

Current-carrying Andreev bound states in a superconductor-ferromagnet proximity system

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We study the ground-state properties of a ferromagnet-superconductor heterostructure on the basis of a quasiclassical theory. We have solved the Eilenberger equations together with Maxwell's equation fully self-consistently and found that due to the proximity effect a Fulde-Ferrel-Larkin-Ovchinnikov (FFLO)-like state is realized in such a system. Moreover, this state has oscillations of the pairing amplitude in either one or two directions, depending on the exchange splitting and thickness of the ferromagnet. In particular, using semiclassical arguments (Bohr-Sommerfeld quantization rule) we show that owing to the presence of the Andreev bound states in the ferromagnet, a spontaneous current in the ground state is generated as a hallmark of the FFLO state in the direction parallel to the interface. We also discuss the effects of the elastic disorder and finite transparency of the interface on the properties of the FFLO state in the system.

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As is well known, ferromagnetism and singlet pairing superconductivity are competing phenomena. While an exchange interaction favors parallel spin alignment, Cooper pairs must be in a singlet (spin) state. But the two phenomena are less mutually exclusive in artificially made ferromagnet-superconductor (FM/SC) heterostructures.¹ In such structures the ferromagnetism and the superconductivity can coexist near the FM/SC interface owing to the proximity effect.² In the case when the normal metal is not a ferromagnet, the proximity effect has been studied for a long time.² By now it is rather well understood in terms of the Andreev reflection processes.³ By contrast, the proximity effect in FM/SC systems has become a center of attention only recently. It is not only important from a scientific point of view, as it allows for study of the interplay between magnetism and superconductivity,⁴ but also from a technological one, as it may find applications in magnetoelectronics⁵ and quantum computing.⁶

A number of new phenomena has been revealed in FM/SC multilayers. The most interesting examples are non-monotonic behavior of the SC transition temperature,⁷ oscillations of a pairing amplitude^{8–10} and the density of states in the FM,^{11–14} paramagnetic Meissner effect,¹⁵ proximity induced very long range triplet superconductivity in FM,¹⁶ or generation of spontaneous currents in the ground state of such systems.¹⁴ These unusual properties, associated with Cooper pairs in an exchange field, can be explained in terms of a phenomenon first identified by Fulde, Ferrell, Larkin, and Ovchinnikov (FFLO).¹⁷ Originally it has been studied in a bulk superconductor with the exchange splitting. It turns out that, although of great conceptual interest, the bulk FFLO state can be realized only in a very small region of the parameter space near the transition to the normal state¹⁷ and usually is destroyed when the exchange splitting E_{ex} is larger than $\sqrt{2}/2\Delta$ (Clogston criterion),¹⁸ where Δ is the SC energy gap. Moreover, the FFLO state is very sensitive to both elastic and spin-orbit scattering.¹⁹ The last two effects make the FFLO state very difficult to observe experimentally in bulk

samples. The situation is much more favorable in FM/SC heterostructures where, due to the proximity effect, the Cooper pairs can survive even if the exchange field in FM is much larger than the SC gap.

According to our current understanding of the FFLO phenomenon in a FM/SC structure, when a Cooper pair enters the ferromagnet it acquires a center of mass momentum $\hbar Q = 2E_{ex}/v_F$,¹⁰ where v_F is Fermi velocity. As a consequence of this, the pairing amplitude picks up a phase $\phi = Q_x x$, and hence oscillates with the distance x from the interface in ferromagnet. It turns out that under certain conditions a 3D-FFLO state, featuring a spatial dependence of the pairing amplitude also along the interface, namely $\phi = Q_y y + Q_z z$, can be realized.^{14,20} It is this latter case that we shall deal with here.

The purpose of the present paper is to demonstrate that in a ferromagnetic layer on a superconducting substrate, for particular values of the exchange splitting E_{ex} and the layer thickness d_F , a FFLO-like state is realized with the pairing amplitude f varying both perpendicular and parallel to the FM/SC interface. It will be shown that such a ground state supports a spontaneously generated current flowing in opposite directions in the FM and the SC regions. The existence of this remarkable state was first predicted on the basis of a simple lattice model.¹⁴ Here we shall deal with the problem by a less model dependent, semiclassical approach and address the issue of the observability of the phenomenon in the presence of disorder within FM layer and at the FM/SC interface.

