Out-of-plane vortex in a two-dimensional easy-plane ferromagnet with a magnetic field along the hard axis

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In a two-dimensional easy-plane Heisenberg model with a magnetic field along the hard axis, switching between different vortex states (parallel and antiparallel to the field) was studied by means of a Monte Carlo simulation. The vortex-core magnetization appears to be conserved, whether a magnetic field is present or not. From conservation of the core magnetization, the switching field between the two vortex states were calculated, in good agreement with the Monte Carlo simulation.

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I. INTRODUCTION

It is well known that the ideal two-dimensional Heisenberg magnet cannot have a long range magnetic order at any finite temperature.[1] With an easy-plane anisotropy, the two-dimensional Heisenberg magnet is known to undergo the Kosterlitz-Thouless transition, i.e., a vortex-antivortex unbinding transition.[2] The two-dimensional Heisenberg ferromagnet with an easy-plane anisotropy can be described by the model Hamiltonian

$$H = -J\sum_{\langle i,j\rangle}(S_i^xS_j^x + S_i^yS_j^y + \lambda S_i^zS_j^z), \tag{1}$$

where J and λ are the exchange coupling constant and the easy-plane anisotropy, respectively, and $\langle i,j\rangle$ indicates nearest-neighbor indices in a two-dimensional lattice. In the model, the so-called in-plane and out-of-plane vortices have been known for $\lambda<\lambda_c$ and $\lambda>\lambda_c$, respectively,[3] where the critical anisotropy $\lambda_c=0.7044$ for the square lattice from a core model,[4] while slightly different values of λ_c were given from numerical calculations.[3,5]

In addition to the *vorticity*, the out-of-plane vortex has two possible directions of magnetization at the origin, i.e., the "polarization" $p=m_o^z=\pm 1$. The polarization determines the vortex dynamics via the gyroforce effect[6] and reversal of the polarization severely affects the dynamics. Switching between states with different polarizations was studied in an easy-plane rotating magnetic field[7] and with thermal noises.[8] In the presence of a magnetic field along the hard axis, the two vortex states, with $p=\pm 1$, respectively, are not equivalent. A vortex with a magnetization parallel to the magnetic field (light vortex) has an energy lower than that of a vortex with a magnetization antiparallel to the field (heavy vortex).[9] As the magnetic field increases, the light (heavy) vortex is compressed (elongated), whose width increases (decreases). For a large enough field, a heavy vortex loses its stability and then the sign of the polarization is switched. The switching field B_c between the two vortex states was obtained as a power of $(1-\lambda)$ in a continuum model.[10] However, the results of the numerical simulation were not consistent with the model, because the continuum limit requires too large a lattice for a numerical calculation and is not suitable for λ near λ_c, where the widthof vortex approaches the lattice size.

In this work, the magnetization curve as a function of magnetic field was obtained by means of a Monte Carlo simulation, from which the core magnetization M_{core} and the switching field B_c were obtained. The core magnetization was shown to be conserved. From conservation of the core magnetization, the switching field was calculated and compared with the results of the Monte Carlo simulation.

II. MONTE CARLO SIMULATION

We performed Monte Carlo simulation for the classical Heisenberg spins $|\vec{S}|=1$ placed on a $L\times L$ square lattice ($L=40-60$) with the model Hamiltonian, Eq. (1). For convenience, the exchange coupling constant J was set to 1. Open boundary conditions were applied to isolate a single vortex. The spin configuration of a single in-plane vortex centered at $(L/2, L/2)$ was used as an initial spin configuration. Starting at temperature $T_{\text{init}}=0.1$, the temperature was lowered down to $T_{\text{end}}=10^{-5}$ by means of a simulated annealing technique.[5] The cooling schedule was $T_n=T_{\text{init}}/n$, where $n=1,2,\ldots,T_{\text{init}}/T_{\text{end}}$.[11] The Metropolis algorithm was employed to update the spin configurations and, at each temperature, 10^2-10^3 MCS's (Monte Carlo steps) were completed. Results for S^x, S^y, and S^z are shown in Fig. 1 for $\lambda=0.71$.

