

Coexistence of spin-density wave and d -wave superconducting order parameter

Zaira Nazario¹ and David I. Santiago^{1,2}¹*Department of Physics, Stanford University, Stanford, California 94305, USA*²*Gravity Probe B Relativity Mission, Stanford, California 94305, USA*

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We study the properties of a spin-density-wave antiferromagnetic mean-field ground state with d -wave superconducting correlations. This ground state always gains energy by Cooper pairing. It would fail to superconduct at half-filling due to the antiferromagnetic gap although its particle-like excitations would be Bogolyubov-BCS quasiparticles consisting of coherent mixtures of electrons and holes. More interesting and relevant to the superconducting cuprates is the case when antiferromagnetic order is turned on weakly on top of the superconductivity. This would correspond to the onset of antiferromagnetism at a critical doping. In such a case a small gap proportional to the weak antiferromagnetic gap opens up for nodal quasiparticles, and the quasiparticle peak would be discernible. We evaluate numerically the absorption by nodal quasiparticles and the local density of states for several ground states with antiferromagnetic and d -wave superconducting correlations.

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I. INTRODUCTION

Ever since the discovery of high temperature superconductivity,¹ it was proposed that the superconducting correlations might already exist in the antiferromagnetic Mott insulator.² The origin of the superconducting correlations was ascribed to the large Coulombic interactions in the undoped materials. The only other large energy scale in the materials is phononic.³

While the microscopic origin of superconductivity remains a matter of debate,⁴⁻⁷ there is growing experimental evidence that the quasiparticles are Bogolyubov-BCS quasiparticles. Bending back of photoemission bands⁸ fits quantitatively the BCS-Bogolyubov model.^{9,10} Scanning tunneling microscopy finds coherent quasiparticles that disperse as a coherent mixture of particles and holes.^{11,12} The particle and hole amplitudes in these experiments and in inverse photoemission experiments^{12,13} fit accurately to the theoretical Bogolyubov-BCS values calculated from the dispersion and gap measured in the normal and superconducting materials, respectively.

Regardless of whether the origin of superconducting correlations is exotic Coulombic physics or some more conventional mechanism, it is clear that the cuprates are BCS paired superconductors. This does not mean that the Coulomb interactions do not matter. Rather, the interesting and contradictory physics for underdoped materials is the result of Coulomb degradation of the superfluid density^{2,4-6} and order parameter competition between superconductivity and correlated electron ground states.^{14,15} The degradation of the superfluid density leads to suppressed T_c due to a phase instability of the superconducting order parameter.^{2,16-19} There are several Coulomb stabilized competing ground states such as orbital antiferromagnetism,¹⁵ stripe or charge density wave ground states,²⁰ and perhaps electronic liquid crystal phases.²¹ Regardless of which of these competing ground states are realized, there is strong experimental evidence for

incommensurate electronic ordering,²² either static or incipient. The evidence seems more consistent with charge-density-wave or stripe order.

In the present work we will study the physics of an antiferromagnet with a strong d -wave Cooper pairing interaction. We do not speculate as to the origin of this superconducting interaction except to point out that in such a model it competes with the Coulombic antiferromagnetic physics. Both the superconductor and the antiferromagnet are studied in the mean field approximation. While one can doubt the validity of such an approximation at a phase transition point, it will be qualitatively correct within the ordered phases.

Cooper pairing leading to a BCS ground state is an instability of a Fermi liquid ground state. In this study we apply the BCS approximation to a spin density wave (SDW) insulating ground state as it exists in the cuprates at half filling. The resulting ground state *has Cooper pairing yet it fails to superconduct* due to the SDW insulating gap. Next we will review some well known facts in order to understand how a state with Cooper pairs does not superconduct. Before doing so we emphasize that this only happens as a consequence of having a *completely filled* insulating band.

When an electric field is applied to a metal, it conducts dissipatively. The way this happens is that the center of mass of the Fermi sea gets displaced upward in the unfilled metallic band.²³ Ohmic dissipation occurs because newly filled electronic states at the top of the Fermi sea get scattered into newly empty electronic states at the bottom of the Fermi sea due to the lack of rigidity of the Fermi liquid ground state (see Fig. 1). When there are Cooper correlations, the electron liquid gets displaced upward in the band too, but as long as the displacement in energy within the band is less than the superconducting gap, Cooper pair correlations make the electron liquid rigid, thus preventing scattering and dissipation. For the case of a SDW ground state at half filling with Cooper pairing correlations there is no superconductivity as the electron fluid *cannot* move upward in the band for the band is full and *there are no electronic states* to be filled