The system we consider is sketched in Fig. 1. It consists of a thin ferromagnet (FM) of thickness d_F deposited on a semi-infinite superconductor (SC) and bounded on the other side by an insulator.

In such an I/FM/SC quantum well there will be bound states corresponding to the closed quasiparticle trajectories.^{14,21} Each trajectory consists of an electron segment, e , which includes an Andreev reflection at the FM/SC interface and an ordinary reflection at the I/FM interface plus

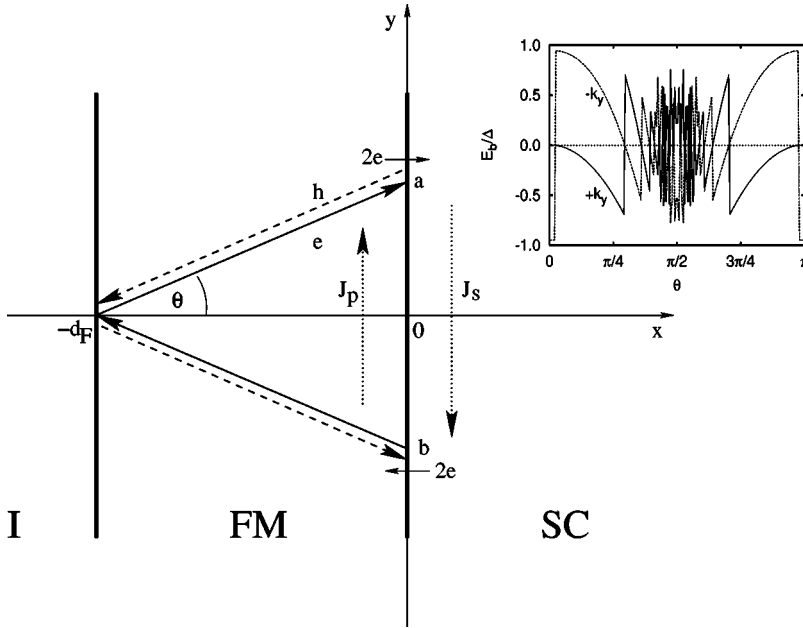


FIG. 1. The (Bohr-Sommerfeld) semiclassical trajectories for quasiparticles which scatter specularly at the I/FM interface and by Andreev reflections at the FM/SC interface. Note that the trajectories have particlelike and holelike segments and they imply an electric current, J_p , in the y direction. Inset: The energy of the Andreev bound states associated with spin up electrons moving in positive ($+k_y$) and negative ($-k_y$) y direction as a function of θ for $\xi_F/d_F=0.425$ and $\phi=0$. Evidently, some of these bound states are at zero energy.

a hole segment, h , retracing back the electron trajectory (see Fig. 1). The Bohr-Sommerfeld quantization rule:²²

$$\int_b^a p_e(\omega) dl - \int_a^b p_h(\omega) dl + \delta\phi - \gamma(\omega) = 2n\pi \quad (1)$$

gives the energies of the bound states. The first and second terms represent the total phase accumulated by a quasiparticle during propagation through the FM region from b to a [$=4d_F(\omega + \sigma E_{ex})/v_F \cos(\theta)$] and back, $\delta\phi$ is the phase difference of the order parameter between point b and a , and $\gamma(\omega) = \arccos(\omega/\Delta)$ is the Andreev reflection phase shift. An example of the θ dependence of the Andreev bound state (ABS) energies for $\xi_F/d_F=0.425$, where $\xi_F = \hbar v_F/E_{ex}$ is the FM coherence length, and $\delta\phi=0$ is shown in the inset of Fig. 1. Clearly, for any exchange splitting $E_{ex} \neq 0$ it is possible to find such a θ that the corresponding ABS is exactly at zero energy. If there is a large number of such zero-energy ABS for some exchange splittings (more precisely for ξ_F/d_F ratio) the density of states (DOS) has a large peak at the Fermi energy ($\epsilon_F=0$). Such a situation turns out to be energetically unfavorable. There is a number of mechanisms which split this peak and thereby lower the energy of the system.²³ One of these is a spontaneous current which ‘‘Doppler’’ shifts the quasiparticle energies by $\delta = ev_F A_y \cos(\theta)$,²⁴ where A_y is a vector potential in the y direction.

