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Gap symmetry of superconductivity in UPd_2Al_3

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The angle-dependent thermal conductivity of the heavy-fermion superconductor UPd_2Al_3 in the vortex state was recently measured by Watanabe *et al.* Here we analyze this data from two perspectives: universal heat conduction and the angle dependence. We conclude that the superconducting gap function $\Delta(\mathbf{k})$ in UPd_2Al_3 has horizontal nodes and is given by $\Delta(\mathbf{k})=\Delta\cos(2\chi)$, with $\chi=ck_z$.

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I. INTRODUCTION

Since the discovery of the heavy-fermion superconductor $CeCu_2Si_2$ in 1979[1] the gap symmetries of unconventional superconductors have become a central issue in condensed-matter physics.[2] In the last few years, the angle-dependent magnetothermal conductivity in the vortex state of nodal superconductors has been established as a powerful technique to address the gap symmetry. This is in part due to the theoretical understanding of the quasiparticle spectrum in the vortex state of nodal superconductors, following the work by Volovik.[3–5] Using this approach, Izawa *et al.* have succeeded in identifying the gap symmetries of superconductivity in Sr_2RuO_4, $CeCoIn_5$, κ-$(ET)_2Cu(NCS)_2$, YNi_2B_2C, and $PrOs_4Sb_{12}$.[6–10]

Superconductivity in UPd_2Al_3 was discovered by Geibel *et al.*[11] in 1991. The reduction of the Knight shift in NMR[12] and the Pauli limiting of H_{c2} (Ref. 13) indicate spin singlet pairing in this compound. Nodal superconductivity with horizontal nodes has been suggested from the thermal-conductivity data[14] and from the c-axis tunneling data of thin-film UPd_2Al_3 samples.[15] Very recently, McHale *et al.*[16] have proposed $\Delta(\mathbf{k})=\Delta\cos(\chi)$ (with $\chi=ck_z$) based on a model where the pairing interaction arises from antiparamagnon exchange with $\mathbf{Q}=(0,0,\pi/c)$.[17] Furthermore, the thermal-conductivity data of UPd_2Al_3 for a variety of magnetic-field orientations have been reported.[18] At first glimpse the experimental data appeared to support the model proposed by McHale *et al.*

The object of the present paper is to show that an alternative model, i.e., $\Delta(\mathbf{k})=\Delta\cos(2\chi)$, describes the thermal-conductivity data more consistently. For this purpose we first generalize the universal heat conduction initially proposed in the context of d-wave superconductivity[19,20] to a variety of nodal superconductors. We limit ourselves to quasi-two-dimensional (quasi-2D) systems with $\Delta(\mathbf{k})=\Delta f$ and $f=\cos(2\phi)$, $\sin(2\phi)$, $\cos\chi$, $e^{i\phi}\cos\chi$, $\cos(2\chi)\sin\chi$, and $e^{i\phi}\sin\chi$. It is found that the in-plane thermal conductivity κ_{xx} is independent of f. On the other hand, the out-of-plane thermal conductivity κ_{zz} can discriminate different f's. Second, we extend an early study of the angle-dependent thermal conductivity[21] for κ_{yy} in a magnetic field rotated in the z-x plane. The comparison of these results with experimental data indicates $\Delta(\mathbf{k})=\Delta\cos(2\chi)$.

II. UNIVERSAL HEAT CONDUCTION

Here we consider the thermal conductivity κ in the limit $T\to 0$ K in the presence of disorder. It is assumed that the impurities are in the unitary scattering limit.[20] We consider the quasi-2D gap functions $\Delta(\mathbf{k})=\Delta f$ with $f=\cos(2\phi)$, $\sin(2\phi)$ (d-wave superconductor as in the high-T_c cuprates), $\cos\chi$, $e^{i\phi}\cos\chi$ [f-wave superconductor as proposed for Sr_2RuO_4 (Ref. 6)], $\cos(2\chi)$, $\sin\chi$, and $e^{i\phi}\sin\chi$. Following Ref. 20, the thermal conductivity within the conducting plane is given by

$$\kappa_{xx}/\kappa_n=\kappa_{yy}/\kappa_n=\frac{\Gamma_0}{\Delta}\left\langle[1+\cos(2\phi)]\frac{C_0^2}{(C_0^2+|f|^2)^{3/2}}\right\rangle, \quad (1)$$

$$=\frac{2\Gamma_0}{\pi\Delta\sqrt{1+C_0^2}}E\left(\frac{1}{\sqrt{1+C_0^2}}\right)=I_1(\Gamma/\Gamma_0), \quad (2)$$

