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Role of nonmagnetic disorder on the stability of the $U(1)$ spin liquid: A renormalization group study

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Recently, Hermele *et al.* claimed that the infrared (IR) fixed point of noncompact QED_3 is stable against instanton excitations in the limit of large flavors of massless Dirac fermions [M. Hermele *et al.*, cond-mat/0404751, Phys. Rev. B (to be published October 2004)]. We investigate an effect of nonmagnetic disorder on the deconfined quantum critical phase dubbed $U(1)$ spin liquid (U1SL) in the context of quantum antiferromagnet. In the case of weak disorder the IR fixed point remains stable against the presence of both the instanton excitations and nonmagnetic disorder and thus the U1SL is sustained. In the case of strong disorder the IR fixed point becomes unstable against the disorder and the Anderson localization is expected to occur. We argue that in this case deconfinement of spinons does not occur since the Dirac fermion becomes massive owing to the localization.

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Recently Hermele *et al.* pointed out that usual random-phase approximation (RPA) treatment of gauge fluctuations[1] is not sufficient in order to examine an instanton effect even in the presence of large flavors (N) of massless Dirac fermions in compact QED_3.[2] They showed that at the infrared (IR) stable fixed point of noncompact spinor QED_3 in large N limit instanton excitations become irrelevant and instanton fugacity goes to zero. This originates from the fact that a magnetic charge has a large value proportional to N at the fixed point. The large fixed point value of a magnetic charge is due to screening of an electric charge by particle-hole excitations of the massless Dirac fermions.[3] The magnetic charge goes to zero in the absence of the massless Dirac fermions at low energy.[2] The larger the magnetic charge, the smaller the probability of instanton excitations. As a consequence they concluded that deconfinement does exist at least at the critical point in the large N limit. In the context of quantum antiferromagnet stable $U(1)$ spin liquid (U1SL) is obtained.[2]

In realistic cases disorder always exists. In the case of noninteracting fermions it was shown by scaling argument that in three spatial dimensions the presence of disorder causes a metal-insulator transition.[4] But in one or two spatial dimensions even weak disorder leads electrons to be localized and only insulating phase is expected to exist.[4] The presence of long-range interaction can change the above picture of noninteracting particles. Herbut studied the role of a random potential resulting from nonmagnetic disorder on a critical field theory of interacting bosons via the Coulomb interaction.[5] In the study he showed that competition between the random potential and the Coulomb interaction leads to a new charged critical point near three spatial dimensions where a dynamical critical exponent z is exactly given by 1.[5] In the absence of the random potential the charged critical point is not expected to exist and only the standard runaway characteristic is found.[5] In the case of Fermi fields Ye investigated the role of disorder on a Chern-Simons field theory of interacting Dirac fermions via Coulomb interaction in two spatial dimensions.[6] In the study he found a line of fixed points which is stable in some cases.[6] In the absence of both the randomness and Chern-Simons interaction only the runaway characteristic was found as the case of bosons.

In this paper we investigate the role of nonmagnetic disorder on the deconfined quantum critical phase of QED_3 in the limit of large flavors of massless Dirac fermions. In the concrete we examine the stability of the IR fixed point of noncompact QED_3 in the presence of both the nonmagnetic disorder and instanton excitations. The existence of the stable IR fixed point in the absence of disorder is a main difference from previous works.[5,6] In the previous studies[5,6] there are no stable IR fixed points in the absence of disorder as discussed above. In the case of weak disorder the IR fixed point is found to remain stable and $U(1)$ spin liquid (U1SL) in the context of quantum antiferromagnet is expected to survive. The stability against the weak disorder results from the existence of the IR fixed point in the absence of disorder in two space and one time dimension. In the case of strong disorder we find that it becomes unstable. We are led to the strong-coupling regime where the Anderson localization is expected to occur. Owing to the localization the Dirac fermions become massive. In the case of massive Dirac fermions the usual RPA treatment may be possible. Instantons are expected to be proliferated. We argue that deconfinement of spinons is not expected to exist in the strong disorder. In addition, we discuss a bosonic field theory in the presence of nonmagnetic disorder and find a difference in the role of disorder on the Dirac fermions and bosons, respectively.