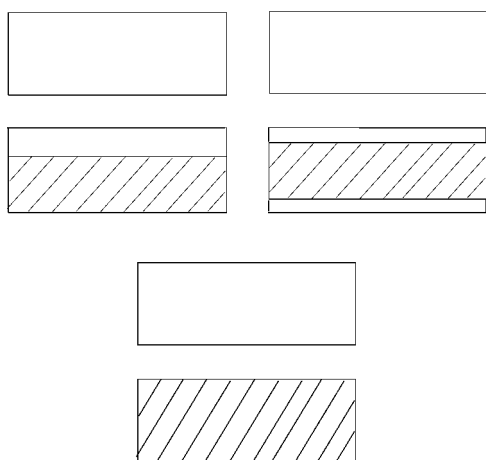


FIG. 1. In the upper left side we draw two bands separated by a gap, with the lower band partially filled appropriate to a metal or superconductor. In the upper right side we draw the situation encountered for the electron fluid under the action of an electric field. The lower sketch illustrates the situation appropriate to an insulator, where the lower band is completely filled, making conduction impossible regardless of Cooper correlations.

unless one excites across the insulating gap and into the conduction band (see Fig. 1).

That the SDW insulating ground state with *d*-wave pairing interactions has Cooper pairing in the ground state can be seen from Fig. 2. In Fig. 2(a) we plot the spectral function for the SDW ground state with no superconducting correlations. In Fig. 2(b) we plot the spectral function for the SDW ground state with superconducting correlations. In the ground state with both superconductivity and antiferromagnetism, the separation between the coherence peaks is bigger as it gets contributions from both the SDW and superconducting gap. A prediction of this model is that the quasiparticles will be coherent with an electron and a hole component in agreement with the BCS-Bogolyubov model.

The SDW ground state with *d*-wave Cooper pairing (SDW-DSC) will become superconducting when doped. At the mean-field level, without worrying about self-consistency, the chemical potential will jump to the appropriate band and there will be a low superfluid density superconductor. Whether this physics is correct for the cuprates is controversial. There is experimental evidence for the chemical potential staying pinned at midgap due to spectral redistribution of states toward midgap states.²⁴ There is also experimental evidence for chemical potential shifts in the cuprates, in the same way as in regular semiconductor materials.²⁵ Independently of whether the SDW-DSC ground state has chemical potential shifts or not, the physics of an insulator with Cooper pairing correlations is interesting. For our study we have the cuprates in mind. For these materials, some phenomenology of this form seems to apply,² but it would be interesting if this physics were to be realized in nature irrespective of the cuprate problem.

In the present work we will flip the problem around. We will start with a *d*-wave superconductor (DSC) and begin turning on SDW antiferromagnetic order on top of the superconductivity. In this limit, the complications mentioned in

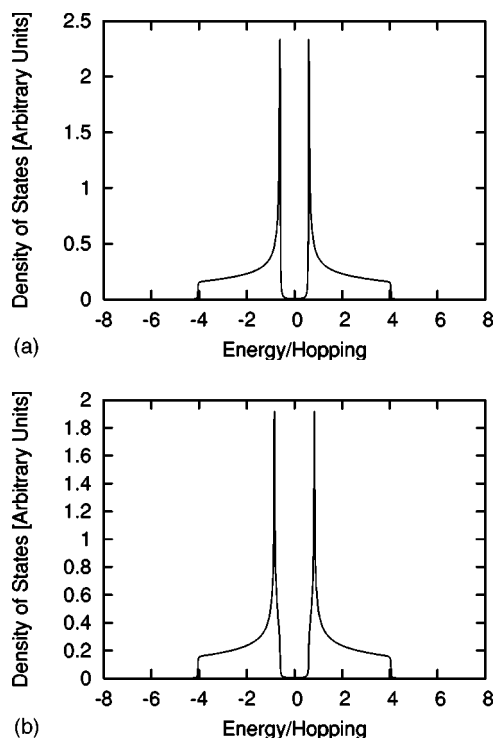


FIG. 2. Spectral function for the SDW ground state without (a) and with (b) superconducting correlations built in.

the previous paragraph are nonexistent. A slow turning on of SDW order on top of the superconductivity will show up as a shift of the antinodal gap and a gapping of the nodal quasiparticles. The latter should be a signal much easier to pick out than the gap shift. The gapping of the nodal quasiparticles is not a unique prediction of antiferromagnetic ordering on top of the superconductivity, as such a gapping can be produced by disorder. On the other hand, the coherence of the gapped “nodal” quasiparticles would be nonexistent for a disordered gap and is thus a unique signature of antiferromagnetic ordering developing on top of the superconductivity. Therefore, if a quasiparticle peak is discernible, and the broadening is less than the disorder-induced broadening ($\geq \hbar^2/2m\Delta x^2 \approx 350$ meV for $\Delta x \sim 1$ nm, appropriate to the cuprates), then the gap is a long range ordered gap and not a disordered gap. Another unique signature of a SDW gap is