An example of such a density of states is depicted in Fig. 2, where one can see a large peak at zero energy (solid line).

This corresponds to zero SC phase difference between points a and b in Fig. 1, namely no spontaneous current. In the inset of Fig. 2 the Bohr-Sommerfeld energy eigenvalue difference between a state with spontaneous current $E_{GS}(\phi)$ and a state with no current $E_{GS}(0)$ is shown. The corresponding DOS, with the zero energy ABS split, is plotted with the dashed line in Fig. 2.

To summarize, we have shown, using a Bohr-Sommerfeld semiclassical argument, that for any exchange splitting, E_{ex} ,

there are Andreev bound states at zero energy and for certain E_{ex} the number of such states is so large that it produces huge zero energy peak in the density of states. Such a peak in turn is split by a spontaneous current and this lowers the total energy of the system. This current carrying state can be regarded as a realization of the FFLO variation of the pairing amplitude in the y direction. So, one can say that the system can be switched between 1D and 2D FFLO-like states as the exchange field or thickness of the ferromagnet is changed. In the following we will show that this spontaneous current can be also obtained within a self-consistent quasiclassical

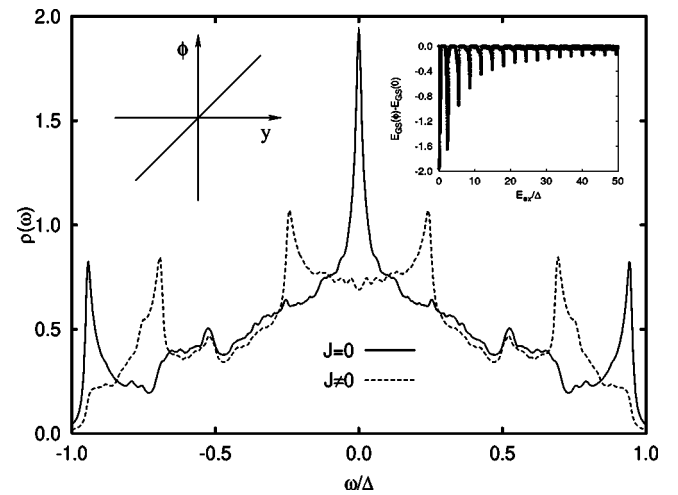


FIG. 2. The total density of states $\rho(\omega)$ for $\xi_F/d_F=0.425$ with no current flow (solid line) and in the presence of the current (dashed line). For $J \propto \partial\phi/\partial y \neq 0$ the zero energy state splits and this lowers the energy of the system. Right inset: The difference between total energies of the ABS with current flow $E_{GS}(\phi)$ and without the current $E_{GS}(0)$ as a function of E_{ex}/Δ ($d_F/\xi_S=1$). Left inset: The phase of the superconducting order parameter ϕ as a function of y . The slope $\partial\phi/\partial y$ implies a supercurrent J_s which is carried by Cooper pairs from point a to b (see Fig. 1).

theory. This approach allows for a treatment of disorder and finite transparency of the interface, and hence provides further useful insights.

