in the relation (5) is connected to E_z by the Poisson equation as

$$\frac{\partial}{\partial z}E_z = \frac{4\pi}{\epsilon_i}\rho_i, \quad (6)$$

where ϵ_i is the medium dielectric constant of the ith superconducting electrode. On the other hand, a combination of the expressions (2) and (4) gives the charge density as

$$\rho_i = -\frac{\epsilon_i}{4\pi\lambda_{e,i}^2}\left(\varphi_i + \frac{\phi_0}{2\pi c}\frac{\partial \gamma_i}{\partial t}\right), \quad (7)$$

where $\lambda_{e,i}[\equiv(\epsilon_i/8\pi e^2 N_{0,i})^{1/2}]$ is the Debye screening length.[7,8]. The above expression (7) without ϵ_i was *a priori* given in Ref. 6, but it is found to be a direct consequence of Eq. (1) with expression (4) in the present case. By taking the gradient (z) on Eq. (6) with Eq. (7), we have the following equation:

$$\frac{\partial^2 E_{z,i}}{\partial z^2} = \frac{1}{\lambda_{e,i}{}^2}E_{z,i}. \quad (8)$$

Here, exactly speaking, $-(1/\lambda_{e,i}^2)(m/2e^2 n_{s,i})(\partial j^z_{s,i}/\partial t)$ should be added to the right-hand side of Eq. (8), but this term can be neglected because the electrochemical potential is assumed to be constant inside the superconducting electrode. One finds that Eq. (8) is mathematically equivalent with the London equation describing the screening effect for the magnetic field,

$$\frac{\partial^2 B_{y,i}}{\partial z^2} = \frac{1}{\lambda_{L,i}{}^2}B_{y,i}, \quad (9)$$

where λ_L is the magnetic London penetration depth. Thus, this equivalency enables us to make a unified formalism by recycling the manner employed in the magnetic field coupling.[5]

Next, let us turn to the multistacked Josephson junction systems. The phase difference between the ith and $(i+1)$th superconductors, $\phi_{i,i-1} = \gamma_i - \gamma_{i-1} - (2\pi/\phi_0)\int_{z_{i-1}}^{z_i} A_z dz$ $(i=1,\ldots,N)$, satisfies the following relations including the space and the time derivatives by using $\lambda_{L,i}$ and $\lambda_{e,i}$:

$$\frac{\phi_0}{2\pi}\frac{\partial \phi_{i,i-1}}{\partial x} = \frac{4\pi\lambda_{L,i}^2}{c}j_i - \frac{4\pi\lambda_{L,i-1}^2}{c}j_{i-1} + \int_{z_{i-1}}^{z_i} B_y dz, \quad (10)$$

$$\frac{\phi_0}{2\pi c}\frac{\partial \phi_{i,i-1}}{\partial t} = -\frac{4\pi\lambda_{e,i}^2}{\epsilon_i}\rho_i + \frac{4\pi\lambda_{e,i-1}^2}{{}^\wedge\epsilon_{i-1}}\rho_{i-1} + \int_{z_{i-1}}^{z_i} E_z dz, \tag{11}$$

where j_i is the in-plane current. On the other hand, Eqs. (8) and (9) can give the in-plane parallel current and the charge density at the surface in the ith superconducting film with thickness t_i.[5] Here, we show only ρ_i^D and ρ_i^U, i.e., the charge density in the upside and the downside surface of the ith superconducting layer as follows:

$$\rho_i^D = \frac{-\epsilon_i}{4\pi\lambda_{e,i}}\frac{E_{i,i-1}\cosh(t_i/\lambda_{e,i}) - E_{i+1,i}}{\sinh(t_i/\lambda_{e,i})}, \tag{12}$$

$$\rho_i^U = \frac{\epsilon_i}{4\pi\lambda_{e,i}}\frac{E_{i+1,i}\cosh(t_i/\lambda_{e,i}) - E_{i,i-1}}{\sinh(t_i/\lambda_{e,i})}, \tag{13}$$