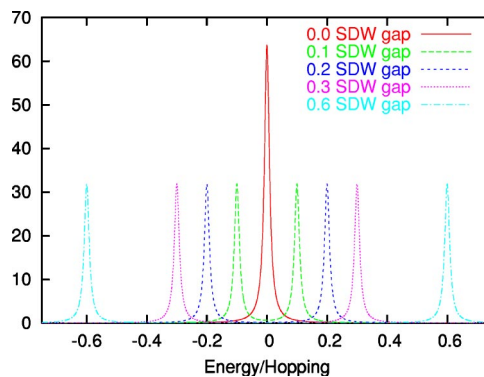


FIG. 3. Gapping of the nodal quasiparticles pole as the SDW order develops.

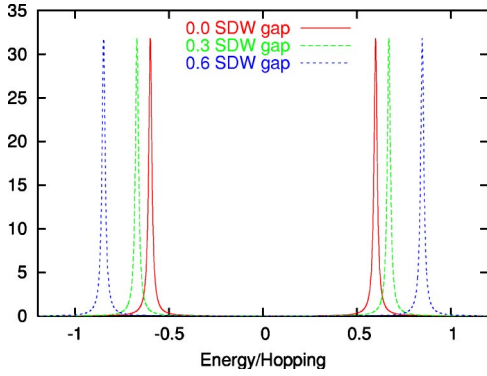


FIG. 4. Shift of the antinodal gap as the SDW order develops.

that the gap will open exactly at the doping where the antiferromagnetism starts.

There are experimental suggestions of antiferromagnetism competing with superconductivity in the deep underdoped regime in the cuprates. For example, measurements show the nodal quasiparticle peaks surviving right up to the doping where antiferromagnetism starts. The spectral weight of such peaks diminishes with decreasing doping, consistent with spectral weight being robbed from the superconducting long range order by a competing long range order such as antiferromagnetism.²⁶ If one looks in the antiferromagnetically ordered dopings, there are experimental suggestions of a competing order parameter that conducts efficiently. Most strikingly, there are measurements of metallic conduction even below the Neel ordering temperature.²⁷

The gapping of the nodal quasiparticles pole as the SDW order develops on top of the superconductivity is shown in Fig. 3 for different values of the SDW gap. The reason we only have a quasiparticle sharp pole is that we have not modeled the realistic electronic self energies relevant to the cuprates as they are irrelevant to the point of principle we are making. Their only effect will be to broaden the quasiparticle peaks and add an incoherent background with the phenomenological features. In Fig. 4 we plot the shift of the antinodal gap as the SDW gap turns on. In Fig. 5 we plot the gap shift versus the SDW gap. In Fig. 6 we plot the spectral density of states in a d -wave superconductor as the SDW gap is turned on. The superconductor with no SDW gap does not

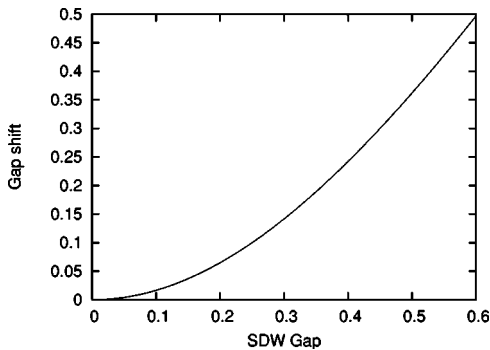


FIG. 5. Shift in the antinodal gap as a function of the SDW gap.

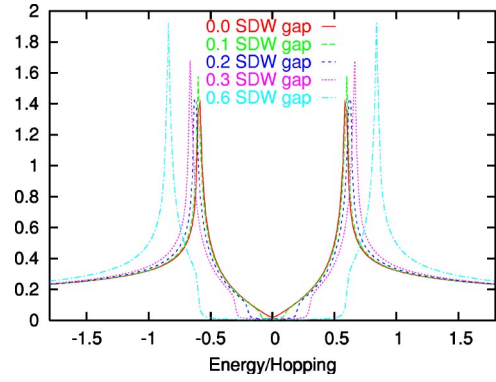


FIG. 6. Spectral density of states for a d -wave superconductor as the SDW gap increases.

have a true gap because of its d -wave symmetry. This is seen in the familiar V-shaped collapse at zero energy. As the SDW gap is turned on, we see the V-shape flatten and expand as a signature of the opening of the antiferromagnetic gap.