First we review deconfinement at the IR fixed point of QED_3 in the absence of nonmagnetic disorder.[2] We consider an effective action usually called QED_3 in imaginary time

$$S = \int d^D x \left[\sum_{\sigma=1}^{N} \bar{\psi}_\sigma \gamma_\mu (\partial_\mu - i a_\mu) \psi_\sigma + \frac{1}{2e^2} |\partial \times a|^2 \right]. \quad (1)$$

Here ψ_σ is a massless Dirac spinor with a flavor index $\sigma = 1, \ldots, N$ and a_μ, a compact U(1) gauge field. $x = (\mathbf{r}, \tau)$ with $(D-1)$-dimensional space $\mathbf{r}$ and imaginary time τ. In the context of U(1) slave boson representation of SU(N) quantum antiferromagnet this action can be considered as an effective action in the π flux phase.[7] In this case the Dirac spinor represents a spinon carrying only the spin quantum number $1/2$. It is well known that QED_3 with the noncompact U(1) gauge field has a stable IR fixed point in the limit of large flavors of massless Dirac fermions.[8] A renormalization group (RG) equation for an electric charge is easily obtained to be in one-loop order,[2,8]

$$\frac{de^2}{dl} = (4-D)e^2 - \lambda N e^4, \tag{2}$$

where λ is a positive numerical constant. The first term represents a bare scaling dimension of e^2. In $(2+1)D$, e^2 is relevant in contrast with $(3+1)D$, where it is marginal. The second term originates from self-energy correction of the U(1) gauge field by particle-hole excitations of massless Dirac fermions. As shown by this RG equation, a stable IR fixed point of $e^{*2}=1/\lambda N$ exists in QED_3. Now our question is if the IR fixed point remains stable after admitting instanton excitations. Using the electromagnetic duality, Hermele *et al.* obtained RG equations of a magnetic charge $g=1/e^2$ and an instanton fugacity y_m,

$$\frac{dg}{dl} = -(4-D)g - \alpha y_m^2 g^3 + \lambda N,$$

$$\frac{dy_m}{dl} = (D-\beta g)y_m, \tag{3}$$

where α and β are positive numerical constants.[2] Owing to the last term λN in the first equation a magnetic charge has a large fixed point value proportional to N, i.e., $g^*=\lambda N$. As a consequence the instanton fugacity goes to zero at this IR fixed point. U1SL in terms of a spinon (Dirac fermion) and noncompact U(1) gauge field is obtained at the quantum critical phase.

Next we investigate the stability of the U1SL fixed point in the presence of nonmagnetic disorder. We reconsider QED_3 in the presence of nonmagnetic disorder

$$S = \int d^D x \left[\sum_{\sigma=1}^{N} \bar{\psi}_\sigma \gamma_\mu (\partial_\mu - iea_\mu)\psi_\sigma + \frac{1}{2}|\partial \times a|^2 + V(x)\sum_{\sigma=1}^{N} \bar{\psi}_\sigma \gamma_0 \psi_\sigma \right]. \tag{4}$$

Here $V(x)$ is a random potential generated by nonmagnetic disorder. It couples to a spinon density owing to the relation of $V\Sigma_{\sigma=1}^N c_\sigma^\dagger c_\sigma = V\Sigma_{\sigma=1}^N f_\sigma^\dagger f_\sigma$.[7] Here c_σ represents an electron with spin σ and f_σ, a spinon with spin σ. A physically relevant case is that the random potential is random only in space but static in time. Thus it does not depend on imaginary time, i.e., $V(x)=V(\mathbf{r})$. We assume that $V(\mathbf{r})$ is a Gaussian random potential with $\langle V(\mathbf{r})V(\mathbf{r}')\rangle = W\delta(\mathbf{r}-\mathbf{r}')$ and $\langle V(\mathbf{r})\rangle=0$.[5,6] Using the standard replica trick to average over the Gaussian random potential, we obtain an effective action in the presence of nonmagnetic disorder

$$S = \int d^{D-1}\mathbf{r} d\tau \left[\sum_{\alpha=1}^{M} \left(\sum_{\sigma=1}^{N} \bar{\psi}_{\sigma,\alpha} \gamma_\mu (\partial_\mu - iea_{\mu,\alpha})\psi_{\sigma,\alpha} + \frac{1}{2}|\partial \times a_\alpha|^2 \right) \right] - \frac{W}{2}\sum_{\alpha,\alpha'=1}^{M}\sum_{\sigma,\sigma'=1}^{N}\int d^{D-1}\mathbf{r} d\tau_1 d\tau_2 \bar{\psi}_{\sigma,\alpha}(\mathbf{r},\tau_1)\gamma_0\psi_{\sigma,\alpha}(\mathbf{r},\tau_1) \times \bar{\psi}_{\sigma',\alpha'}(\mathbf{r},\tau_2)\gamma_0\psi_{\sigma',\alpha'}(\mathbf{r},\tau_2). \tag{5}$$

Here α,α' are replica indices and the limit $M\to 0$ is to be taken at the end.