The quasiclassical matrix Eilenberger equation²⁵ reads

$$\mathbf{v}_F \nabla \hat{g}_\sigma(\mathbf{v}_F, \mathbf{r}) + \left[\tilde{\omega}_\sigma(\mathbf{r}) \hat{\tau}_3 + \hat{\Delta}(\mathbf{r}) + \frac{1}{2\tau} \langle \hat{g}_\sigma(\mathbf{v}_F, \mathbf{r}) \rangle, \hat{g}_\sigma(\mathbf{v}_F, \mathbf{r}) \right] = 0 \quad (2)$$

where

$$\hat{g}_\sigma = \begin{pmatrix} g_\sigma & f_\sigma \\ f_\sigma^+ & -g_\sigma \end{pmatrix}, \quad \hat{\Delta} = \begin{pmatrix} 0 & \Delta \\ \Delta^* & 0 \end{pmatrix} \quad (3)$$

and

$$\tilde{\omega}_\sigma(\mathbf{r}) = \omega + i\sigma E_{ex} + ie\mathbf{v}_F \mathbf{A}(\mathbf{r}). \quad (4)$$

Here $\hat{\tau}_3$ is the Pauli matrix, $\omega = \pi T(2n+1)$ is the Matsubara frequency, $\sigma (= \pm 1)$ labels the electron spin, $\mathbf{A}(\mathbf{r})$ is the vector potential, and $\langle \cdots \rangle$ denotes averaging over directions of the Fermi velocity \mathbf{v}_F . The matrix Green's function has to obey the normalization condition $\hat{g}_\sigma^2(\mathbf{v}_F, \mathbf{r}) = \hat{1}$. The exchange splitting E_{ex} is nonzero and constant in the ferromagnet only while $\Delta(\mathbf{r})$ is nonzero in the superconductor and is calculated self-consistently from

$$\Delta(\mathbf{r}) = U\pi\rho_0 T \sum_{\omega, \sigma} \langle f_\sigma(\mathbf{v}_F, \mathbf{r}) \rangle, \quad (5)$$

where we have assumed that the coupling constant $U < 0$ in the SC and $= 0$ in the FM. ρ_0 is the normal state DOS and T stands for temperature.

The spontaneous current has to be determined self-consistently together with the Maxwell equation (Ampere's law), which couples the electron current to the magnetic field. The total current in the y direction at each point x measured from the interface is given by

$$J_y^{tot}(x) = 2ie\pi\rho_0 T \sum_{\omega, \sigma} \langle \mathbf{v}_F g_\sigma(\mathbf{v}_F, x) \rangle, \quad (6)$$

where e is the electron charge, while the Maxwell equation (in the Landau gauge) reads

$$\frac{d^2 A_y(x)}{dx^2} = -\mu_0 J_y^{tot}(x), \quad (7)$$

with μ_0 being the permeability of free space.

We have solved the Eilenberger equation (2) numerically along each quasiparticle 2D trajectory using the Riccati parametrization (Schopohl-Maki transformation)²⁶ together with the self-consistency relations (5)–(7).

The most remarkable feature of the self-consistent solution is that the iterations of the Eilenberger equations frequently converge to a solution with a finite value of the current even though there is no external vector potential. These solutions have lower total energy than the corresponding solutions where the constraint $A=0$ is imposed. The current flows in one direction over the whole ferromagnet and flows back on the SC side on the scale of the coherence length ξ_S (see solid curve in Fig. 3), so the total current is zero, as it should in the true ground state. The fact that the current

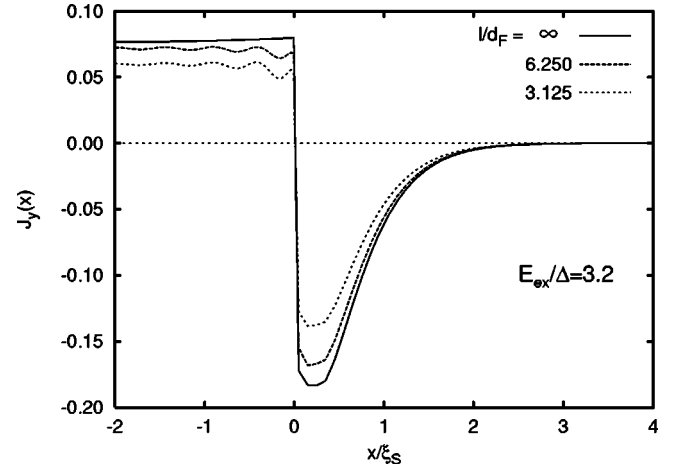


FIG. 3. The total spontaneous current as a function of x for a number of mean free path values l . Note that disorder introduces oscillations of the current.

flows over the whole FM is due to the extended nature of the ABS. There is also magnetic flux associated with such current distribution. Typically the spontaneous magnetic field produced by this current is of the order of $0.1 B_{c2}$, where B_{c2} is the upper critical field of the bulk SC.