After the temperature was lowered to T_{end}, a magnetic field B was applied along the hard axis (z axis), where the Zeeman interaction $H_Z=-B\Sigma_i S_i^z$ was added to the model-Hamiltonian Eq. (1). At each magnetic field, the initial 10^4 MCS's were discarded and the following 10^4 MCS's were used to calculate physical quantities. The magnetic field step ΔB was usually 10^{-5} to 10^{-6} and some larger steps were used for rough calculation of the overall hysteresis loop.

III. RESULTS AND DISCUSSION

Figure 2 shows a representative magnetic hysteresis loop for $\lambda=0.91$. Figure 2(a) shows an overall hysteresis loop calculated with $\Delta B=10^{-3}$. The vortex creation is clearly observed at $B_{\text{vc}}\sim 0.15$. The vortex annihilation field of $B_{\text{va}}\sim 0.43$ appears to be equal to the saturation field, where all the spins are aligned along the field direction and the light vortex is annihilated. In Fig. 2(b), the initial magnetization

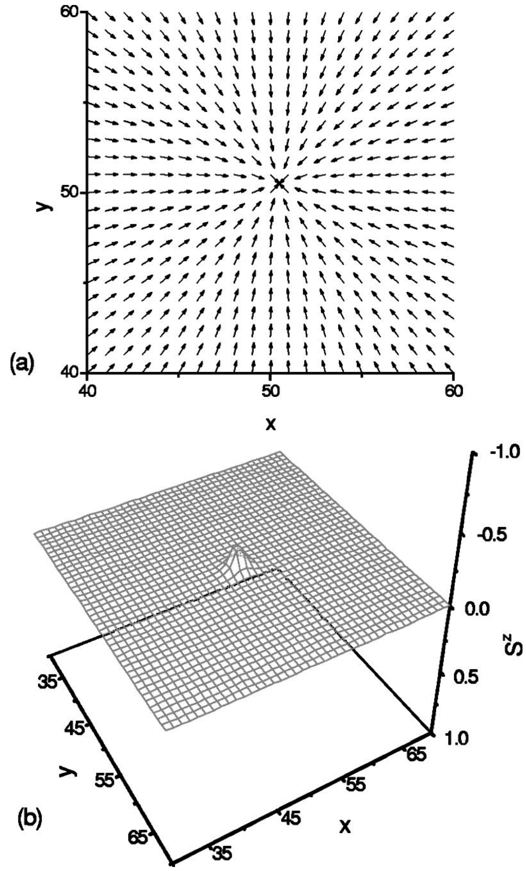

FIG. 1. (a) In-plane and (b) out-of-plane spin components for $\lambda = 0.71$ and $L = 100$, where only the central region is shown for better visualization.