Introducing renormalized field variables of $\psi_\sigma = e^{-[(D+z-2)/2]l} Z_k^{1/2}\psi_{\sigma,r}$ and $a_\mu = e^{-[(D+z-3)/2]l} Z_a^{1/2} a_{\mu,r}$, we obtain renormalized couplings of $e^2 = e^{-(5-D-z)l} Z_a^{-1} e_r^2$ and $W = e^{-(4-D-z)l} Z_k^{-2} Z_W W_r$. Here z is a dynamical critical exponent. Z_k, Z_a, Z_W are usual renormalization constants of a Dirac fermion, gauge field, and strength of a random potential, respectively. A subscript r represents "renormalized." Equation (5) is obtained to be in terms of renormalized variables

$$S = \int d^{D-1}\mathbf{r}' d\tau' \left[\sum_{\alpha=1}^{M} \left(\sum_{\sigma=1}^{N} Z_k \bar{\psi}_{\sigma,\alpha} \gamma_\mu (\partial'_\mu - iea_{\mu,\alpha})\psi_{\sigma,\alpha} + \frac{Z_a}{2}|\partial' \times a_\alpha|^2 \right) \right] - Z_W\frac{W}{2}\sum_{\alpha,\alpha'=1}^{M}\sum_{\sigma,\sigma'=1}^{N}\int d^{D-1}\mathbf{r}' d\tau'_1 d\tau'_2 \bar{\psi}_{\sigma,\alpha}(\mathbf{r}',\tau'_1) \times \gamma_0\psi_{\sigma,\alpha}(\mathbf{r}',\tau'_1)\bar{\psi}_{\sigma',\alpha'}(\mathbf{r}',\tau'_2)\gamma_0\psi_{\sigma',\alpha'}(\mathbf{r}',\tau'_2) \tag{6}$$

with rescaled space $\mathbf{r}'=e^{-l}\mathbf{r}$ and time $\tau'=e^{-zl}\tau$. In the above we omitted a subscript r for a simple notation. Calculating the renormalization constants Z_k, Z_a, Z_W in one-loop order, we obtain RG equations

$$\frac{de^2}{dl} = (5-z-D)e^2 - \lambda N e^4,$$

$$\frac{dW}{dl} = (4-z-D-\chi e^2)W + \zeta(N+c)W^2 \tag{7}$$

with positive numerical constants λ,χ,ζ,c. Here z is determined by the condition of $e^{-2zl}Z_\omega = e^{-2l}Z_k$ with Z_k, a renormalization constant of a Dirac fermion in momentum and Z_w, that in energy,[5] which gives

$$z = 1 + AW \tag{8}$$

with a positive numerical constant A. Our interest is the case of $D=2+1$, i.e., two space and one time dimensions. These RG equations basically coincide with those of Ref. 6 in $D=2+1$ if the term $(4-D)e^2$ in the first RG equation is neglected. The presence of this term leads to the stable IR fixed point as discussed earlier. Precise values of the positive numerical constants are not important in our consideration. We solve the above RG equations with arbitrary positive numerical constants in order to understand a general structure.

First we check the case of noninteracting Dirac fermions in the presence of a random potential. In zero charge limit $(e \to 0)$ a RG equation is given by

$$\frac{dW}{dl} = (4 - z - D)W + \zeta(N + c)W^2. \tag{9}$$

In $D=3+1$ (three spatial dimensions) there is an unstable fixed point of $W_c=1/\zeta(N+c)$ with $z=1+[A/\zeta(N+c)]$ to separate a metal and an insulator. In $D=2+1$ of present interest the unstable fixed point becomes zero, i.e., $W_c=0$. Thus only insulating phase is expected to exist[4] as discussed in the Introduction.

In the case of interacting Dirac fermions via long-range "electromagnetic" interaction[9] these RG equations [Eq. (7)] show three fixed points in $D=2+1$; the first is $W_{1c}=0$ and $e_{1c}^2=0$ with $z=1$, the second, $W_{2c}=0$ and $e_{2c}^2=1/\lambda N$ with $z=1$, and the third, $W_{3c}=O(1/N^2)$ and $e_{3c}^2=(1/\lambda N)+O(1/N^3)$ with $z=1+O(1/N^2)$. The first is a fixed point of free Dirac fermions which is unstable for nonzero charge e^2.[10] The RG flow goes to the second, the IR fixed point of noncompact QED_3. The third is an unstable fixed point. This fixed point does not exist in the absence of the gauge interaction. The existence of the new unstable fixed point originates from the term $(4-D)e^2$ of the first RG equation in Eq. (7), representing the relevance of an electric charge in $D=2+1$. In the case of small strength of the random potential, i.e., $W<W_{3c}$, the random potential becomes irrelevant and the usual IR fixed point (the second) remains stable. The stability against the weak disorder results from the existence of the stable IR fixed point in the absence of disorder in $D=2+1$, as will be shown below. In the case of large strength of the random potential, i.e., $W>W_{3c}$, the random potential becomes stronger. We are led to the strong-coupling regime where our perturbative calculation does not apply. In this case the Anderson localization is expected to occur and thus the Dirac fermions become massive.