where $E_{i,i-1}$ is the electric field inside the insulating layer between the $(i-1)$th and ith superconducting layers and t_i is the superconducting layer thickness. Here, it is found that expanding $\sinh(t_i/\lambda_{e,i})$ and $\cosh(t_i/\lambda_{e,i})$ by $t_i/\lambda_{e,i}$ in the above equations (12) and (13) simplifies them into the following equation:

$$\frac{4\pi}{\epsilon_i}t_i\rho_i = E_{i,i-1} - E_{i+1,i}, \tag{14}$$

where ρ_i is assumed to be homogeneous inside the superconducting layer. Starting with the above simplified Poisson equation (14), we can reach the electric field coupling formalism in the zero thickness limit.[6–8] However, we note that the above expansion is valid only for $t_i \ll \lambda_{e,i}$ but t_i is comparable to $\lambda_{e,i}$ or t_i is rather larger even in intrinsic Josephson junctions. Furthermore, we find that the formalism in the zero thickness limit does not distinguish between the upside and the downside surface in a superconducting layer at all. Such a treatment is an oversimplification, especially for cases with $t_i > \lambda_e$. From these expressions (12) and (13), the time and the space derivatives of the phase difference $\phi_{i,i-1}$ between the upper surface of the $(i-1)$th superconducting layer and the lower surface of the ith one are written as

$$\frac{\phi_0}{2\pi}\frac{\partial \phi_{i,i-1}}{\partial x} = D_{i,i-1}^L B_{i,i-1} + s_i^L B_{i+1,i} + s_{i-1}^L B_{i-1,i-2}, \tag{15}$$

$$\frac{\phi_0}{2\pi c}\frac{\partial \phi_{i,i-1}}{\partial t} = D_{i,i-1}^C E_{i,i-1} + s_i^C E_{i+1,i} + s_{i-1}^C E_{i-1,i-2}, \tag{16}$$

where

$$D_{i,i-1}^L = D_{i,i-1} + \lambda_{L,i}\coth\left(\frac{t_i}{\lambda_{L,i}}\right) + \lambda_{L,i-1}\coth\left(\frac{t_{i-1}}{\lambda_{L,i-1}}\right), \tag{17}$$

$$D_{i,i-1}^C = D_{i,i-1} + \lambda_{e,i}\coth\left(\frac{t_i}{\lambda_{e,i}}\right) + \lambda_{e,i-1}\coth\left(\frac{t_{i-1}}{\lambda_{e,i-1}}\right), \tag{18}$$

$$s_i^L = -\frac{\lambda_{L,i}}{\sinh(t_i/\lambda_{L,i})},\; s_i^C = -\frac{\lambda_{e,i}}{\sinh(t_i/\lambda_{e,i})}, \tag{19}$$

where $D_{i,i-1}$ is the thickness of the insulating layer, and the superscripts L and C stand for the relevance to the magnetic and the electric field coupling, respectively. Here, we would like to note that Eq. (16) coincides with the expression by Koyama and Tachiki [Eq. (10) in Ref. 6] with the approximation as $\sinh(t_i/\lambda_{e,i}) \sim t_i/\lambda_{e,i}$ if ϵ_i is dropped in expression (7).

By combining these above equations with the following Maxwell equation:

$$\frac{\partial B_{i,i-1}^y}{\partial x} = \frac{\epsilon_{i,i-1}}{c}\frac{\partial E_{i,i-1}^y}{\partial t} + \frac{4\pi}{c}j_{i,i-1}^z, \tag{20}$$

where $\epsilon_{i,i-1}$ is the dielectric constant of the insulating layer between the ith and $(i-1)$th superconducting layers, we have an equation for an array of the phase difference $\hat{\phi}$, $\hat{\phi}^t \equiv (\cdots\phi_{i,i-1}\cdots)$ as