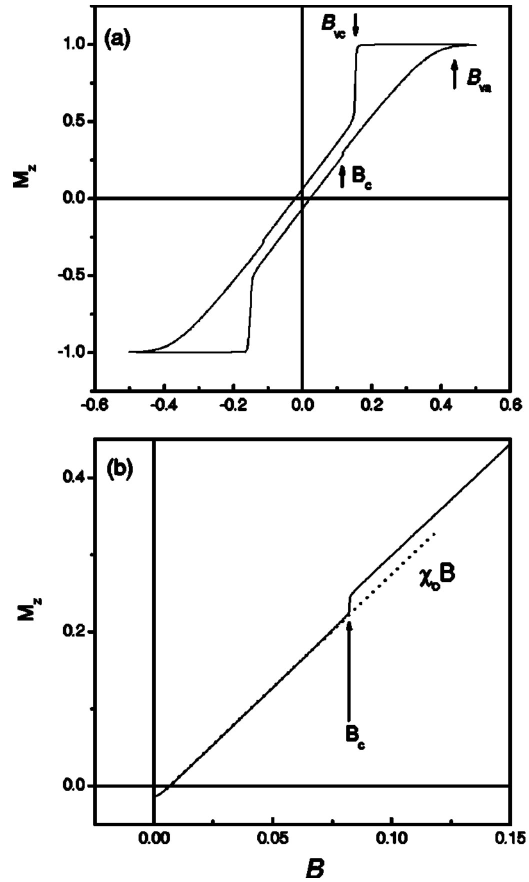

FIG. 2. For $\lambda = 0.91$ and $L = 50$, (a) magnetic hysteresis loop with the field step of $\Delta B = 10^{-3}$ and (b) initial magnetization curve with $\Delta B = 10^{-5}$. B_{vc}, B_{va}, and B_c indicate the vortex creation field, the vortex annihilation field, and the switching field, respectively.

curve was calculated with $\Delta B = 10^{-5}$, where the core reversal of the heavy vortex was clearly observed as a step. Far below B_{va}, the magnetization M_z is linear to the applied field B, $M_z \propto \chi_o B$, where $\chi_o = dM_z/dB$ corresponds to the magnetic susceptibility of the spins far from the core of vortex. For a heavy vortex, the increase of $\chi_o B$ corresponds to the elongation of the vortex. We can observe the core reversal as a step in the magnetization curve for $\lambda \geqslant 0.708$ and therefore λ_c is believed to be lower than 0.708.

Figure 3 shows the magnetic susceptibility χ_o as a function of λ. χ_o was obtained by a linear fit of the initial $M_z(B)$ curve below (solid symbols) and above (open symbols) B_c, corresponding to the heavy and the light vortex, respectively. In both cases, the magnetic susceptibility is well described by $\chi_o = 0.266/(1-\lambda)$. A slight inconsistency was observed when $\lambda \rightarrow 1$ and $\lambda \rightarrow \lambda_c$. Near λ_c, B_c is so small that an accurate estimate of χ_o is not easy. When $\lambda \rightarrow 1$ above B_c, a strong broadening of the vortex core may be responsible for the inconsistency.

Figure 4 shows the core magnetization of the out-of-plane vortex. The solid symbols correspond to $L^2 M_z$ just after the simulated annealing without a magnetic field. The open symbols correspond to $L^2 \Delta M_z/2$ at B_c, where ΔM_z is a difference between the magnetizations just below and above B_c. Although some discrepancies are apparent for large λ, the two values may be taken to agree with each other. When the magnetic field increases, the heavy vortex is elongated along the field direction, whose core width is narrowed, and the light vortex is compressed, whose core width is broadened. When the magnetic field increases through B_c, the heavy vortex changes to the light one at B_c, accompanied by an abrupt broadening of the core width. The apparent discrepancy between the two values for large λ appears to be due to an abrupt broadening of the core width, as the lattice size may not be large enough to include the whole vortex. The core magnetization of the out-of-plane vortex appears to be conserved.

The out-of-plane vortex takes a conelike shape, whose volume corresponds to the core magnetization. The height corresponding to S^z at the center of the vortex was described by a power of $(\lambda - \lambda_c)$, i.e., $S_o^z \sim (\lambda - \lambda_c)^{\nu}$ with $\nu = 0.3925$.[5] The core width was described, in a continuum model, by $r_v(0) = 0.5[\lambda/(1-\lambda)]^{\theta}$ with $\theta = 0.5$ (the lattice constant was set to 1).[3] The core magnetization will be proportional to $S_o^z \times r_v(0)^2$ and then

$$M_{\mathrm{core}} = 0.25C(\lambda - \lambda_c)^{\nu}[\lambda/(1-\lambda)]^{2\theta}, \tag{2}$$

where C depends on the shape of the out-of-plane vortex and $\lambda_c = 0.7044$ as obtained from the core model.[4] Because, in a continuum model, $S_0^z = 1$ regardless of the value of λ, the