where κ_n is the thermal conductivity in the normal state when $\Gamma=\Gamma_0$, and Γ is the quasiparticle scattering rate in the normal state. Here $\langle\ldots.\rangle$ denotes the average over ϕ and χ, and Eq. (1) tells us that the planar thermal conductivity is independent of the gap functions given above. Also $\Gamma_0=(\pi/2\gamma)T_c=0.882T_c$ and T_c is the superconducting transition temperature of the pure system. However, the quasiparticle scattering rate at $E=0$ is given by ΔC_0, and C_0 is determined by[20]

$$\frac{C_0^2}{\sqrt{1+C_0^2}}K\left(\frac{1}{\sqrt{1+C_0^2}}\right)=\frac{\pi\Gamma}{2\Delta} \quad (3)$$

and $\Delta=\Delta(0,\Gamma)$ has to be determined self-consistently as in Ref. 20. Here $K(k)$ and $E(k)$ are the complete elliptic integrals. We show $I_1(\Gamma/\Gamma_0)$ in Fig. 1. Now let us look at the out-of-plane thermal conductivity κ_{zz}. This is given by

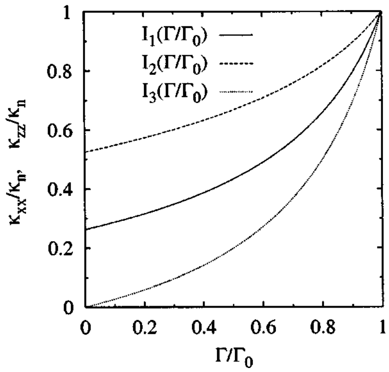

FIG. 1. The functions I_1, I_2 and I_3.

$$\kappa_{zz}/\kappa_n = \frac{\Gamma_0}{\Delta}\left\langle [1-\cos(2\chi)]\frac{C_0^2}{(C_0^2+|f|^2)^{3/2}}\right\rangle, \tag{4}$$

$$= I_1(\Gamma/\Gamma_0) \tag{5}$$

for $f=\cos(2\phi)$, $\sin(2\phi)$, and $\cos(2\chi)$, but

$$\frac{\kappa_{zz}}{\kappa_n} = \frac{4\Gamma_0}{\pi\Delta\sqrt{1+C_0^2}}\left[E\left(\frac{1}{\sqrt{1+C_0^2}}\right) - C_0^2\left(K\left(\frac{1}{\sqrt{1+C_0^2}}\right) - E\left(\frac{1}{\sqrt{1+C_0^2}}\right)\right)\right], \tag{6}$$

$$= I_2\left(\frac{\Gamma}{\Gamma_0}\right) \tag{7}$$

for $f=\cos\chi$, $e^{\pm i\phi}\cos\chi$, and

$$\kappa_{zz}/\kappa_n = \frac{2\Gamma_0\Gamma}{\Delta}\left(1-\frac{E\left(\frac{1}{\sqrt{1+C_0^2}}\right)}{K\left(\frac{1}{\sqrt{1+C_0^2}}\right)}\right) \equiv I_3\left(\frac{\Gamma}{\Gamma_0}\right) \tag{8}$$

for $f=\sin\chi$, $e^{i\phi}\sin\chi$. These functions are shown in Fig. 1.

In Fig. 2 we show $\kappa_{yy}(H)$ and κ_{zz} for $\mathbf{H}\|\hat{z}$ taken for UPd_2Al_3 (Ref. 18). In particular $(\kappa_{00})_{yy}=(\kappa_{00})_{zz}$ indicates $\Delta(\mathbf{k})\sim\cos(2\chi)$. Of course the effect of the magnetic field is not equivalent to the effect of impurities. But this comparison points to $\Delta(\mathbf{k})\sim\cos(2\chi)$ for UPd_2Al_3. We note also that for $f=\sin\chi$ and $e^{i\phi}\sin\chi$, there will be no universal heat conduction in κ_{zz}.

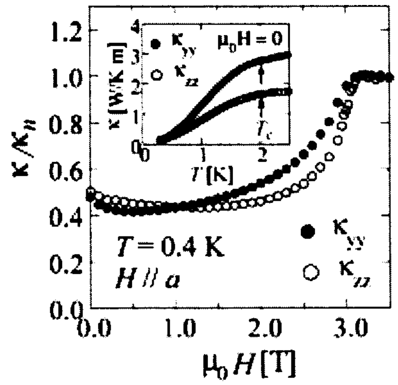

FIG. 2. $\kappa_{yy}(H)$ and $\kappa_{zz}(H)$ for UPd_2Al_3.