$$\frac{\phi_0}{2\pi}\frac{\partial^2}{\partial x^2}\hat{\phi} = \mathbf{L}\left(\frac{\epsilon\phi_0}{2\pi c^2}\mathbf{C}^{-1}\frac{\partial^2}{\partial t^2}\hat{\phi} + \frac{2\phi_0\hat{\sigma}}{c^2}\mathbf{C}^{-1}\frac{\partial}{\partial t}\hat{\phi} + \frac{4\pi}{c}\hat{j}_c\sin\hat{\phi}\right), \tag{21}$$

where $\hat{\sigma}$ and $\hat{j}_c$ are arrays giving the site-dependent quasiparticle conductivity and critical current density, and the matrices $\mathbf{L}$ and $\mathbf{C}$ are, respectively, described as

$$\mathbf{L} = \begin{pmatrix} D_{1,0}^L & s_1^L & & & & \mathbf{0} \\ s_1^L & D_{2,1}^L & s_2^L & & & \\ & \cdots & \cdots & & & \\ & & \cdots & \cdots & & \\ & & & & D_{N-1,N-2}^L & s_{N-1}^L \\ \mathbf{0} & & & & s_{N-1}^L & D_{N,N-1}^L \end{pmatrix}$$

and

$$\mathbf{C} = \begin{pmatrix} D_{1,0}^C & s_1^C & & & & \mathbf{0} \\ s_1^C & D_{2,1}^C & s_2^C & & & \\ & \cdots & \cdots & & & \\ & & \cdots & \cdots & & \\ & & & & D_{N-1,N-2}^C & s_{N-1}^C \\ \mathbf{0} & & & & s_{N-1}^C & D_{N,N-1}^C \end{pmatrix}.$$

In the following, we focus on a case in which all of the superconducting and the insulating layers are equivalent, i.e., s_i^L, s_i^C, $D_{i,i-1}^L$, and $D_{i,i-1}^C$ are replaced by site-independent constants. The case is realized in the intrinsic Josephson junctions. Then, we have a covenient form for further calculations as

$$\mathbf{C}\frac{\phi_0}{2\pi}\frac{\partial^2}{\partial x^2}\hat{\phi} = \mathbf{L}\frac{\epsilon\phi_0}{2\pi c^2}\frac{\partial^2}{\partial t^2}\hat{\phi} + \mathbf{L}\frac{2\phi_0\sigma}{c^2}\frac{\partial}{\partial t}\hat{\phi} + \mathbf{LC}\frac{4\pi j_c}{c}\sin\hat{\phi}, \tag{22}$$

where we use a relation $\mathbf{LC}=\mathbf{CL}$ which holds only under the above special condition.

Let us demonstrate how Eq. (22) contains the electric and magnetic field coupling models suggested previously. First, we examine a case in which $\hat{\phi}$ is spatially independent in the in-plane direction, i.e., $\hat{\phi}(x',t')=\hat{\phi}(t')$. Then, the equation is reduced to the following one:

$$\frac{\epsilon}{c^2}\frac{\partial^2}{\partial t^2}\hat{\phi}+\frac{4\pi\sigma}{c^2}\frac{\partial}{\partial t}\hat{\phi}+\frac{\mathbf{C}}{D\lambda_c{}^2}\sin\hat{\phi}=0, \tag{23}$$

where $\lambda_c \equiv \sqrt{c\phi_0/8\pi^2 D j_c}$. This equation is equivalent with the electric field coupling model[6–8] in the equation form. However, we claim that the present model, Eq. (23), is more general since it is applicable for stacked junction systems with any superconducting layer thickness. Second, we drop the electric field coupling, i.e., $\mathbf{C}$ is replaced by the unit matrix $\mathbf{I}$. Then, we have the coupled sine-Gordon equation obtained by Sakai *et al.* as follows:[5]

$$\frac{\partial^2}{\partial x^2}\hat{\phi}-\mathbf{L}\frac{\epsilon}{c^2}\frac{\partial^2}{\partial t^2}\hat{\phi}-\mathbf{L}\frac{4\pi\sigma}{c^2}\frac{\partial}{\partial t}\hat{\phi}-\frac{\mathbf{L}}{D\lambda_c{}^2}\sin\hat{\phi}=0. \tag{24}$$