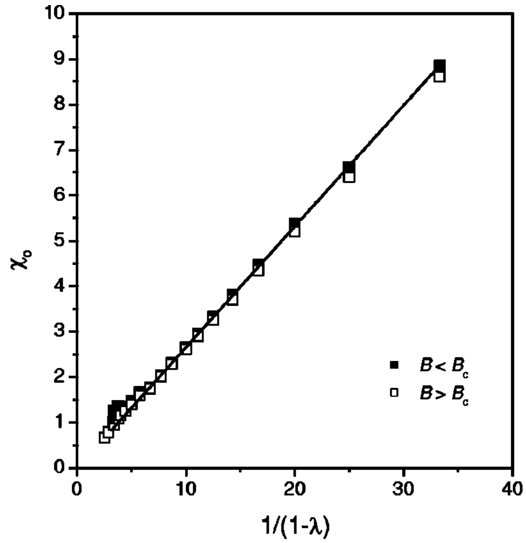

FIG. 3. χ_o versus λ, where $\chi_o = dM_z/dB$. The solid symbols were obtained for heavy vortices ($B < B_c$) and the open symbols for light vortices ($B > B_c$). The solid line shows a linear fit to the solid symbols with a slope of 0.266(±0.001).

above approximation for the core magnetization is believed to overcome the limit of the continuum model to some degrees. The solid line in Fig. 4 shows a fit to Eq. (2) with $\nu = 0.39 \pm 0.05$, $\theta = 0.45 \pm 0.01$, and $C = 21 \pm 3$. Equation (2) is in fairly good agreement with the simulation data, except very near λ_c. The exponent $\nu = 0.39$ is quite consistent with the previous work and the exponent $\theta = 0.45$ is a little smaller than the continuum value.

Figure 5 shows the switching field B_c as a function of λ. From conservation of the core magnetization, we can calculate B_c. In the presence of B, the core magnetization of the heavy vortex is

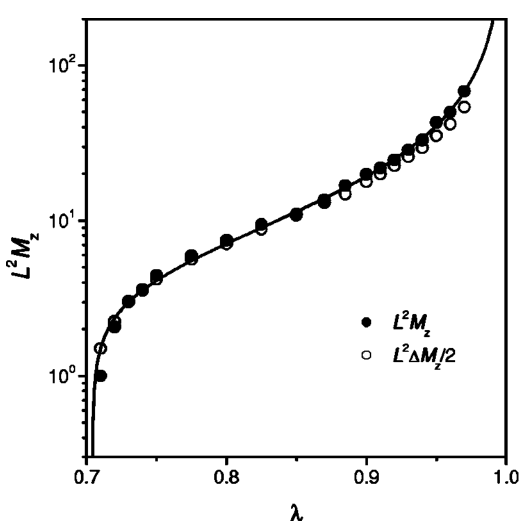

FIG. 4. Core magnetization of the out-of-plane vortex as a function of λ. The solid symbols correspond to L^2M_z in the absence of a magnetic field. The open symbol corresponds to $L^2\Delta M_z/2$ at B_c. The solid line is a fit to Eq. (2).

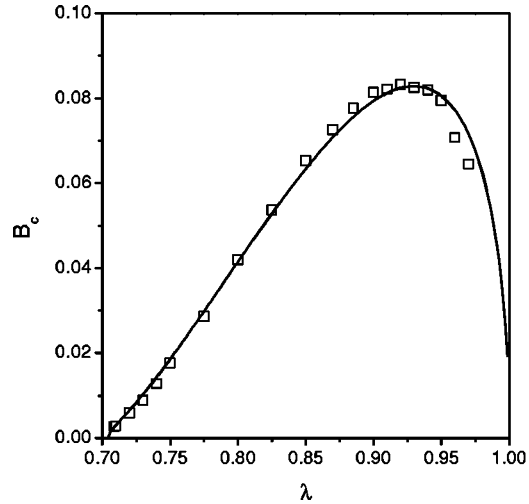

FIG. 5. The vortex-core reversal field B_c as a function of λ. The solid line corresponds to Eq. (7).