II. HUBBARD MODEL WITH d -WAVE ATTRACTIVE INTERACTIONS

For the cuprate problem, the two large effects are the antiferromagnetic, or Coulombic, physics, and the strong superconductivity. Hence we will start from a phenomenological Hamiltonian which is a Hubbard model with a d -wave electronic interaction. This interaction will give rise to d -wave superconductivity when we make the mean-field BCS approximation. The Hamiltonian is

$$\mathcal{H} = \sum_{\vec{k}, \sigma} (\epsilon_{\vec{k}} - \mu) c_{\vec{k}, \sigma}^{\dagger} c_{\vec{k}, \sigma} + \frac{U}{N} \sum_{\vec{k}_1, \vec{k}_2, \vec{q}} c_{\vec{k}_1, \uparrow}^{\dagger} c_{\vec{k}_1 + \vec{q}, \uparrow} c_{\vec{k}_2 + \vec{q}, \downarrow}^{\dagger} c_{\vec{k}_2, \downarrow} + \sum_{\vec{k}_1, \vec{k}_2} V(\vec{k}_1, \vec{k}_2) c_{\vec{k}_1, \uparrow}^{\dagger} c_{-\vec{k}_1, \downarrow}^{\dagger} c_{-\vec{k}_2, \downarrow} c_{\vec{k}_2, \uparrow}, \quad (1)$$

where $c_{\vec{k}, \sigma}^{\dagger}, c_{\vec{k}, \sigma}$ are the electronic creation and destruction operators with momentum \vec{k} and spin σ , $\epsilon_{\vec{k}}$ is the kinetic energy, μ the chemical potential, and U is the Hubbard repulsion. We are working in a spatial lattice with N sites. The last term is an electronic interaction chosen in the reduced BCS form,⁹ which will be used to stabilize superconductivity. In order to have d -wave superconductivity we choose $V(\vec{k}_1, \vec{k}_2) = V_0(\cos k_{1x} - \cos k_{1y})(\cos k_{2x} - \cos k_{2y})$. This phenomenological Hamiltonian can have a mean-field SDW ground state and a mean-field DSC ground state. It can be used to study the turning on of DSC correlations on top of an SDW ground state, or the turning on of SDW order on top of the superconductivity.

We will analyze this Hamiltonian by imposing a SDW mean-field condition, which is stabilized by the Hubbard term. This will be followed by a DSC mean-field condition, which is stabilized by the reduced BCS d -wave interaction. While the use of two mean-field conditions is not common, it has important precedents. It was used by P. W. Anderson²⁸ in his study of the role of plasmons in restoring gauge invari-

ance to the BCS ground state. In this work he invented the Anderson-Higgs mechanism.²⁹ He solved for the properties of the electron system imposing a mean-field condition on the electron density, as in the study of electron correlations by Sawada *et al.*³⁰ and a BCS electron pairing mean-field condition.⁹

The Hubbard interaction stabilizes the mean-field order

$$\sigma SN \equiv \sum_{\vec{k}} \langle c_{\vec{k}+\vec{Q},\sigma}^{\dagger} c_{\vec{k},\sigma} \rangle, \quad (2)$$

where S is a measure of the magnetic moment per site contributing to the antiferromagnetism and $\vec{Q}=(\pi, \pi)$ is the commensurate ordering wave vector and S is the average magnetic moment per site. Other ordering wave vectors are possible for spin and/or charge, i.e., stripe, order parameters but we do not consider them in our study. When we impose this condition on the Hamiltonian and neglect fluctuation terms, the Hamiltonian becomes

$$\begin{aligned} \mathcal{H} = & \sum_{\vec{k},\sigma} (\epsilon_{\vec{k}} - \mu) c_{\vec{k},\sigma}^{\dagger} c_{\vec{k},\sigma} + UNS^2 - US \sum_{\vec{k},\sigma} \sigma c_{\vec{k}+\vec{Q},\sigma}^{\dagger} c_{\vec{k},\sigma} \\ & + \sum_{\vec{k}_1, \vec{k}_2} V(\vec{k}_1, \vec{k}_2) c_{\vec{k}_1, \uparrow}^{\dagger} c_{-\vec{k}_1, \downarrow}^{\dagger} c_{-\vec{k}_2, \downarrow} c_{\vec{k}_2, \uparrow}. \end{aligned} \quad (3)$$

We see that by ordering antiferromagnetically we gain variational energy $-UNS^2$ if self-consistency can be achieved. We next impose the mean-field d -wave Cooper pairing

$$\begin{aligned} \Delta_{\vec{k}_2} & \equiv (\cos k_{2x} - \cos k_{2y}) V_0 \sum_{k_1} (\cos k_{1x} - \cos k_{1y}) \langle c_{\vec{k}_1, \uparrow}^{\dagger} c_{-\vec{k}_1, \downarrow}^{\dagger} \rangle \\ & \equiv \Delta_0 (\cos k_{2x} - \cos k_{2y}) \equiv V_0 x (\cos k_{2x} - \cos k_{2y}), \end{aligned} \quad (4)$$

where x is a measure of the electrons contributing to the superconductivity and hence proportional to the superfluid density. Then the Hamiltonian becomes