III. ANGLE-DEPENDENT MAGNETOTHERMAL CONDUCTIVITY

First let us recapture the quasiparticle density of states in the vortex state of nodal superconductors. For simplicity we consider f's with horizontal nodes: $f=e^{i\phi}\cos\chi$, $\cos\chi$, $\cos 2\chi$, $\sin\chi$, and $e^{i\phi}\sin\chi$ (Ref. 21). Then the first two f's have nodes at $\chi_0=\pm\pi/2$, whereas $f=\cos 2\chi$ at $\chi_0=\pm\pi/4$ and $f=\sin\chi$ and $e^{i\phi}\sin\chi$ at $\chi_0=0$.

In an arbitrary field orientation we obtain the quasiparticle density of states

$$\mathcal{G}(\mathbf{H}) \equiv \frac{N(0,\mathbf{H})}{N_0} = \frac{2}{\pi^2}\frac{v_a\sqrt{eH}}{\Delta}I_1(\theta) \tag{9}$$

for the superclean limit and

$$\mathcal{G}(\mathbf{H}) \simeq \left(\frac{2\Gamma}{\pi\Delta}\right)^{1/2}\left[\log\left(4\sqrt{\frac{2\Delta}{\pi\Gamma}}\right)\right]^{1/2}\left[1+\frac{v_a^2 eH}{8\pi^2\Gamma\Delta}\times\log\left(\frac{\Delta}{v_a\sqrt{eH}}\right)I_2(\theta)\right] \tag{10}$$

for the clean limit, where

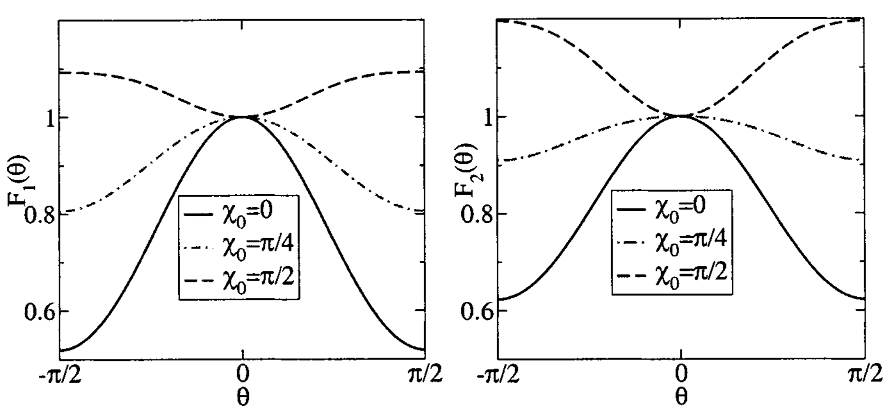

FIG. 3. The angular functions $F_1(\theta)$ (left) and $F_2(\theta)$.

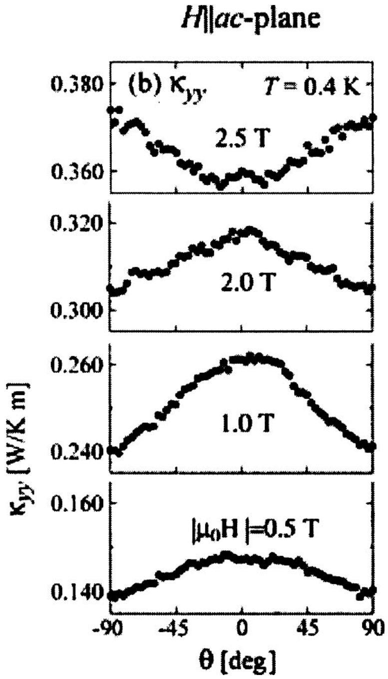

FIG. 4. Angular-dependent magnetothermal conductivity κ_{yy} of UPd_2Al_3.