$$M_{\text{core}} = CS_o^z(B)r_v(B)^2 = CS_o^z r_v(0)^2, \tag{3}$$

and

$$r_v(B)^2 = \frac{S_o^z r_v(0)^2}{S_o^z(B)}. \tag{4}$$

It can be assumed that S_o^z does not change even in the presence of B and the elongation of the heavy vortex is due to $\chi_o B$. Then, the height of the elongated vortex becomes $S_o^z(B) \sim S_o^z + C_B\chi_o B$. At B_c, the magnitude of the vortex core width becomes order of $r_v(B_c) \sim \sqrt{C_a r_v(0)}$.[10] Thus, Eq. (4) gives

$$r_v(B_c)^2 = C_a r_v(0) = \frac{S_o^z r_v(0)^2}{S_o^z + C_B\chi_o B_c}. \tag{5}$$

Finally, B_c becomes

$$B_c = \frac{S_o^z}{C_B\chi_o}\left(\frac{r_v(0)}{C_a} - 1\right). \tag{6}$$

Inserting $S_o^z \sim (\lambda - \lambda_c)^\nu$ and $r_v(0) = 0.5[\lambda/(1-\lambda)]^\theta$ into Eq. (6),

$$B_c = \frac{(1-\lambda)(\lambda-\lambda_c)^\nu}{0.266C_B}\left[\frac{1}{2C_a}\left(\frac{\lambda}{1-\lambda}\right)^\theta - 1\right]. \tag{7}$$

In the fitting of our simulation data to Eq. (7), the values of $\lambda_c = 0.7044$ as well as $\nu = 0.39$ and $\theta = 0.45$, obtained from Fig. 4, were used, and the two parameters C_B and C_a were obtained from the fitting. The solid line in Fig. 5 shows a fit to Eq. (7) with the fitting parameters $C_B = 2.4 \pm 0.05$ and $C_a = 0.71 \pm 0.006$, in very good agreement with the simulation results.

In Eqs. (6) and (7), $\chi_o = 0.266/(1-\lambda)$ obtained from Fig. 3 is close to the inverse of the anisotropy field $4(1-\lambda)$ in the continuum model, and the value of $\theta = 0.45$, obtained from Fig. 4, is very close to that of the continuum model ($\theta = 0.5$). Assuming $S_o^z = 1$, $\chi_o = 1/4(1-\lambda)$, and $\theta = 0.5$ as in the continuum model, Eq. (7) becomes $B_c = 4(1-\lambda)$

$\times[\sqrt{\lambda/(1-\lambda)}/2C_a-1]/C_B$. B_c becomes zero when $\lambda=1$ or $\sqrt{\lambda_c/(1-\lambda_c)}/2=C_a$. Then, C_a is equal to the core width $r_v(0)$ at λ_c, according to $r_v(0)=0.5[\lambda/(1-\lambda)]^{\theta}$ ($\theta=0.5$) in the continuum model. Thus, the parameter C_a may be taken to possess an obvious physical meaning. Assuming $\lambda_c=0.8$ as expected in the continuum model, C_a becomes 1, which is quite consistent with the continuum model. If we take $\theta=0.45$ and $\lambda_c=0.7044$, C_a becomes 0.74, which is quite close to our value of $C_a=0.71$. The exponent $\nu=0.39$, obtained in a previous work, is quite compatible with our data, and finally the only free parameter, in the true sense, in the fitting to Eq. (7) may be taken to be C_B.

In summary, we have performed a Monte Carlo simulation for a two-dimensional easy-plane Heisenberg model with a magnetic field. From the magnetization curve, the core magnetization and the switching field between the two vortex states were obtained. The core magnetization appeared to be conserved. The switching field calculated from conservation of the core magnetization was consistent with the Monte Carlo results.

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