$$\begin{aligned} \mathcal{H} = & \sum_{\vec{k},\sigma} (\epsilon_{\vec{k}} - \mu) c_{\vec{k},\sigma}^{\dagger} c_{\vec{k},\sigma} + UNS^2 - US \sum_{\vec{k},\sigma} \sigma c_{\vec{k}+\vec{Q},\sigma}^{\dagger} c_{\vec{k},\sigma} - \frac{\Delta_0^2}{V_0} \\ & + \sum_{\vec{k}} \Delta_{\vec{k}} (c_{\vec{k}, \uparrow}^{\dagger} c_{-\vec{k}, \downarrow}^{\dagger} + c_{-\vec{k}, \downarrow} c_{\vec{k}, \uparrow}). \end{aligned} \quad (5)$$

We see that if the phenomenological d -wave interaction is attractive, i.e., $V_0 < 0$, we gain variational energy Δ_0^2/V_0 by Cooper pairing *regardless* of whether we have ordered antiferromagnetically or not. Of course, if we are at half filling, the material will be insulating *irrespective* of the presence of Cooper pairs, as we would have to excite quasiparticles across the SDW insulating gap in order for conduction to take place.

III. BOGOLYUBOV DIAGONALIZATION OF THE MEAN-FIELD HAMILTONIAN

We now diagonalize the Hamiltonian by the Bogolyubov method.¹⁰ We will do this in two steps. First we diagonalize the SDW part. In order to do this more conveniently, we will split the momentum sums into sums over the reduced magnetic zone. The Hamiltonian is then

$$\begin{aligned} \mathcal{H} = & \sum_{\vec{k},\sigma} \{ (\epsilon_{\vec{k}}^{\pm} - \mu) (c_{\vec{k},\sigma}^{\dagger} c_{\vec{k},\sigma} + c_{\vec{k}+\vec{Q},\sigma}^{\dagger} c_{\vec{k}+\vec{Q},\sigma}) + \epsilon_{\vec{k}}^{-} (c_{\vec{k},\sigma}^{\dagger} c_{\vec{k},\sigma} \\ & - c_{\vec{k}+\vec{Q},\sigma}^{\dagger} c_{\vec{k}+\vec{Q},\sigma}) - 2\sigma US c_{\vec{k}+\vec{Q},\sigma}^{\dagger} c_{\vec{k},\sigma} \} + UNS^2 - \frac{\Delta_0^2}{V_0} \\ & + \sum_{\vec{k}} \Delta_{\vec{k}} (c_{\vec{k}, \uparrow}^{\dagger} c_{-\vec{k}, \downarrow}^{\dagger} + c_{-\vec{k}, \downarrow} c_{\vec{k}, \uparrow} - c_{\vec{k}+\vec{Q}, \uparrow}^{\dagger} c_{-\vec{k}-\vec{Q}, \downarrow} \\ & - c_{-\vec{k}-\vec{Q}, \downarrow} c_{\vec{k}+\vec{Q}, \uparrow}), \end{aligned} \quad (6)$$

where the prime on the summation sign means that the sum is restricted to the wave vectors in the magnetic zone. $\epsilon_{\vec{k}}^{\pm} \equiv (\epsilon_{\vec{k}} + \epsilon_{\vec{k}+\vec{Q}})/2$ and $\epsilon_{\vec{k}}^{-} \equiv (\epsilon_{\vec{k}} - \epsilon_{\vec{k}+\vec{Q}})/2$. The last term in the superconducting interaction is negative because $\Delta_{\vec{k}+\vec{Q}} = -\Delta_{\vec{k}}$. In order to diagonalize the magnetic part we define the Bogolyubov operators

$$b_{\vec{k},\sigma} = \alpha_{\vec{k}} c_{\vec{k},\sigma} - \sigma \beta_{\vec{k}} c_{\vec{k}+\vec{Q},\sigma}, \quad (7)$$

$$b_{\vec{k}+\vec{Q},\sigma} = \alpha_{\vec{k}} c_{\vec{k}+\vec{Q},\sigma} + \sigma \beta_{\vec{k}} c_{\vec{k},\sigma}. \quad (8)$$