Within the present self-consistent calculations we were also able to study the effect of elastic scattering in the FM. The disorder on the superconducting side is ineffective on account of Anderson's theorem.²⁷ By contrast it plays a significant role on the FM side. The current for a number of mean free paths is shown in Fig. 3.

As one can see disorder introduces oscillations of the current. The spontaneous current is proportional to the DOS at the Fermi energy¹⁴ and so the oscillations of the current are related to the oscillatory behavior of the DOS in the disordered sample.^{12,13} In the clean limit the DOS is constant in the whole FM. This is a well-known property of the Eilenberger equations in the clean limit.¹³ Moreover, disorder also suppresses the current, as expected, since it introduces decoherence of electron-hole pairs in the FM. Finally if the mean free path l is shorter than the FM thickness d_F the current is completely suppressed. If $l < d_F$ the Andreev reflected particles cannot reach the I/FM interface, which is a necessary condition for the formation of the current carrying ABS, because they are scattered on the impurities and the electron-hole coherence is lost. In this regard the FFLO variation of the pairing amplitude in the y direction is very sensitive to the elastic disorder. However, in the x direction the FFLO state persists until $l < \xi_F$,¹⁰ even if $l < d_F$.

To take into account the effect of specular reflections at the FM/SC interface we adapt the approach proposed by Zareyan *et al.*,¹² where a certain probability distribution was associated with each semiclassical trajectory (for details see Ref. 12). In Fig. 4 we show the current for two different transparencies $0 < \eta < 1$.

As we would expect transparency $\eta < 1$ suppressed the current because it suppresses Andreev reflection processes and at the same time introduces normal (specular) reflections at the FM/SC interface. The principle effect of increasing disorder is to reduce the mean free path and hence make the

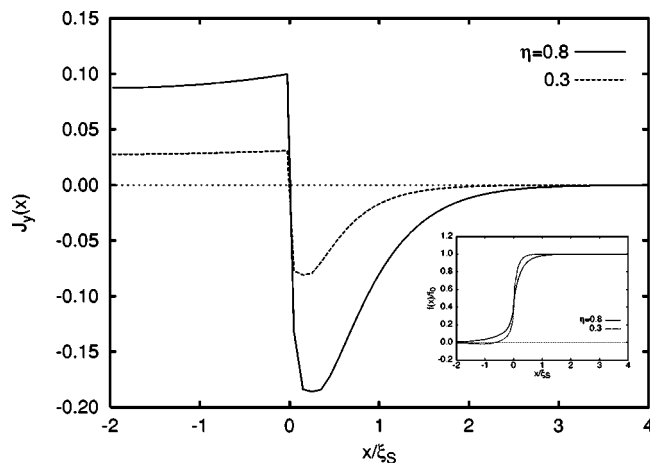


FIG. 4. Effect of finite transparency η on the spontaneous current $E_{ex}/\Delta=0.2$. $\eta=1$ means a reflectionless interface, while $\eta=0$ means a perfectly reflective one. Inset: The corresponding pairing amplitude normalized to its bulk value. Note that η changes the period of oscillations of $f(x)$.

formation of Andreev bound states more difficult. But even for $\eta=0.3$, in the present case, we still get the current. It turns out that η has similar influence on the properties of the system as E_{ex} does. As we can read from the inset of Fig. 4, it changes the period of oscillations of the pairing amplitude. Moreover, if we changed η only, for certain of its values we get solutions with a current flowing, while for others there is no current. For the above set of parameters we have found that current flows in the regions where $0.7 < \eta < 1$ and $0.2 < \eta < 0.35$. So we can also switch between the 1D and 2D FFLO state by changing the transparency.