Now, let us study collective plasma modes based on Eq. (22). In this paper, we concentrate on the plasma modes under the zero field. The excitation modes under the presence of Josephson vortices will be discussed elsewhere. We find that the dispersion relations of the plasma excitation modes in an N junction stacked system are analytically given as

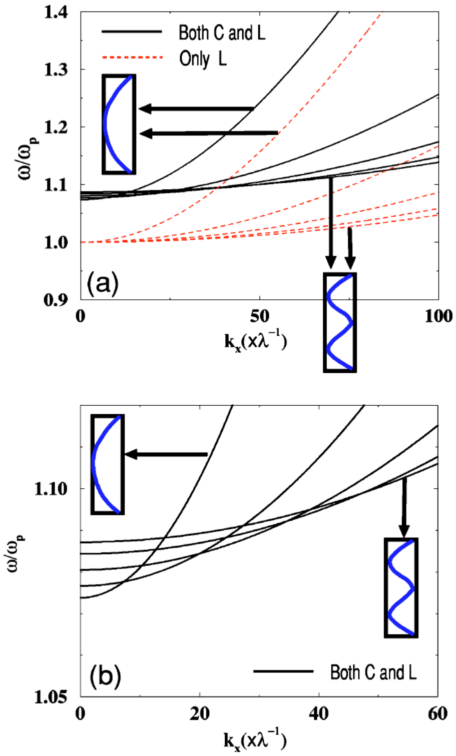

FIG. 2. (Color online) (a) The frequency vs the in-plane wave number (k_x) of the collective plasma modes in the unified model (22) (both C and L) and in the magnetic field coupling only (only L). The mode profiles along the stacked direction are schematically sketched in the insets. The used parameters D, λ_e, λ_c, and t are assumed to be 13 Å, 1 Å, 125 μm, and 2 Å, respectively. These values follow an intrinsic Josephson junction, i.e., $Bi_2Sr_2CaCu_2O_8$. (b) The enlarged picture for the frequency vs k_x in the smaller k_x region. Only the case considering both couplings is presented, and the mode crossing region is displayed.

$$\omega=\omega_p\sqrt{\left[\frac{D^C}{D}+2\frac{s^C}{D}\cos\left(\frac{\ell\pi}{N+1}\right)\right]\left(1+\frac{Dk_x{}^2\lambda_c^2}{D^L+2s^L\cos\left(\frac{\ell\pi}{N+1}\right)}\right)}(\ell=1\sim N), \tag{25}$$

where k_x is the in-plane wave vector, $\omega_p(\equiv c/\sqrt{\epsilon}\lambda_c)$ is the Josephson plasma frequency, and ℓ stands for the ℓth eigenmode whose eigenvector is as follows:

$$A^{\ell}_{i,i-1}=\sqrt{\frac{1}{N+1}}\sin\left(\frac{i\ell\pi}{N+1}\right). \tag{26}$$

In the relation (25), the presence of the electric field coupling gives the frequency splitting in $k_x=0$ by the mode difference along the stacked direction, while if it is not present, then $(D^C/D)+2(s^C/D)\cos[\ell\pi/(N+1)]\to 1$ and all dispersions converge into ω_p in $k_x=0$. These features are presented in Figs. 2(a) and 2(b), which shows the dispersion curves $[\omega(k_x)]$ in a five junction stacked system whose parameters correspond to $Bi_2Sr_2CaCu_2O_8$ for two cases of both couplings (the real five lines) and the magnetic field coupling only (the dotted five lines). This frequency splitting at $k_x=0$ in the case of both couplings [see Fig. 2(b)] clearly suggests that the longitudinal plasma mode is dispersive due to the electric field coupling. Also, we notice that the inclusion of the electric field coupling shifts up all dispersions to the high-frequency side, as seen in Fig. 2(a). The hardening reflects that the electric field penetration into the superconducting layer effectively increases the insulator thickness D [see also Eqs. (18) and (25)]. In fact, the plasma excitation mode frequency is rewritten as