If we choose

$$\alpha_{\vec{k}}^2 = \frac{1}{2} \left(1 + \frac{\epsilon_{\vec{k}}^{-}}{E_{\vec{k}}^{-}} \right) \quad \beta_{\vec{k}}^2 = \frac{1}{2} \left(1 - \frac{\epsilon_{\vec{k}}^{-}}{E_{\vec{k}}^{-}} \right), \quad (9)$$

$$E_{\vec{k}}^2 = (\epsilon_{\vec{k}}^{-})^2 + U^2 S^2 \quad (10)$$

the Hamiltonian becomes

$$\begin{aligned} \mathcal{H} = & \sum_{\vec{k},\sigma} \{ (\epsilon_{\vec{k}}^{\pm} - \mu) (b_{\vec{k},\sigma}^{\dagger} b_{\vec{k},\sigma} + b_{\vec{k}+\vec{Q},\sigma}^{\dagger} b_{\vec{k}+\vec{Q},\sigma}) + E_{\vec{k}} (b_{\vec{k},\sigma}^{\dagger} b_{\vec{k},\sigma} \\ & - b_{\vec{k}+\vec{Q},\sigma}^{\dagger} b_{\vec{k}+\vec{Q},\sigma}) \} + UNS^2 - \frac{\Delta_0^2}{V_0} + \sum_{\vec{k}} \Delta_{\vec{k}} (b_{\vec{k}, \uparrow}^{\dagger} b_{-\vec{k}, \downarrow}^{\dagger} \\ & + b_{-\vec{k}, \downarrow} b_{\vec{k}, \uparrow} - b_{\vec{k}+\vec{Q}, \uparrow}^{\dagger} b_{-\vec{k}-\vec{Q}, \downarrow}^{\dagger} - b_{-\vec{k}-\vec{Q}, \downarrow} b_{\vec{k}+\vec{Q}, \uparrow}). \end{aligned} \quad (11)$$

Our last step to diagonalize the full Hamiltonian is the Bogolyubov diagonalization of the leftover superconducting part by defining the canonical operators

$$B_{\vec{k},\sigma} = u_{\vec{k}}^{\dagger} b_{\vec{k},\sigma} + \sigma v_{\vec{k}}^{\dagger} b_{\vec{k},\sigma}^{\dagger}, \quad (12)$$

$$B_{\vec{k}+\vec{Q},\sigma} = u_{\vec{k}} b_{\vec{k}+\vec{Q},\sigma} - \sigma v_{\vec{k}} b_{\vec{k}+\vec{Q},\sigma}^{\dagger}, \quad (13)$$

If we choose

$$(u_{\vec{k}}^{\pm})^2 = \frac{1}{2} \left(1 + \frac{\epsilon_{\vec{k}}^{\pm} - \mu \pm E_{\vec{k}}^{\pm}}{E_{\vec{k}}^{\pm}} \right), \quad (14)$$

$$(v_{\vec{k}}^{\pm})^2 = \frac{1}{2} \left(1 - \frac{\epsilon_{\vec{k}}^{\pm} - \mu \pm E_{\vec{k}}^{\pm}}{E_{\vec{k}}^{\pm}} \right), \quad (15)$$

$$(E_{\vec{k}}^{\pm})^2 = (\epsilon_{\vec{k}}^{\pm} - \mu \pm E_{\vec{k}}^{\pm})^2 + \Delta_{\vec{k}}^2, \quad (16)$$

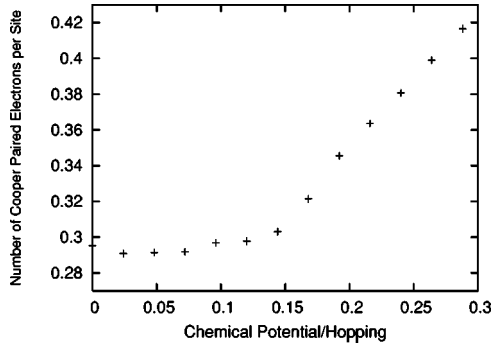


FIG. 7. Electrons contributing to the “superfluid density” as a function of the chemical potential.

the Hamiltonian then becomes

$$\mathcal{H} = \sum_{\vec{k}, \sigma} [E_k^+ B_{k, \sigma}^\dagger B_{k, \sigma}^- + E_k^- B_{k+\vec{Q}, \sigma}^\dagger B_{k+\vec{Q}, \sigma}^-] + UNS^2 - \frac{\Delta_0^2}{V_0} + \text{constants.} \quad (17)$$

We see that we have two separate superconducting bands with dispersions E_k^+ and E_k^- . This happens because the SDW ordering has split the noninteracting band, i.e., the system with $U=0$. Of course, if the SDW gap were to collapse, the two bands would merge into one superconducting band. If we look at it from the opposite perspective, we see that when we turn on the SDW order, there will be an insulating gap. We have shown in Fig. 1 how this gap opens up at the node as calculated in the later section on Green’s functions.