Finally we note that so far we took no account of the direct magnetic field B_{FM} due to the ferromagnet. We esti-

mated B_{FM} by numerically summing the dipole fields of the ferromagnetic layers. For thickness, d_F , and magnetization, M (which in the Stoner model corresponds to $E_{ex}=IM$, where I is a phenomenological parameter), we estimate B_{FM} to be considerably less than B_{sp} due to the spontaneous current. Thus the magnetic field induced by the spontaneous current in the ground state should be directly observable.²⁸ Furthermore it should be stressed that in all our calculations the magnetization was constrained to point in the z direction. Thus the direction of the spontaneous current was determined by the condition that B_{sp} is parallel to M . In a more general theory where Andreev orbits also occur in the x - z plane and spin orbit coupling is taken into account these issues would need to be reexamined. Also the above calculations were two-dimensional, but preliminary studies of a 3D system indicate that there are no qualitative changes when orbits in the x - z plane are included.

In summary we have demonstrated that under certain, quite general conditions the ground state of a I/FM/SC trilayer features a spontaneous current flowing in opposite directions in the FM and SC layers. We argued that this state can be viewed as a 2D FFLO proximity state and hence the observation of the above current would be a decisive proof that the surprising behavior of such heterostructures is governed by the FFLO phenomenon. We also showed that this state persists only in the clean limit where the mean free path is longer than FM thickness and investigated the effect of low transparency of the interface on the observability of the ground-state current.

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¹C. L. Chien and D. H. Reich, J. Magn. Magn. Mater. **200**, 83 (1999); Y. A. Izyumov, Y. N. Proshin, and M. G. Khusainov, Phys. Usp. **45**, 109 (2002).

²C. J. Lambert and R. Raimondi, J. Phys.: Condens. Matter **10**, 901 (1998); C. W. J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997); B. Pannetier and H. Courtois, J. Low Temp. Phys. **118**, 599 (2000).

³A. F. Andreev, Sov. Phys. JETP **19**, 1228 (1964).

⁴N. F. Berk and J. R. Schrieffer, Phys. Rev. Lett. **17**, 433 (1966); C. Pfeleiderer, M. Uhlarz, S. M. Hyden, R. Vollmer, H. v. Lohneysen, N. R. Bernhoeft, and G. G. Lonzarich, Nature (London) **412**, 58 (2001); D. Aoki, A. Huxley, E. Ressouche, D. Braithwaite, J. Floquet, J.-P. Brison, E. Lhotel, and C. Paulsen, *ibid.* **413**, 613 (2001).

⁵G. E. W. Bauer, Y. V. Nazarov, D. Huertas-Hernando, A. Brataas, K. Xia, and P. J. Kelly, Mater. Sci. Eng., B **84**, 31 (2001); S. Oh, D. Youm, and M. R. Beasley, Appl. Phys. Lett. **71**, 2376 (1997); L. R. Tagirov, Phys. Rev. Lett. **83**, 2058 (1999); Y. N. Proshin, Y. A. Izyumov, and M. G. Khusainov, Supercond. Sci. Technol.

15, 285 (2002).

⁶G. Blatter, V. B. Geshkenbein, and L. B. Ioffe, Phys. Rev. B **63**, 174511 (2001).

⁷H. K. Wong, B. Y. Jin, H. Q. Yang, J. B. Ketterson, and J. E. Hilliard, J. Low Temp. Phys. **63**, 307 (1986); J. S. Jiang, D. Davidović, D. H. Reich, and C. L. Chien, Phys. Rev. Lett. **74**, 314 (1995); V. Marcaldo, C. Attanasio, C. Coccorese, L. Maritato, S. L. Prischepa, and M. Salvato, Phys. Rev. B **53**, 14 040 (1996); Th. Mühge, N. N. Garifyanov, Yu. V. Goryunov, G. G. Khaliullin, L. R. Tagirov, K. Westerholt, I. A. Garifullin, and H. Zabel, Phys. Rev. Lett. **77**, 1857 (1996).