$$\omega = \overline{\omega_p}\sqrt{\left[1+2\frac{s^C}{D^C}\cos\left(\frac{\ell\pi}{N+1}\right)\right]\left(1+\frac{Dk_x{}^2\lambda_c^2}{D^L+2s^L\cos\left(\frac{\ell\pi}{N+1}\right)}\right)}(\ell = 1 \sim N), \tag{27}$$

where $\overline{\omega_p}(\equiv \omega_p\sqrt{D^C/D}) = \sqrt{8\pi^2 cD^C j_c \epsilon\phi_0}$. This clearly indicates that the Josepshon plasma frequency itself is renormalized by $\sqrt{D^C/D}$ compared to the conventional expression, and means that the plasma frequency is dependent on how the electric field penetrates into the superconducting layer. We note that the effect is not included in the thin limit theory[6–8] because the thin limit theory does not care about how the electric field penetrates and therefore the plasma frequency is always given by $\omega_p = (\equiv c/\sqrt{\epsilon}\lambda_c)$. Thus, the present theory is found to be essentially different from the thin limit theory. In addition, we find that the splitting at $k_x = 0$ and the hardening become remarkable as t is comparable to λ_e or decreases below λ_e, i.e., the case of intrinsic Josephson junctions.

Now, let us focus on two points in the dispersion relations, i.e., $k_x \to 0$ and $k_x \to \infty$. First, when $k_x \to 0$, we find that the frequency of the out-of-phase mode ($\ell = N$) is larger than that of the in-phase mode ($\ell = 1$) in contrast to no mode difference without the electric field coupling. On the other hand, at $k_x \to \infty$ the magnetic field coupling surpasses the electric field one, and it raises up the frequency of the in-phase-like mode contrary to $k_x = 0$. Thus, the frequency order dependent on the mode profile is found to invert at small k_x. This causes the dispersion lines to cross at finite k_x as seen in Fig. 2(b), while the dispersion in the large k_x region, i.e., mode frequencies excited under the high field, is very similar to that without the electric field coupling. In real systems, the former crossings result in the mode conversion between different plasma modes and induce energy flows from electrostatic oscillations to electromagnetic waves. This nature enables us to excite the longitudinal plasma by the irradiation of the electromagnetic wave and to radiate the electromagnetic wave from the longitudinal plasma oscillation. In fact, such mode conversion processes occur in experimental situations as the microwave absorption and the radiation of the electromagnetic wave.

In conclusion, we formulated the electric field penetration into the superconducting layer based on the semimicroscopic method and derived the unified equation applicable for any stacked Josephson junctions. Furthermore, we gave the analytic form of the frequency dispersions of the collective plasma waves as a function of the in-plane wave number and found that the Josephson plasma hardens as the superconducting layer thickness decreases and the dispersion curves cross with increasing the in-plane wave number. The present model covers a rich variety of electromagnetic dynamics in stacked Josephson junctions.

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[17] In the conventional theory, z_1 and z_2 in the phase difference $\phi_{2,1}(z_1,z_2)$ are implicitly assumed to be the inside of the superconducting electrode where the supercurrent decays enough. However, we find that $\phi_{2,1}(z_1,z_2)$ is almost independent of the choice of z_1 and z_2 even though they are close to the interfaces because, e.g., a different choice of z_2 gives $\phi_{2,1}(z_1,z_2') - \phi_{2,1}(z_1,z_2) = (2\pi c/\phi_0)(m/2e^2 n_s)\int_{z_2}^{z_2'} j_s^z dz$, whose right-hand side is within an order of 10^{-5} and much smaller compared to the notable phase difference in the Josepshon effects.