IV. SELF-CONSISTENT DETERMINATION OF THE SUPERFLUID DENSITY

The Bogolyubov transformations can of course be inverted to yield the electron creation and destruction operators in term of the Bogolyubov eigenoperators ($B_{\vec{k}}$) of the system. From this we can evaluate the self-consistency or “gap” Eqs. (2) and (4). We obtain

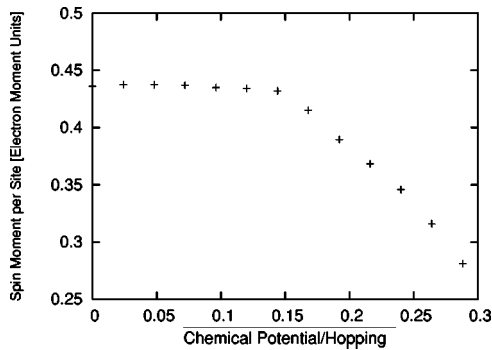


FIG. 8. Spin moment per site as a function of the chemical potential.

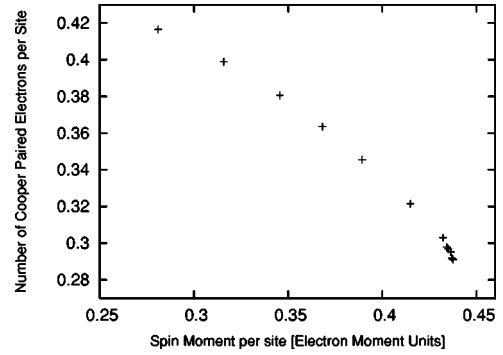


FIG. 9. Electrons contributing to the “superfluid density” as a function of the spin moment per site.

$$\frac{2N}{U} = \sum_{\vec{k}} \left(\frac{\epsilon_k^+ - \mu + E_k^-}{E_k^+ E_k^-} - \frac{\epsilon_k^- - \mu - E_k^+}{E_k^- E_k^+} \right) \quad (18)$$

from the antiferromagnetic self-consistency condition (2) and

$$-\frac{2}{V_0} = \sum_{\vec{k}} (\cos k_x - \cos k_y)^2 \left(\frac{1}{E_k^+} + \frac{1}{E_k^-} \right) \quad (19)$$

from the superconducting self-consistency or gap Eq. (4). The negative sign on the left is consistent with $V_0 < 0$ as is necessary to stabilize DSC. We see that these are two coupled equations for the superconducting number of electrons x , a measure of the superfluid density, and the spin moment magnitude S in the antiferromagnet. Their solution will contain information about how the two orders compete and how they rob spectral weight from each other as one dopes the material, i.e., as one changes the chemical potential.

In Fig. 7 we plot the self-consistent numerical determination of x versus chemical potential μ . Figure 8 shows the spin moment S versus μ , and Fig. 9 is a graph of x versus S . These three figures were plotted for interactions leading to zero doping antiferromagnetic gap of 0.6 in units of the hopping strength and zero doping superconducting gap of 0.3 in the same units. These values were chosen because they were the same values we used for illustrations in the previous graphs.

From the figures of superfluid density versus chemical potential and spin moment versus chemical potential, we see that the superfluid density grows as the material is doped away from half filling. At the same time, the spin moment decreases as μ increases. We remind the reader that this is the exact behavior seen in the cuprates. Despite the pairing interaction at half filling, the material is an insulating antiferromagnet due to the SDW gap. As one dopes it, the DSC starts stealing spectral weight from the SDW as evidenced by the figures in this section. In this way we see that near half filling the DSC can be very tenuous becoming more robust away from half filling. This is strikingly similar to the gossamer superconductivity ideas of Laughlin and collaborators.^{5,6}

V. GREEN'S FUNCTION FOR THE SDW-DSC HAMILTONIAN

In the present section we will write down the expressions for the Green's functions for a system with SDW order and d -wave Cooper pairing in each of the SDW bands. The expression for the retarded Green's function or the propagator is³¹

$$G(\vec{x}, \vec{x}', t) = -i \sum_{n, \sigma} \{ \theta(t) e^{-iE_n t/\hbar} \langle \psi_0 | c_{\vec{x}, \sigma} | \psi_n \rangle \langle \psi_n | c_{\vec{x}', \sigma}^\dagger | \psi_0 \rangle + \theta(-t) e^{iE_n t/\hbar} \langle \psi_0 | c_{\vec{x}', \sigma}^\dagger | \psi_n \rangle \langle \psi_n | c_{\vec{x}, \sigma} | \psi_0 \rangle \}, \quad (20)$$

$$\theta(t) = -\frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{d\omega}{\omega + i\eta} e^{-i\omega t} \quad \eta = 0^+, \quad (21)$$

where

$$c_{\vec{x}, \sigma} = \frac{1}{\sqrt{N}} \sum_{\vec{k}} c_{\vec{k}, \sigma} e^{-i\vec{k} \cdot \vec{x}}. \quad (22)$$

n labels the eigenstates of the system with energies E_n and the ground state energy E_0 has been chosen to be 0.