$$
\begin{aligned}
I_1(\theta) &= (\cos^2\theta + \alpha\sin^2\theta)^{1/4}\frac{1}{\pi}\int_0^{\pi} d\phi[\cos^2\theta + \sin^2\theta \\
&\quad \times(\sin^2\phi + \alpha\sin^2\chi_0) + \sqrt{\alpha}\sin(\chi_0)\cos\phi\sin(2\theta)]^{1/2} \\
&\simeq (\cos^2\theta + \alpha\sin^2\theta)^{1/4} \\
&\quad \times\left[1 + \sin^2\theta\left(-\frac{1}{2} + \alpha\sin^2\chi_0\right)\right]^{1/2} \\
&\quad \times\left[1 - \frac{1}{64}\frac{\sin^2\theta(\sin^2\theta + 16\alpha\sin^2\chi_0\cos^2\theta)}{\left(1 + \sin^2\theta\left(-\frac{1}{2} + \alpha\sin^2\chi_0\right)\right)^2}\right]
\end{aligned}
$$

and

$$
I_2(\theta) = (\cos^2\theta + \alpha\sin^2\theta)^{1/2}\left[1 + \sin^2\theta\left(-\frac{1}{2} + \alpha\sin^2\chi_0\right)\right]. \tag{11}
$$

Here $\alpha=(v_c/v_a)^2$ and θ is the angle $\mathbf{H}$ makes from the z axis. Then the specific heat, the spin susceptibility, and the planar superfluid density in the vortex state in the limit $T\to 0$ K are given by[22]

$$
C_s/\gamma_N T = \mathcal{G}(\mathbf{H}), \quad \frac{\chi_S}{\chi_N} = \mathcal{G}(\mathbf{H}), \tag{12}
$$

$$
\frac{\rho_{s\parallel}(\mathbf{H})}{\rho_{s\parallel}(0)} = 1 - \mathcal{G}(\mathbf{H}). \tag{13}
$$

Similarly the thermal conductivity κ_{yy} when the magnetic field is rotated in the z-x plane is given by

$$
\frac{\kappa_{yy}}{\kappa_n} = \frac{2}{\pi^3}\frac{v_a^2 eH}{\Delta^2}F_1(\theta) \tag{14}
$$

in the superclean limit and

$$
\frac{\kappa_{yy}}{\kappa_{00}} = 1 + \frac{v_a^2(eH)}{6\pi^2\Gamma\Delta}F_2(\theta)\log\left(2\sqrt{\frac{2\Delta}{\pi\Gamma}}\right)\log\left(\frac{2\Delta}{v_a\sqrt{eH}}\right) \tag{15}
$$

in the clean limit where

$$
F_1(\theta) = \sqrt{\cos^2\theta + \alpha\sin^2\theta}\left[1 + \sin^2\theta\left(-\frac{3}{8} + \alpha\sin^2\chi_0\right)\right], \tag{16}
$$

$$
F_2(\theta) = \sqrt{\cos^2\theta + \alpha\sin^2\theta}\left[1 + \sin^2\theta\left(-\frac{1}{4} + \alpha\sin^2\chi_0\right)\right]. \tag{17}
$$

We show in Fig. 3, $F_1(\theta)$ and $F_2(\theta)$ for $\alpha=0.69$ (the value appropriate for UPd_2Al_3) and $\chi_0=0$, $\pi/4$, and $\pi/2$, which is compared with the experimental data[18] taken at $T=0.4$ K shown in Fig. 4. Except for the data taken for $H=2.5$ T, the data for $H=0.5$, 1, and 2 T are consistent with $\chi_0=\pi/4$, indicating again $f=\cos 2\chi$. We note also the sign of the twofold term in κ_{yy} at $T=0.4$ K changes sign at $H=0.36$ T. This is consistent with the fact that for $T < v\sqrt{eH}$ the nodal excitations are mostly due to the Doppler shift while for $T > v\sqrt{eH}$ the thermal excitations dominate.[23]

IV. CONCLUDING REMARKS

We have analyzed recent thermal conductivity data[18] of UPd_2Al_3 from two perspectives: universal heat conduction and the angle dependence. The present study indicates $\Delta(\mathbf{k})=\Delta\cos(2\chi)$. This is different from the conclusion reached in Ref. 18. Also we have extended the universal heat conduction for a class of superconducting order parameters $\Delta(\mathbf{k})$, which will be useful for identifying the gap symmetry of new superconductors, such as URu_2Si_2 and UNi_2Al_3.

Furthermore, we have worked out the expressions for κ_{yy} when the magnetic field is rotated within the z-x plane. The angle dependence of κ_{yy} is extremely useful to locate the nodal lines when all nodal lines are horizontal. Perhaps κ_{yy} in Sr_2RuO_4 will help to identify the precise position of the horizontal nodal lines in $\Delta(\mathbf{k})$, if a further study of nodal lines is necessary. Also after UPt_3 and UPd_2Al_3 we expect many of the U-compound superconducting energy gaps to have horizontal lines.

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