⁸A. I. Buzdin, L. N. Bulaevskii, and S. V. Panyukov, JETP Lett. **35**, 178 (1982); A. I. Buzdin and M. V. Kuprianov, *ibid.* **52**, 487 (1990).

⁹Z. Radović, M. Ledvij, L. Dobrosavljević-Grujić, and J. C. Clem, Phys. Rev. B **44**, 759 (1991).

¹⁰E. A. Demler, G. B. Arnold, and M. R. Beasley, Phys. Rev. B **55**, 15 174 (1997).

¹¹T. Kontos, M. Aprili, J. Lesueur, and X. Grison, Phys. Rev. Lett. **86**, 304 (2001).

- ¹²M. Zareyan, W. Belzig, and Y. V. Nazarov, *Phys. Rev. Lett.* **86**, 308 (2001).
- ¹³A. I. Buzdin, *Phys. Rev. B* **62**, 11 377 (2000); I. Baladié and A. I. Buzdin, *ibid.* **64**, 224514 (2001); K. Halterman and O. T. Valls, *ibid.* **65**, 014509 (2002); F. S. Bergeret, A. F. Volkov, and K. B. Efetov, *ibid.* **65**, 134505 (2002).
- ¹⁴M. Krawiec, B. L. Györfy, and J. F. Annett, *Phys. Rev. B* **66**, 172505 (2002); *Eur. Phys. J. B* **32**, 163 (2003).
- ¹⁵S. F. Lee, J. J. Liang, T. M. Chuang, S. Y. Huang, J. H. Kuo, and Y. D. Yao, cond-mat/0301313.
- ¹⁶A. F. Volkov, F. S. Bergeret, and K. B. Efetov, *Phys. Rev. Lett.* **90**, 117006 (2003); M. Eschrig, J. Kopu, J. C. Cuevas, and G. Schön, *ibid.* **90**, 137003 (2003).
- ¹⁷P. Fulde and A. Ferrell, *Phys. Rev.* **135**, A550 (1964); A. Larkin and Y. Ovchinnikov, *Sov. Phys. JETP* **20**, 762 (1965).
- ¹⁸M. A. Clogston, *Phys. Rev. Lett.* **9**, 266 (1962).
- ¹⁹S. Takada, *Prog. Theor. Phys.* **43**, 27 (1970).
- ²⁰Y. A. Izyumov, Y. N. Proshin, and M. G. Khusainov, *JETP Lett.* **71**, 138 (2000); M. G. Khusainov, Y. A. Izyumov, and Y. N. Proshin, *Physica B* **284-288**, 503 (2000).
- ²¹P. G. de Gennes and D. Saint-James, *Phys. Lett.* **4**, 151 (1963).
- ²²K. P. Duncan and B. L. Györfy, *Ann. Phys. (N.Y.)* **298**, 273 (2002).
- ²³M. Sigrist, *Prog. Theor. Phys.* **99**, 899 (1998); M. Fogelström and S.-K. Yip, *Phys. Rev. B* **57**, R14 060 (1998).
- ²⁴S. Higashitani, *J. Phys. Soc. Jpn.* **66**, 2556 (1997).
- ²⁵G. Eilenberger, *Z. Phys.* **214**, 195 (1968).
- ²⁶N. Schopohl and K. Maki, *Phys. Rev. B* **52**, 490 (1995); N. Schopohl, cond-mat/9804064 (unpublished).
- ²⁷R. Moradian, J. F. Annett, B. L. Györfy, and G. Litak, *Phys. Rev. B* **63**, 024501 (2001).
- ²⁸A. K. Geim, S. V. Dubonos, J. G. S. Lok, I. V. Grigorieva, J. C. Maan, L. T. Hansen, and P. E. Lindelof, *Appl. Phys. Lett.* **71**, 2379 (1997).