From the time Fourier transform of the Green's function above we obtain the local retarded propagator in the energy representation

$$G(\vec{x}, \vec{x}, E) = \frac{1}{\pi N} \sum_{\vec{k}} \left\{ \frac{(u_{\vec{k}}^+)^2}{E - E_{\vec{k}}^+ + i\eta} + \frac{(u_{\vec{k}}^-)^2}{E - E_{\vec{k}}^- + i\eta} - \frac{(v_{\vec{k}}^+)^2}{E + E_{\vec{k}}^+ - i\eta} - \frac{(v_{\vec{k}}^-)^2}{E + E_{\vec{k}}^- - i\eta} \right\}. \quad (23)$$

The local spectral density function follows from³¹

$$A(\vec{x}, E) = -\frac{1}{\pi} \text{Im} G(\vec{x}, \vec{x}, E). \quad (24)$$

All of our density of states are calculated from these expressions in a 1000×1000 momentum lattice with an energy resolution of 0.01. We choose the hopping energy scale to be 1, so all energies are measured in hopping units. When we have superconductivity we choose the gap to be 0.3. The antiferromagnetic gap is chosen anywhere between 0 and 0.6, usually with jumps of 0.1. We have nearest neighbor hopping only. These values need not be realistic; they are just chosen to illustrate the effect.

Similarly, if we Fourier transform the Green's function (20) in both time and space, we obtain the retarded propagator in the wave-vector energy representation.

$$G(\vec{k}, E) = \frac{1}{\pi} \left\{ \frac{(u_{\vec{k}}^+)^2}{E - E_{\vec{k}}^+ + i\eta} + \frac{(u_{\vec{k}}^-)^2}{E - E_{\vec{k}}^- + i\eta} - \frac{(v_{\vec{k}}^+)^2}{E + E_{\vec{k}}^+ - i\eta} - \frac{(v_{\vec{k}}^-)^2}{E + E_{\vec{k}}^- - i\eta} \right\}. \quad (25)$$

From this formula we calculate the absorption strength versus energy for the nodal quasiparticles. We do this by simply fixing \vec{k} to be at the node and plotting the spectral density³¹

$$A(\vec{k}, E) = -\frac{1}{\pi} \text{Im} G(\vec{k}, E) \quad (26)$$

versus energy. Energy units, values and uncertainties are chosen as described for the local density of states.

VI. CONCLUSIONS

Since early in the high temperature superconductivity game the coexistence of antiferromagnetism and the putative resonating valence bond (RVB) superconducting state has been the object of theoretical consideration.^{33,34} As mentioned in the Introduction, there is growing but not strictly direct evidence that the low energy quasiparticles in the superconducting cuprates are coherent BCS-Bogolyubov quasiparticles. This suggests a BCS Cooper pair ground state. In the present work, as opposed to earlier work, we have concentrated on the coexistence and competition of BCS and SDW order.

We studied a mean-field Hamiltonian with two mean-field order parameters. The Hamiltonian contains a spin-density-wave antiferromagnetic mean field stabilized by a Hubbard interaction and a d -wave Cooper pairing mean field stabilized by a phenomenological d -wave interaction. The two order parameters can coexist and the SDW ground state *always* gains energy by Cooper pairing when the d -wave interaction is attractive and nonzero. The SDW ground state with Cooper pairing *fails* to superconduct at half-filling due to the antiferromagnetic gap. Its particle-like excitations are Bogolyubov-BCS quasiparticles consisting of coherent mixtures of electrons and holes.

Of greater interest and relevance to the superconducting cuprates is the case when antiferromagnetic order is turned on weakly on top of the superconductivity. This would correspond to the onset of antiferromagnetism at a critical doping. In such a case a small gap proportional to the weak antiferromagnetic gap opens up for nodal quasiparticles, and the quasiparticle peak would be discernible. While the gapping of the nodal quasiparticle could be caused by a large enough disorder, such a disorder would broaden the quasiparticle peak so much as to make it invisible. A unique signature of antiferromagnetic gapping of the nodal quasiparticles is that it will turn on always at the doping when antiferromagnetism starts while disorder gapping will turn on at different sample dependent dopings. Such an effect has recently been observed in photoemission studies of the high temperature superconducting cuprates.³²

We solved the superconducting and antiferromagnetic gap equations self-consistently and found the number of superconducting electrons x and the spin magnetic moment per site S . The superfluid density x increases with doping stealing spectral weight from the antiferromagnetism, which leads to a corresponding decrease in spin moment S . This is the same behavior observed in the cuprate superconductors.

Finally, we wrote down the exact expressions for the

Green's function for the system with coexisting SDW and DSC order parameters. These are evaluated numerically in a 1000×1000 momentum lattice with 0.01 energy resolution in units of the lattice hopping. From the imaginary parts of the Green's functions we obtained the absorption by nodal quasiparticles and the local density of states.

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