In order to see the stability of the fixed points we expand the RG equations near each fixed point. Inserting $e^2=e_{2c}^2+f$ and $W=W_{2c}+h$ to Eq. (7), we obtain linearized RG equations near the IR fixed point

$$\frac{df}{dl} = -f - \frac{A}{\lambda N}h,$$

$$\frac{dh}{dl} = -\frac{\chi}{\lambda N}h. \tag{10}$$

As shown by these RG equations, it is clear that the IR fixed point is stable against the weak disorder. The stability originates from the finite fixed-point value of an electric charge, i.e., $e_{2c}^2=1/\lambda N$. Expanding the RG equations near the third fixed point, we obtain linearized RG equations to the order of $1/N$

$$\frac{df}{dl} = -f - \frac{A}{\lambda N}h,$$

$$\frac{dh}{dl} = \frac{\chi}{\lambda N}h. \tag{11}$$

The second equation shows the instability of this fixed point.

Now we examine an instanton effect on these fixed points. Using the electromagnetic duality, we obtain RG equations of a magnetic charge $g=e^{-2}$ and an instanton fugacity y_m in the presence of a random potential

$$\frac{dg}{dl} = -(5 - z - D)g - \alpha y_m^2 g^3 + \lambda N,$$

$$\frac{dW}{dl} = \left(4 - z - D - \chi\frac{1}{g}\right)W + \zeta(N + c)W^2,$$

$$\frac{dy_m}{dl} = (z - 1 + D - \beta g)y_m \tag{12}$$

with positive numerical constants α,β the same as those in Eq. (3). The term $-\alpha y_m^2 g^3$ in the first RG equation is added owing to the screening of a magnetic charge by instanton excitations.[2] In the case of weak disorder, i.e., $W<W_{3c}$, the IR fixed point of $g^*=\lambda N$ remains stable as discussed above. Thus the U1SL is sustained by the same reason as the case of no disorder. In the case of strong disorder, i.e., $W>W_{3c}$, the IR fixed point becomes unstable. The strength of disorder becomes larger. An important question is whether the magnetic charge goes to zero or not as the strength of disorder gets larger. At first glance of Eq. (11) the fixed point value of an electric charge seems to be sustained. Thus one may conclude that U1SL is still expected to occur. In this case, more precisely, gapped U1SL may appear owing to the Anderson localization which is expected to occur in the strong disorder. But this is an illusion. As discussed above, the Dirac fermions are expected to be massive owing to the localization. The above calculation cannot apply to this strong-coupling regime. Further, this phase is not expected to be critical any more. In this "insulating" phase of spinons the screening of the internal charge becomes negligible. The internal charge is expected to go to infinity following its bare scaling dimension. The magnetic charge goes to zero. In this case instanton excitations are relevant perturbation and the instanton fugacity gets larger to go to infinity. Thus deconfinement of spinons is not expected to occur in the strong disorder.

Now we discuss an effect of nonmagnetic disorder on a critical-field theory of interacting bosons via long-range electromagnetic interaction. The critical-field theory usually dubbed scalar QED_3 can be considered to describe interacting holons via internal gauge interaction in the context of $U(1)$ slave boson theory.[7,11] In the scalar QED_3 a stable IR fixed point called charged or IXY fixed point is also found in the limit of large N[12] as the case of the spinor QED_3. Here N is the flavor number of boson fields. The charged fixed point governs a superconductor to insulator transition. The charged fixed point is shown to be unstable in the presence of weak disorder.[13] A new stable fixed point is expected to appear in association with a random potential. This fixed point seems to be related with a Bose glass to a superconductor transition,[5,13] not a Mott insulator to superconductor transition governed by the charged fixed point.[12] But its nature is not clear. The emergence of the new stable fixed point is a main difference between the scalar and spinor QED_3. This is because even weak disorder is a relevant perturbation only in a bosonic field theory. The relevance of weak disorder originates from a different scaling between bosons and Dirac fermions.[14] This different scaling is due to a difference in the equation of motion, the Klein-Gordon equation, and the Dirac equation, respectively. At this new stable fixed point the fixed point value of an electric charge square is still proportional to inverse of the flavor number of bosons, i.e., $e_c^2 \sim N^{-1}$.[13] Thus a magnetic charge is proportional to N as the case of the charged fixed point. In the limit of large flavor number instanton fugacity goes to zero at this new fixed point. Deconfinement of boson fields (holons) is expected to occur. Bosonic U(1) liquid is sustained at the new fixed point associated with nonmagnetic disorder.

In this paper we considered SU(N) quantum antiferromagnet described by N flavors of massless Dirac fermions interacting via compact U(1) gauge fields. But a real antiferromagnet has SU(2) symmetry. Thus the flavor number of the Dirac fermions is given by $N=2$.[7] In this case it is not clear whether the present result can be applicable. It is known that there exists a critical flavor number N_c associated with spontaneous chiral symmetry breaking (SχSB) in QED_3[7]. But a precise value of the critical number is still in debate.[15] If the critical value is larger than 2, the SχSB is expected to occur for the physical $N=2$ case.[7,16] The Dirac fermions become massive. In the SχSB phase the massive Dirac fermions are confined to form mesons, here, magnons.[7] As a result, in the case of $N_c > N=2$ the U1SL is not expected to exist and the present consideration is not applied. But there is a cure even in this case. We consider hole doping to the U1SL. Doped holes are represented by holons carrying only a charge degree of freedom.[17] Recently Senthil and Lee claimed that critical fluctuations of holons at a quantum critical point associated with a superconducting transition can result in the suppression of instanton excitations and thus the quantum critical point can be described by the U1SL for a spin degree of freedom.[17] This argument is based on the fact that the SχSB does not occur owing to critical fluctuations of holons. The critical fluctuations increase a flavor number of massless fluctuations.[18] If a total flavor number of massless spinons and holons exceeds the critical value N_c, the SχSB is not expected to occur.[18,19] As a result the present scenario for the role of nonmagnetic disorder in the U1SL has a chance to be applicable.

To summarize, we showed that the $U(1)$ spin liquid is sustained against the presence of weak nonmagnetic disorder. However, strong disorder leads the fixed point to be unstable and the RG flow goes to the strong-coupling regime. In the strong-coupling regime our perturbative RG does not work any more. Thus a more refined calculation is required. We argued that deconfinement of spinons does not occur in this regime since the Dirac fermions become massive owing to localization. Lastly, we compared the case of Dirac fermions with that of bosons. In a bosonic field theory a new stable fixed point associated with disorder emerges in contrast with a fermionic field theory. At this new stable fixed point an electric charge is sufficiently screened and deconfinement of bosons is expected to occur.

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[2] M. Hermele, T. Senthil, M. P. A. Fisher, P. A. Lee, Nasto Nagaosa, and X.-G. Wen, cond-mat/0404751, Phys. Rev. B (to be published October 2004).

[3] Consider the electromagnetic duality of $eg=1$ where e is an electric charge and g, a magnetic charge. If an electric charge becomes small owing to screening effect by massless Dirac fermions, a magnetic charge gets large.

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[8] H. Kleinert *et al.*, Phys. Rev. Lett. **88**, 232001 (2002); Nucl. Phys. B **666**, 361 (2003).

[9] Here the electric charge e is related not with a real electromagnetic field A_μ but an internal U(1) gauge field a_μ (Ref. 7).

[10] In the case of Coulomb interaction the fixed point of free Dirac fermions is stable against small charge (Ref. 6). This is because there is no term like $(4-D)e^2$ of the first RG equation in Eq. (7).

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[14] From a bosonic action of $S=\int d^{D-1}r d\tau |\partial_\mu \phi|^2$ a scaling is easily determined by $\phi = e^{-[(D+z-3)/2]l} Z_k^{1/2} \phi_r$ with a renormalized field ϕ_r and a renormalization constant Z_k. We note the difference between the Dirac fermion and boson by the exponent $(D+z-2)/2$ and $(D+z-3)/2$, respectively. This difference changes $-\chi e^2 W$ at $D=3$ in Eq. (7) into $(2-\chi e^2)W$ in Eq. (6) of Ref. 5.

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[17] T. Senthil and P. A. Lee, cond-mat/0406066 (unpublished).

[18] A flavor number of massless fluctuations is given by $N=N_f+N_b$. Here N_f is the flavor number of massless Dirac fermions and N_b, that of critical holon fluctuations. In a SU(2) slave boson theory (Ref. 17) N_b is given by 2. Thus the total flavor number is $N=4$. This exceeds a known critical value $N_c \approx 3.24$ (Ref. 7). Chiral symmetry breaking is not expected to occur.

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